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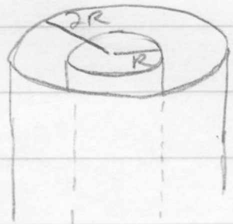
two long solenoids of length l, radii $2R, R$, n turns/length w/ coincident axes, what is the mutual inductance?

$$M = \frac{\phi_2}{I_1}$$



$$\phi_{2R} = \overbrace{nl}^{\text{number of turns}} \underbrace{B_R \pi R^2}_{\text{flux through one turn}}$$, flux through $2R$ (note $B_R = 0$ for $r > R$)

$$B_R = \mu_0 n I_R \Rightarrow I_R = \frac{B_R}{\mu_0 n}$$, current in R solenoid

$$M = \frac{\phi_{2R}}{I_R} = \frac{nl \cancel{B_R} \pi R^2}{\cancel{B_R}/\mu_0 n} = \boxed{\mu_0 n^2 l \pi R^2}$$

You should convince yourself that M is the same no matter which solenoid you call 1 or 2.

$$\phi_R = nl\, B_{2R} \pi R^2$$

$$B_{2R} = \mu_0 n I_{2R} \Rightarrow I_{2R} = \frac{B_{2R}}{\mu_0 n}$$

$$M = \frac{nl \cancel{B_{2R}} \pi R^2}{\cancel{B_{2R}}/\mu_0 n} = \boxed{\mu_0 n^2 l \pi R^2}$$

This should always be true.

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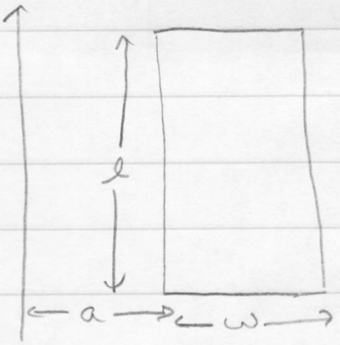


$$M = \frac{\Phi_2}{I_1}$$

$$\Phi_2 = \int B\,dA = \frac{\mu_0 I_1 l \ln\left(\frac{a+w}{a}\right)}{2\pi}$$

(check the last HW)

$$\boxed{M = \frac{\mu_0 l \ln\left(\frac{a+w}{a}\right)}{2\pi}}$$

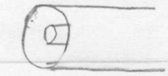
#11

solenoid, $l = 0.5\,m$, $R = 0.02\,m$, $n = 1000/0.5\,m$

What is the self inductance?

$$L = \frac{\Phi}{I} = \frac{\underbrace{(n\,l)}_{\#\text{ turns}}\underbrace{(\mu_0 n \cancel{I})}_{B}\underbrace{(\pi R^2)}_{A}}{\cancel{I}} = \mu_0 n^2 l \pi R^2 = \boxed{3.16\times10^{-3}\,H}$$

#22 coaxial cable, inner radius a, outer radius b

What is the inductance per unit length? 

$$\frac{L}{l} = \frac{\Phi}{l I}$$

$$\Phi = \int B\,dA = \int_{R=a}^{R=b} \frac{\mu_0 I}{2\pi R}\, l\,dR = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\boxed{\frac{L}{l} = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)}$$

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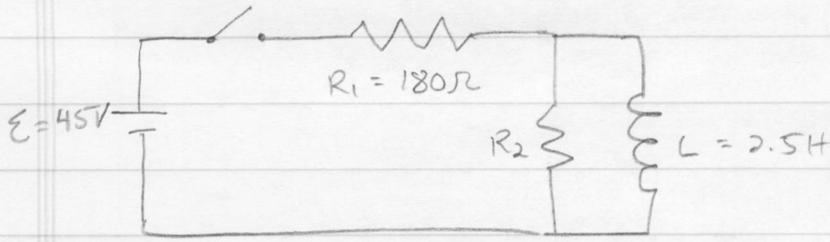

what is R_2 if $\mathcal{E}_{L,max} = 100V$?

