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What is the current density in the filament? (Example 27-6)

$$J = \frac{I}{A} = \frac{0.833\,A}{\pi(0.025\,mm)^2} = \boxed{4.24\times10^8\ A/m^2}$$

"Current density" is the current per cross-sectional area.

What about a wire w/ diameter 0.21cm?

You should already know that, because the wire is much thicker, the density will be much lower for the same current. That is good, because you don't want your wire glowing white hot!

$$J' = \frac{I}{A'} = \frac{0.833A}{\pi(0.105cm)^2} = \boxed{2.41\times10^5\ A/m^2}$$

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Another expression for the current density is given by $\vec{J} = nq\vec{v}_d$, where n is the number of charged particles, q is the charge on those particles and $\vec{v}_d$ is the drift velocity.

In this problem, we will have two contributions: one from protons and one from electrons. They will add because their "q"s have opposite sign and their "$\vec{v}_d$"s have opposite direction.

$$J_{total} = n(-e)(-v_d) + n(e)(v_d)$$

$$= (5\times10^{18})(-1.6\times10^{-19}C)(40\,m/s) + (5\times10^{18})(1.6\times10^{-19}C)(6.5\,m/s)$$

$$= \boxed{37.2\ A/m^2}$$

What fraction of the current is carried by the electrons?

$$\frac{I_e}{I_p + I_e} = \frac{n_e q_e V_{d,e}}{(n_e q_e V_{d,e}) + (n_p q_p V_{d,p})} = \frac{V_{d,e}}{(V_{d,e} + V_{d,p})} = \frac{40\,m/s}{46.5\,m/s} = \boxed{0.86}$$

The electrons have the same number and magnitude charge as the protons, but drift more quickly.

The drift velocity is the effective velocity of a charge through a material. It depends on the nature of the material.

For the wire, from 27-3a:

$$J = nqv_d \Rightarrow v_d = \frac{J}{nq} = \frac{I/A}{nq} = \frac{100\times10^{-3}A/\pi(0.05\times10^{-3}m)^2}{1.1\times10^{29}m^{-3}\,(1.6\times10^{-19}C)} = \boxed{7.2\times10^{-4}\,m/s}$$

For the solution,

$$J = J_+ + J_- = n_+(q_+)v_{d+} + n_-(q_-)v_{d-}$$

$$= 2nqv_d \quad (n, q, v_d \text{ are assumed to be the same for the two different species})$$

$$J = \frac{I}{A} = 2nqv_d \Rightarrow v_d = \frac{I}{2nqA} = \frac{100\times10^{-3}A}{2(6.1\times10^{23}m^{-3})(2e)\pi(0.5cm)^2} = \boxed{3.2\times10^{-3}\,m/s}$$

For the vacuum,

$$J = nqv_d \Rightarrow v_d = \frac{J}{nq} = \frac{I/A}{nq} = \frac{100\times10^{-3}A/\pi(0.5\times10^{-3}m)^2}{2.2\times10^{16}m^{-3}\;1.6\times10^{-19}C} = \boxed{3.6\times10^{7}\,m/s}$$

Obviously the e^- are moving fastest through the vacuum. In fact, they are approaching the speed of light!

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Table 27-1 lists materials based on their ρ (resistivity)
So that is what we are going to be solving for.

We are given $l = 2.4\text{cm}$, $A = \pi(1\text{mm})^2$
and we know $R = \frac{\rho l}{A}$. Finally, Ohms law: $V = IR$.

$$R = \frac{\rho l}{A} = \frac{V}{I} \Rightarrow \rho = \frac{AV}{lI} = \frac{\pi(1\text{mm})^2(9\text{V})}{(2.4\text{cm})(2.6\text{mA})} = \boxed{0.45\ \Omega\text{m}}$$

Looks like we got some $\boxed{\text{GERMANIUM}}$ on our hands! Woohoo!

