Example:

A student measures a quantity x many times and calculates the mean as $\bar{x} = 10$ and the standard deviation as $\sigma_x = 1$. What fraction of his readings would you expect to find between 11 and 12?

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}$$

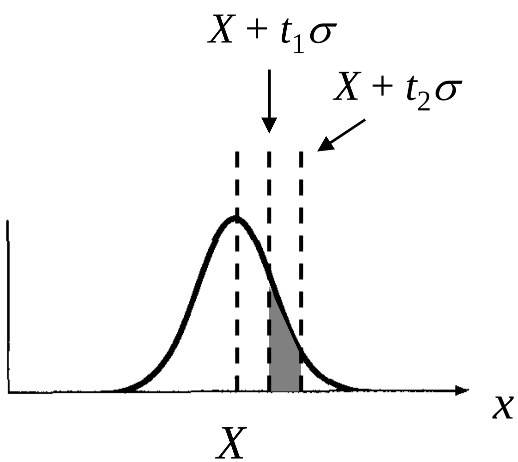
$$t = \frac{x - X}{\sigma}$$

Table B

$$\text{Prob}(X \le x \le X + t\sigma) = \int_X^{X+t\sigma} G_{X,\sigma}(x)dx = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-z^2/2} dz$$

The probability of a measurement to be between $X + t_1\sigma$ and $X + t_2\sigma$

$$\text{Prob}(X + t_1\sigma \le x \le X + t_2\sigma) = \frac{1}{\sqrt{2\pi}} \int_0^{t_2} e^{-z^2/2} dz - \frac{1}{\sqrt{2\pi}} \int_0^{t_1} e^{-z^2/2} dz$$



$$t_1 = \frac{x_1 - X}{\sigma} = \frac{11 - 10}{1} = 1$$

$$t_2 = \frac{x_2 - X}{\sigma} = \frac{12 - 10}{1} = 2$$

$$\text{Prob}(X + \sigma \le x \le X + 2\sigma) = \frac{1}{\sqrt{2\pi}} \int_0^{2} e^{-z^2/2} dz - \frac{1}{\sqrt{2\pi}} \int_0^{1} e^{-z^2/2} dz = 48\% - 34\% = \underline{14\%}$$

Table B

Acceptability of a Measured Answer

(value of x) = $x_{best} \pm \sigma$

x_{exp} - expected value of x, e.g. based on some theory

x_{best} differs from x_{exp} by t standard deviations → $$t = \frac{|x_{best} - x_{\exp}|}{\sigma}$$

Table A.

$$\text{Prob(outside } t\sigma) = 1 - \text{Prob(within } t\sigma)$$

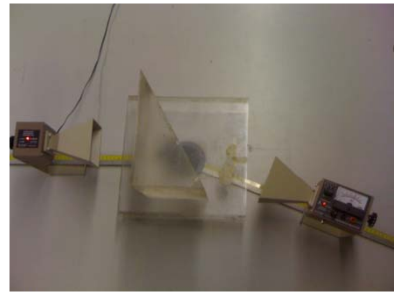
< 5 % - significant discrepancy, $t > 1.96$

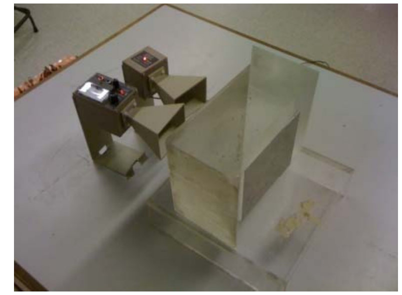
< 1 % - highly significant discrepancy, $t > 2.58$

boundary for unreasonable improbability

the result is beyond the boundary for unreasonable improbability → the result is unacceptable

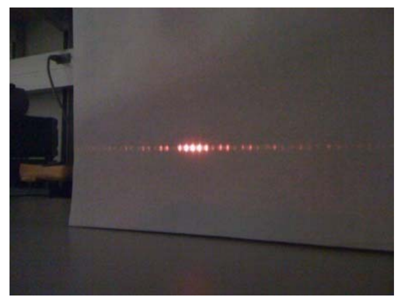
Experiment 4: Refraction and Interference with Microwaves