R_2 and L are in parallel $\Rightarrow$ the voltage drop across them is the same. After the switch has been closed for a long time, the L acts as a wire and no voltage drops across R_2 (or L). The current is given by $I = \frac{\mathcal{E}}{R_1}$. When the switch is opened, the circuits is effectively reduced to L and R_2 in series. The current cannot change instantaneously so all of I flows through $R_2 \Rightarrow$ the voltage drop is

$$IR_2 = \frac{\mathcal{E} R_2}{R_1} = \mathcal{E}_{L,max} \Rightarrow R_2 = \frac{R_1 \mathcal{E}_{L,max}}{\mathcal{E}} = \boxed{400\Omega}$$

36 $\mathcal{E}_0 = 12V$, $R_1 = 4.0\Omega$, $R_2 = 8\Omega$, $R_3 = 2\Omega$, $L = 2.0H$

What is I_2 (a) when the switch is closed? (b) much later? (c) when the switch is opened again?

The thing to keep in mind here is that I_3 cannot change instantaneously.

(a) $I_1 = I_2 = \mathcal{E}_0/(R_1+R_2) = 12V/(12\Omega) = \boxed{1A}$

(b) "a long time after" $\Rightarrow L$ is a wire

$R_{eff} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 5.6\Omega$

$I_{tot} = \mathcal{E}_0/R_{eff} = 45V/5.6\Omega = 8A$

$I_2R_2 = I_3R_3 = (I_{tot} - I_2)R_3 = I_{tot}R_3 - I_2R_3$

$\Rightarrow I_2 = \frac{I_{tot} R_3}{(R_2+R_3)} = \boxed{1.6A}$

(c) $I_2 = I_3 = I_{tot} - I_2 = \boxed{6.4A}$

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$L = 220\times10^{-3}\,H$, $I = 350\times10^{-3}A$

How much energy must be supplied to raise I to 800mA?

$$U = \tfrac{1}{2}LI^2 \Rightarrow \Delta U = \tfrac{1}{2}L(\Delta I)^2 = \tfrac{1}{2}(220\times10^{-3}H)(800mA - 350mA)^2$$

$$= \boxed{2.23\times10^{-2}\,J}$$

#46

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\mu_0 n^2 A l I^2 = \tfrac{1}{2}\mu_0\left(\frac{N}{l}\right)^2\pi\left(\frac{d}{2}\right)^2 l I^2$$

$$= \tfrac{1}{2}\mu_0\left(\frac{500}{.23m}\right)^2\pi\left(\frac{.015m}{2}\right)^2 .23m\,(65\times10^{-3}A)$$

$$= \boxed{7.85\times10^{-6}\,J}$$

#55

A toroidal coil, inner radius R, square $l \times l$ crossection

N turns of wire, current I. What is the magnetic energy?

$$U = \int u_B\,dV = \int \frac{B^2}{2\mu_0}\,dV = \frac{1}{2\mu_0}\int_{r=R}^{r=R+l}\left(\frac{\mu_0 NI}{2\pi r}\right)^2 2\pi r l\,dr \leftarrow \text{Eq 30-12}$$

$$= \frac{\mu_0^2 N^2 I^2 l}{2\mu_0 2\pi}\int_{r=R}^{r=R+l}\frac{1}{r}\,dr$$

$$\boxed{= \frac{\mu_0 N^2 I^2 l}{4\pi}\ln\left(\frac{R+l}{R}\right)}$$

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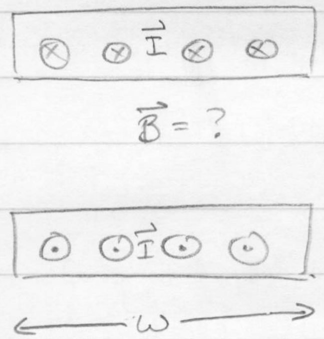

$$B = \frac{\mu_0 J_s}{2} + \frac{\mu_0 J_s}{2} = \mu_0 J_s = \frac{\mu_0 I}{w}$$

(Note that they add by right hand rule)

$$\frac{U}{l} = \frac{u_B V}{l} = \frac{B^2}{2\mu_0} dw = \frac{\left(\frac{\mu_0 I}{w}\right)^2 dw}{2\mu_0}$$

$$= \frac{1}{2}\mu_0 I^2 \frac{d}{w}$$

$$U = \frac{1}{2} L I^2 = \frac{1}{2}\left(\mu_0 \frac{dl}{w}\right) I^2 \Rightarrow \frac{L}{l} = \boxed{\frac{\mu_0 d}{w}}$$