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Resistance per unit length: $\frac{R}{l} = 50 m\Omega/km$

We have $R = \frac{\rho l}{A} \Rightarrow \frac{R}{l} = \frac{\rho}{A} = \frac{\rho}{\pi r^2} = \frac{\rho}{\pi (\frac{d}{2})^2} = \frac{4\rho}{\pi d^2}$

We want to solve for d (diameter):

$$d = \sqrt{\frac{4\rho}{\pi (R/l)}}$$

$$(\text{for copper}) = \sqrt{\frac{4(1.68\times10^{-8}\,\Omega m)}{\pi(50 m\Omega/km)}} = \boxed{0.021m}$$

$$(\text{for aluminum}) = \sqrt{\frac{4(2.65\times10^{-8}\,\Omega m)}{\pi(50 m\Omega/km)}} = \boxed{0.026m}$$

There are many ways to answer this, but let's compute the cost of 1m for each.

$$\text{cost for one meter} = \frac{\$}{kg}\ \frac{kg}{m^3}\ \frac{m^3}{m}$$

$$(\text{for copper}) = \frac{\$1.53}{kg}\ \frac{8.9g}{cm^3}\ \frac{\pi(0.0105m)^2\,1m}{m} = \boxed{4.58\ \$/m}$$

$$(\text{for aluminum}) = \frac{\$1.34}{kg}\ \frac{2.7g}{cm^3}\ \frac{\pi(0.013\,m)^2\,1m}{m} = \boxed{1.92\ \$/m}$$

Apparently $\boxed{\text{ALUMINUM}}$ is more economical.

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$$P = IV \Rightarrow I = \frac{P}{V} = \frac{1000\,MW}{120\,V} = 8.33\times10^{6}\,A$$

for the whole country, $\frac{8.33\times10^{6}\,A}{10^{8}} = \boxed{0.083\,A \text{ per TV}}$

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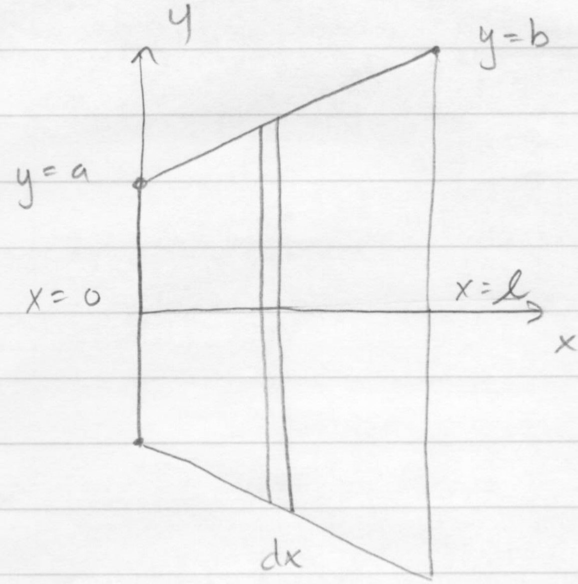
Here we are, adding up more little things using integration. Let's setup a coordinate system.



$$R = \int_{\text{cone}} dR = \int_{x=0}^{x=l} \frac{\rho\, dx}{A} \qquad A = \pi y^2, \; y = \left(\frac{b-a}{l}\right)x + a$$

$$A = \pi\left[\left(\frac{b-a}{l}\right)x + a\right]^2$$

$$R = \int_{x=0}^{x=l} \frac{\rho\, dx}{\pi\left[\left(\frac{b-a}{l}\right)x+a\right]^2}, \quad u = \left(\frac{b-a}{l}\right)x + a \quad \frac{du}{dx} = \left(\frac{b-a}{l}\right)$$

$$\Rightarrow dx = \left(\frac{b-a}{l}\right)^{-1} du$$

$$R = \int_{u=a}^{u=b} \frac{\rho}{\pi} \frac{\left(\frac{b-a}{l}\right)^{-1} du}{u^2} = \frac{\rho}{\pi\left(\frac{b-a}{l}\right)} \int_{u=a}^{u=b} \frac{du}{u^2} = \frac{\rho}{\pi\left(\frac{b-a}{l}\right)} \left[-\frac{1}{u}\right]_{u=a}^{u=b}$$

$$= \frac{\rho}{\pi\left(\frac{b-a}{l}\right)}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho l}{\pi}\left(\frac{\frac{1}{a} - \frac{1}{b}}{b-a}\right) = \boxed{\frac{\rho l}{\pi a b}}$$