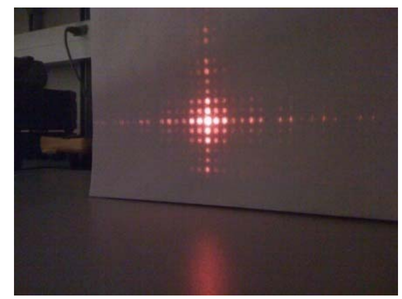




Experiment 5: Measurements Magnetic Fields

Experiment 6: Diffraction and Interference with Coherent Light





Experiment 7: Lenses and the Human Eye

Experiment 5:   Measurements Magnetic Fields

Goal: use fundamental electromagnetic equations and principles to measure the magnetic fields

1. Flip Coil Method

2. Current Balance Technique

1. Flip Coil Method

Basic Equations

Faraday's law $$V = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = -\frac{d\phi}{dt}$$

V is the integral around a closed contour = the voltage around a circuit

ϕ is the magnetic flux passing through the surface contained by the circuit

↑

the integral of the magnetic field over the area of the circuit A: $\phi = \int \mathbf{B}\cdot d\mathbf{A}$

the Lorentz force on a point charge: $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$

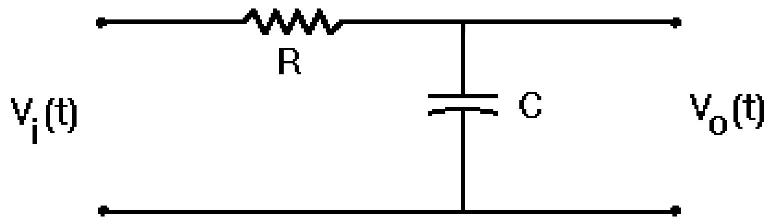
the force produced by a small current element $I\,d\mathbf{l}$: $d\mathbf{F} = I\,d\mathbf{l}\times\mathbf{B}$

the units: V (Volts), ϕ (Tesla-m^2=Weber), $\mathbf{F}$ (Newtons), I (Amperes), $\mathbf{B}$ (Tesla), length (m)

An RC integrator

how do we find the magnitude of ***B*** (or ϕ) from its derivative?

integrate the equation $V = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = -\frac{d\phi}{dt}$



$$V = \frac{Q}{C} \longrightarrow V_o(t_1) = \frac{1}{C}\int_0^{t_1} I\,dt$$

$$I = \frac{V_i - V_o}{R} \longrightarrow$$

$$V_o(t_1) = \frac{1}{RC}\int_0^{t_1}\left[V_i(t) - V_o(t)\right]dt$$

$$= \frac{1}{RC}\int_0^{t_1} V_i(t)\,dt - \frac{1}{RC}\int_0^{t_1} V_o(t)\,dt$$

for large *RC*
$RC >> t_1$ $\longrightarrow$

$$V_o(t_1) = \frac{1}{RC}\int_0^{t_1} V_i(t)\,dt$$

circuit acts an **integrator**

connect an integrator to a loop of wire located in a time-varying magnetic field

increasing from $B = 0$ and obtain an output $$V_o(t) = \frac{1}{RC}\int_0^t \frac{d\phi}{dt}dt = \frac{\phi(t)}{RC}$$