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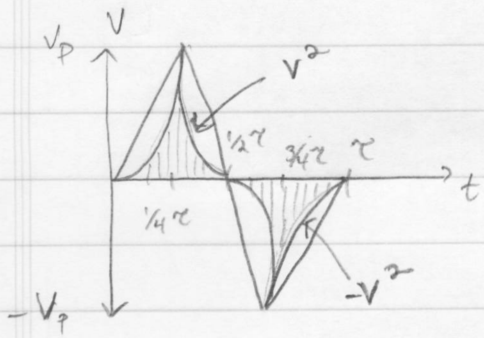
$V = V_p \sin \omega t$

$$V_{rms}^2 \equiv \frac{1}{\tau}\int v^2\,dt = \frac{1}{\tau}\int_{t'=0}^{t'=\tau} V_p^2 \sin^2(\omega t')\,dt'$$

$$= \frac{V_p^2}{\tau}\left(\frac{\tau}{2} - \cancelto{0}{\frac{\sin(2\omega\tau)}{4\omega}}\right) \qquad \omega\tau = 2\pi$$

$$= \frac{V_p^2}{2} \Rightarrow \boxed{V_{rms} = \frac{V_p}{\sqrt{2}}}$$

#9



$$\int_0^{\tau} v^2\,dt = 4\int_0^{\tau/4} v^2\,dt = 4\int_0^{\tau/4}\left(\frac{4V_p}{\tau}\right)^2 t^2\,dt$$

$$= 4\left(\frac{4V_p}{\tau}\right)^2 \frac{1}{3}\left(\frac{\tau}{4}\right)^3$$

$$= \frac{1}{3} V_p^2 \tau$$

$$V_{rms}^2 = \frac{1}{3}V_p^2 \Rightarrow \boxed{V_{rms} = \frac{V_p}{\sqrt{3}}}$$

#11 $I_1 \sin(\omega t) + I_2 \cos(\omega t) = I_p \sin(\omega t + \varphi)$

must be true at all times $\Rightarrow$ consider $t=0$ and $t = \pi/2$

$\omega t = 0 \Rightarrow I_2 = I_p \sin(\varphi)$

$\omega t = \pi/2 \Rightarrow I_1 = I_p \sin(\frac{\pi}{2} + \varphi) = I_p \cos(\varphi)$

$$\sin^2\varphi + \cos^2\varphi = \left(\frac{I_2}{I_p}\right)^2 + \left(\frac{I_1}{I_p}\right)^2 = 1 \Rightarrow \boxed{I_p = \sqrt{I_1^2 + I^2}}$$

$$\frac{\sin\varphi}{\cos\varphi} = \tan\varphi = \frac{(I_2/I_p)}{(I_1/I_p)} = \frac{I_2}{I_1} \Rightarrow \boxed{\varphi = \tan^{-1}(I_2/I_1)}$$

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$I_p = V_p \omega C = \frac{V_p}{R} \Rightarrow C = \frac{1}{R\omega} = \boxed{1.47 \times 10^{-6} F}$

#20

$X_C = 1 \times 10^3 \Omega$, $C = 2 \times 10^{-6} F$

$X_C = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{X_C C} = 500 \text{ rad/s} = \boxed{79.6 \text{ Hz}}$

$X_L = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{1 \times 10^3 \Omega}{500 \text{rad/s}} = \boxed{2 H}$

$\frac{X_C}{X_L} \approx \frac{1}{\omega^2} \Rightarrow \boxed{X_L' = 4 X_C'}$

#24

$\omega = 2\pi \times 10 \times 10^3 Hz$

$X_L = 10 X_C$

$\omega L = \frac{10}{\omega C} \Rightarrow CL = \frac{10}{\omega^2}$

$X_L' = X_C'$

$\omega' L = \frac{1}{\omega' C} \Rightarrow \omega' = \sqrt{\frac{1}{CL}} = \frac{\omega}{\sqrt{10}} \Rightarrow F' = \frac{\omega}{2\pi\sqrt{10}} = \boxed{3.16 \times 10^3 Hz}$

#37

Eqs in section 33-3 (note $\frac{\omega \tau}{8} = 45°$)

(a) $q(t) = q_p \cos(\omega t) \Rightarrow q(\tau/8) = q_p \cos(\omega \tau/8)$

$q/q_p = \boxed{1/\sqrt{2}}$

(b) $U_E / U_{E,p} = \cos^2(\omega \tau/8) = \boxed{1/2}$

(c) $I/I_p = -\sin(\omega \tau/8) = \boxed{-1/\sqrt{2}}$

(d) $U_B / U_{B,p} = \sin^2(\omega \tau/8) = \boxed{1/2}$