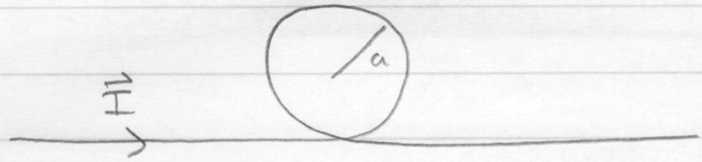


What is $\vec{B}$ at the center of the loop?

The key here is to realize that $\vec{B}$ obeys the principle of superposition. Also, the above situation is analagous to one long straight wire and one loop of current.

For a long straight wire, we have (Eq. 30-5):

$$B = \frac{\mu_0 I}{2\pi y}, \quad y = a \text{ in our case.}$$

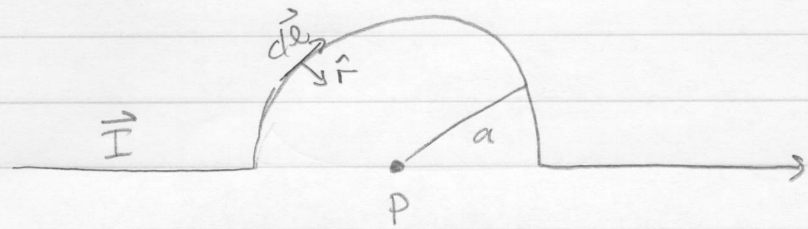
The direction, given by the right hand rule, is out of the page.

For a loop of current, we have (Eq. 30-3):

$$B = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}, \quad x = 0 \text{ in our case.}$$

The direction is also out of the page (right hand rule).

Their sum is then $\boxed{B_{total} = \frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{2a}, \text{ out of the page.}}$



What is $\vec{B}$ @ P?

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, dl \times \hat{r}}{r^2} \quad \text{(Biot-Savart Law)}$$

First notice that only the semicircular section contributes to $\vec{B}$ at P because for the straight sections, dl is either parallel or antiparallel to $\hat{r}$.

However, on the semicircle, $d\vec{l}$ and $\hat{r}$ are perpendicular, as shown on the drawing. Furthermore, $d\vec{l} \times \hat{r}$ is a constant vector of length dl pointing into the page, by the right hand rule. We then have:

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, dl}{a^2} = \frac{\mu_0 I}{4\pi a^2} \int dl = \frac{\mu_0 I}{4\pi a^2} \underbrace{(\pi a)}_{\text{half the circumference of a circle w/ } r = a} = \boxed{\frac{\mu_0 I}{4a}}$$

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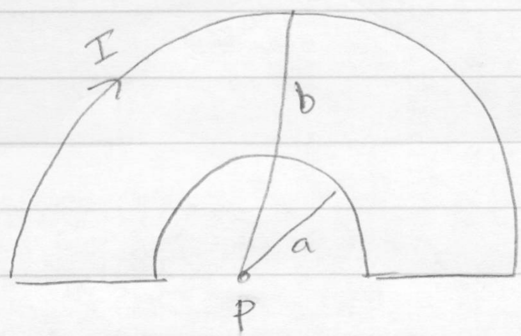

What is $\vec{B}$ at P?

Notice that there will be no contributions from the straight portions because $d\vec{l}$ is parallel or anti-parallel to $\hat{r}$. The semicircle of radius a will give a $\vec{B}$ out of the page and the semicircle of radius b will give a $\vec{B}$ into the page, so our final result will be:

$$B_{total} = B_a - B_b \text{, out of the page.}$$

From the last problem, we have $B_a = \frac{\mu_0 I}{4a}$ and $B_b = \frac{\mu_0 I}{4b}$

So, all told we have

$$\boxed{B_{total} = \frac{\mu_0 I}{4a} - \frac{\mu_0 I}{4b} \text{, out of the page.}}$$

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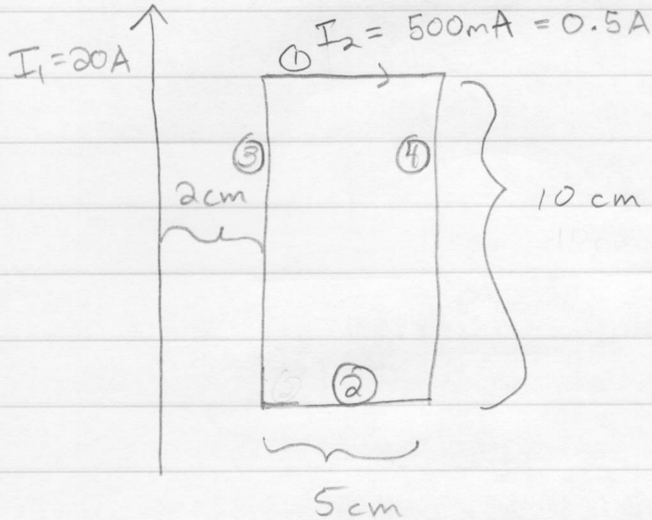

What is $\vec{F}_B$ on the loop?

In the book (Eq. 30-6), they have worked out for you the interaction of two current carrying wires that are parallel. This will take care of 3 and 4, but what about 1 and 2. The right hand rule tells us that the force on 1 will be up and the force on 2 will be down, but otherwise they are identical so they will cancel exactly.