$RC >> t$

$$V_i = -\frac{d\phi}{dt}$$

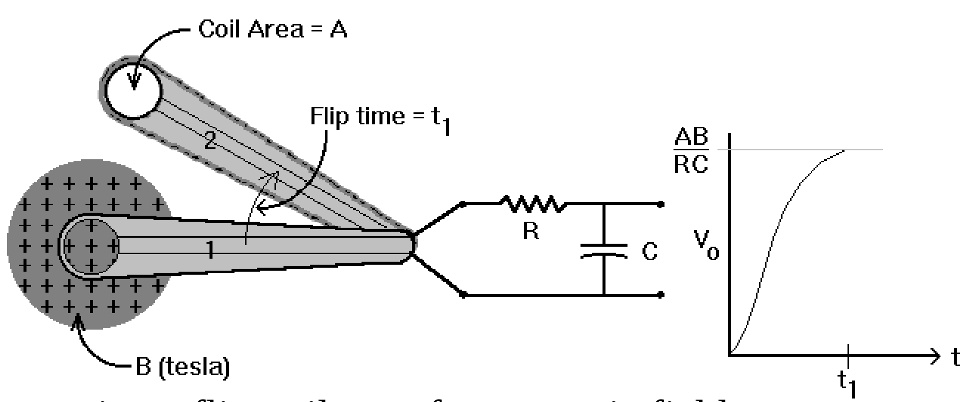
Faraday's law

$$V_o(t_1) = \frac{1}{RC}\int_0^{t_1} V_i(t)dt$$

RC integrator

for uniform B: $\phi = AB$ ⟶ $V_o(t) = \frac{A}{RC}B(t)$

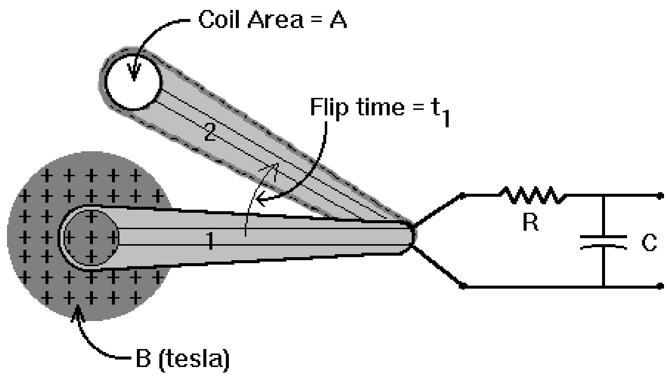
flip-coil technique: take our loop which initially rests in a place where $B = 0$ and thrust it quickly ($t << RC$) into the magnetic field region (or visa versa)

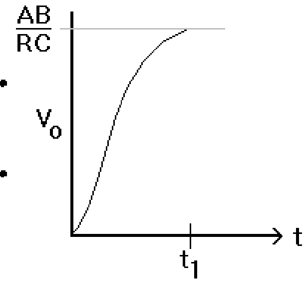


$$\Delta V_o = \frac{nA}{RC}\Delta B$$

for n coils

moving a flip-coil out of a magnetic field,
while integrating the induced currents with an RC circuit