For 3, $F = \frac{\mu_0 I_1 I_2 l}{2\pi d} = \frac{\mu_0 I_1 I_2 (0.1m)}{2\pi (0.02m)}$ (left)

and 4, $F = \frac{\mu_0 I_1 I_2 (0.1m)}{2\pi (0.07m)}$ (right)

$F_{net} = F_{③} - F_{④}$, left

$= 1\times10^{-5} N - 2.86\times10^{-6} N$, left

$= \boxed{7.14\times10^{-6} N, \text{ left}}$

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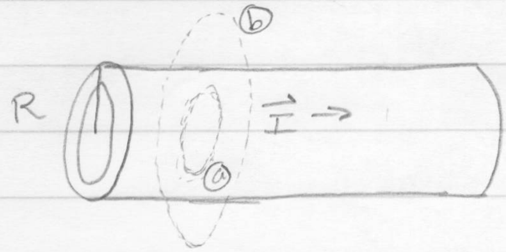

What is $\vec{B}$ inside and outside the pipe?

Ampere's law (Eq. 30-7): $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encircled}}$

For the amperian loop (a), $\vec{B} \cdot d\vec{l} = B\,dl$ is constant and $B \oint dl = \mu_0 I_{enc} \Rightarrow B 2\pi r = \mu_0 (0)$ for $r < R$

$$\Rightarrow \boxed{B = 0 \text{ inside}} \quad (r<R)$$

For (b), $\vec{B} \cdot d\vec{l} = B\,dl$ is also constant, but we have

$$B 2\pi r = \mu_0 I \text{ for } r > R$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \text{ outside}} \quad (r > R)$$

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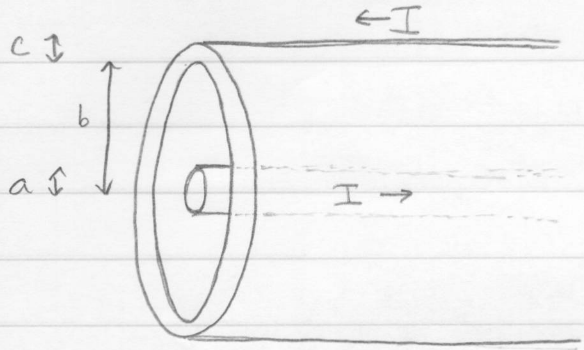

What is $\vec{B}$ for
(a) $r < a$?
(b) $a < r < b$?
(c) $r > b+c$?

Use Ampere's law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encircled}$

(a) $B 2\pi r = \mu_0 I \left(\dfrac{\text{Area encircled}}{\text{Total Area}} \right) = \mu_0 I \dfrac{\pi r^2}{\pi a^2}$ $(r < a)$

$\Rightarrow \boxed{B = \dfrac{\mu_0 I r}{2\pi a^2}}$ $(r < a)$

(b) $B 2\pi r = \mu_0 I$ $(a < r < b)$

$\Rightarrow \boxed{B = \dfrac{\mu_0 I}{2\pi r}}$ $(a < r < b)$

(c) $B 2\pi r = \mu_0 (I - I) = 0$ $(r > b+c)$

$\Rightarrow \boxed{B = 0}$ $(r > b+c)$

The currents are equal in magnitude but opposite in direction, so they add to zero.

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Copper wire, $l = 10\,m$, $d = 0.5 \times 10^{-3}\,m$, $I = 15\,A$

What is B inside (a) a 2cm diameter solenoid?

(b) a single circular loop (@ the center)?

(a) $B = \mu_0 n I$ (Eq 30-11) for a solenoid, where $n = \frac{N}{L}$ is the number of turns per unit length.

$$N = \frac{l}{2\pi r} = \frac{10\,m}{2\pi(1\times10^{-2}\,m)} \doteq 159$$

$$L = dN = (0.5\times10^{-3}\,m)(159) = 7.95\times10^{-2}\,m$$

$$B = \mu_0 \frac{(159)(15A)}{(7.95\times10^{-2}\,m)} = \boxed{3.77\times10^{-2}\,T}$$

(b) $B = \frac{\mu_0 I}{2r}$, $2\pi r = l \Rightarrow r = \frac{l}{2\pi}$

$$B = \frac{\mu_0 I}{2\left(\frac{l}{2\pi}\right)} = \frac{\mu_0 \pi I}{l} = \frac{\mu_0 \pi (15A)}{10\,m} = \boxed{5.92\times10^{-6}\,T}$$

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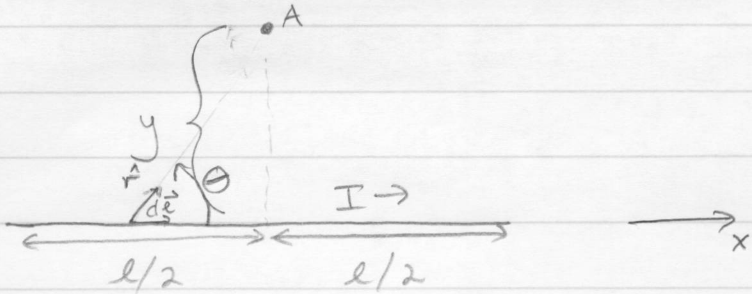

What is $\vec{B}$ at A?

Right hand rule $\Rightarrow$ $\boxed{\text{out of the page}}$

Biot-Savart $\vec{B} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2}$

($d\vec{l} \times \hat{r}$ is always out of the page)

$$= \frac{\mu_0 I}{4\pi} \int_{-\ell/2}^{\ell/2} \frac{dx}{(\sqrt{x^2+y^2})^2} \left(\frac{y}{\sqrt{x^2+y^2}} \right)$$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$$= \frac{\mu_0 I}{4\pi} \int_{-\ell/2}^{\ell/2} \frac{y\, dx}{(\sqrt{x^2+y^2})^{3/2}}$$

$$= \frac{\mu_0 I y}{4\pi} \left. \frac{x}{y^2\sqrt{x^2+y^2}} \right|_{-\ell/2}^{\ell/2} = \boxed{\frac{\mu_0 I}{4\pi y} \left(\frac{\ell}{\sqrt{(\frac{\ell}{2})^2+y^2}} \right)}$$