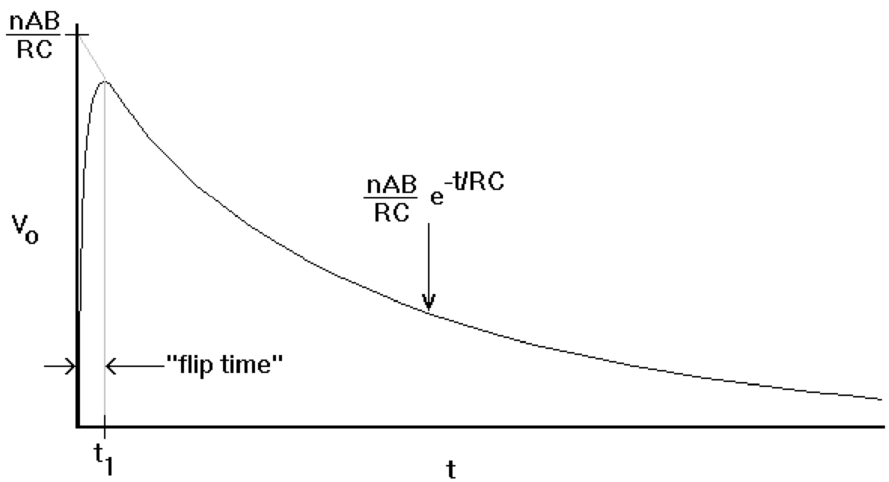




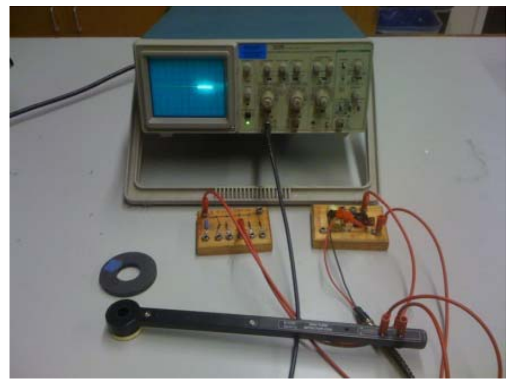
$$\Delta V_o = \frac{nA}{RC}\Delta B$$

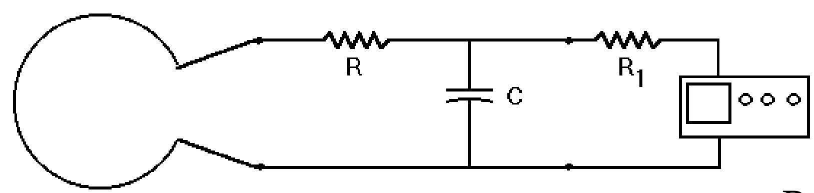
after the "flip" has occurred and the integrator output voltage has jumped to ΔV_o

the capacitor discharges through R and its voltage decays as $V(t) = V_o e^{-t/RC}$



the output voltage for a flip coil moved rapidly out of a magnetic field region and integrated by an RC circuit followed by discharge of the capacitor





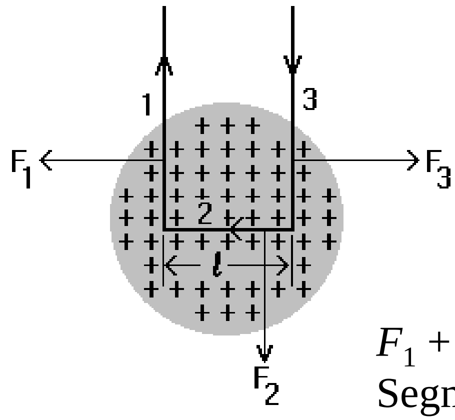
actual decay time constant is R_2C, where $R_2 = \dfrac{R_1 R}{R_1 + R}$

keep the flip time t_1 small ($t_1 << R_2C$)

2. Current Balance Technique

a wire carrying current I feels a force: $F_x = \int I_y B_z(y)dy$

$$dF_x = I_y B_z dl$$



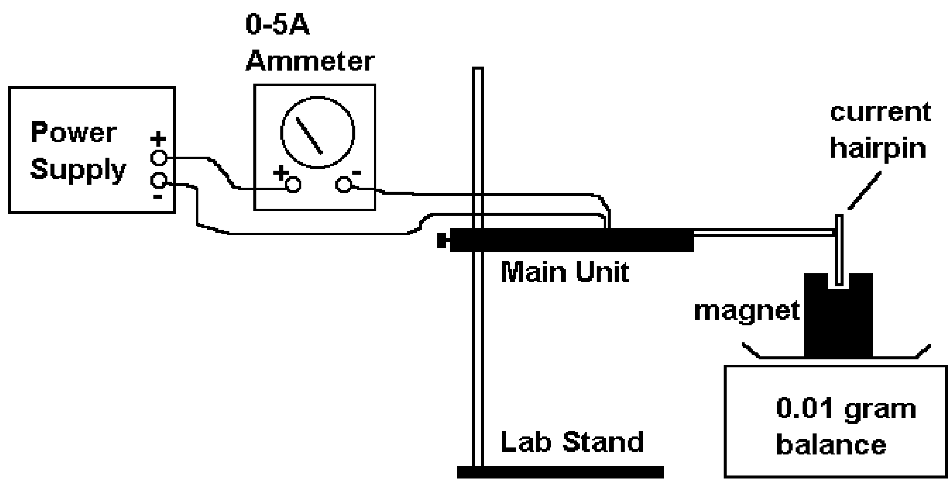
the total force on the hairpin $F = IBl$

$\downarrow$

$$B = \frac{F}{Il}$$

$F_1 + F_3 = 0$

Segment 2 is short enough to be everywhere in a uniform field.



Determine the magnetic field of the magnet by measuring the force on a current in the field. Choose several hairpins of different lengths. For each hairpin choose several different currents. Measure the mass difference from the scale and calculate the Lorentz force.