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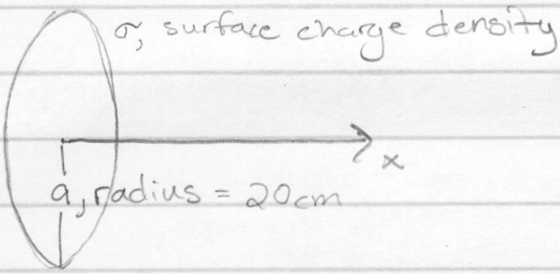
Uniformly charged disk, on axis $\Rightarrow E_{disc} = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)$



For what values of x does treating the disc as an infinite sheet provide an approximation of E that is good to within 10%?

for an infinite sheet (Eq. 24-9) $E_{sheet} = \frac{\sigma}{2\varepsilon_0} = \frac{4\pi k\sigma}{2} = 2\pi k\sigma$.

So, for $x \simeq 0$ (very near the disk) the expressions are essentially equivalent, as you would expect. However, going farther from the disk E_{disk} decreases. So the question is asking us to find where E_{sheet} is 10% "too large": $E_{sheet} = (1.1)E_{disc}$ or $\frac{E_{sheet}}{E_{disc}} = 1.1$

$$\frac{2\pi k\sigma}{2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)} = 1.1 \Rightarrow 1 - \frac{x}{\sqrt{x^2+a^2}} = \frac{1}{1.1}$$

$$\Rightarrow 1 - \frac{1}{1.1} = \frac{x}{\sqrt{x^2+a^2}} \Rightarrow \left(1 - \frac{1}{1.1}\right)\sqrt{x^2+a^2} = x$$

$$\Rightarrow \left(1 - \frac{1}{1.1}\right)^2(x^2+a^2) = x^2 \Rightarrow x^2\left(1 - \left(1 - \frac{1}{1.1}\right)^2\right) = a^2\left(1 - \frac{1}{1.1}\right)^2$$

$$x = a\sqrt{\frac{\left(1 - \frac{1}{1.1}\right)^2}{\left(1 - \left(1 - \frac{1}{1.1}\right)^2\right)}} = 0.0913a$$

, for $a = 20$cm, $x = 1.83$cm, which means the approximation is good to within 10% for

$$\boxed{0\text{cm} < x < 1.83\text{cm}}$$

For what range of x is a point charge approximation good to within 10%?

for a point charge, $E_{point} = \frac{kq_T}{x^2}$, but what is q_T?

We are given σ, a surface charge density, and a, the radius of a disc. So, $q_T = \sigma A = \sigma \pi a^2$.

Now we have $E_{point} = \frac{k\sigma\pi a^2}{x^2}$. For $x \approx 0$, E_{point} is very large, so we are looking for the point where E_{point} "comes down" to 110% of E_{disc} $\Rightarrow$

$$E_{point} = (1.1) E_{disc} \Rightarrow \frac{E_{point}}{E_{disc}} = 1.1$$

$$\Rightarrow \frac{k\sigma\pi a^2}{x^2} \frac{1}{2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)} = \frac{a^2}{2x^2\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)} = 1.1$$

This is a mess to try to solve by hand, so solve it using a computer or your calculator. You will not be asked to solve such a thing on a quiz, don't worry.

Mathematica gives me: $x = 2.724a$

For $a = 20$ cm, $x = 54.48$ cm, meaning the approximation is good to within 10% for

$$\boxed{x > 54.48\text{cm}}$$

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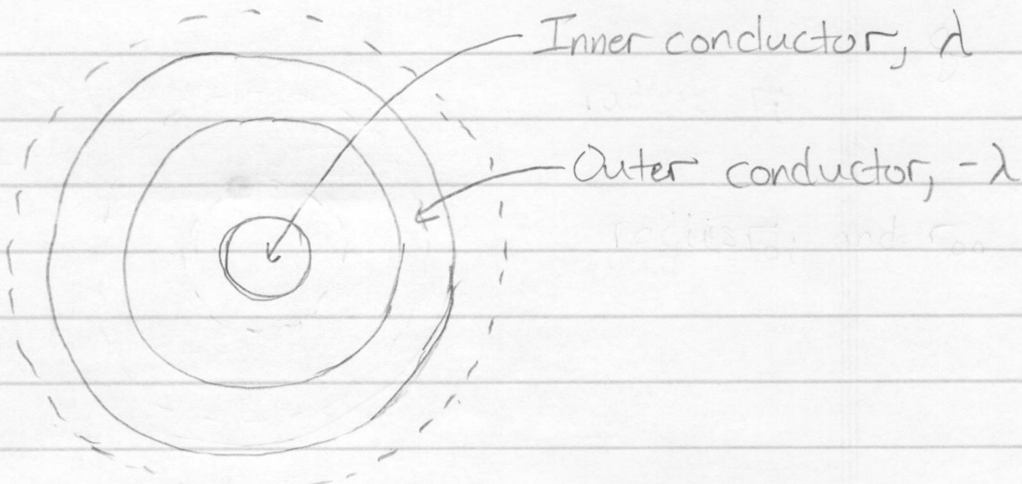
A coaxial cable, inner and outer conductor, carry equal and opposite charges.
What is the net charge on the outside of the outer conductor?



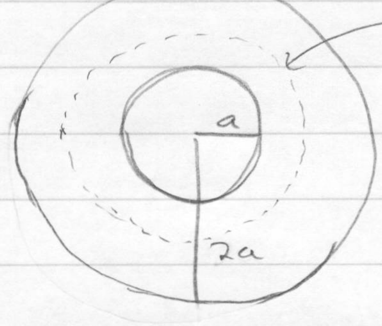
Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

We know that a Gaussian surface outside of the outer conductor encloses $\lambda + (-\lambda) = 0 \ \frac{C}{m}$ net charge per unit length. By Gauss's law

$$EA = \frac{q_{enc}}{\varepsilon_0}$$

this implies that $E = 0$ (because A and ε_0 are not 0).
For E to be zero everywhere outside the outer conductor, σ (the surface charge density) must also be zero.

Not convinced? Very close to the outside of the outer conductor it looks like an infinite sheet of charge, for which we know $\sigma = 2\varepsilon_0 E$ (Eq. 24-10).
Obviously here $E = 0$ implies $\sigma = 0$.



Sphere of radius $2a$ with hole of radius a

uniform charge density ρ

Calculate E for $a < r < 2a$.

Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

Choose spherical gaussian surface of radius R: $a < R < 2A$

$\Rightarrow$ E is everywhere parallel to dA $\Rightarrow$ $\oint \vec{E} \cdot d\vec{A} = EA$

where $A = 4\pi R^2$. $q_{enc} = \rho V_{enc} = \rho\left(\frac{4}{3}\pi R^3 - \frac{4}{3}\pi a^3\right)$

Now we have $E\left(4\pi R^2\right) = \frac{\rho \frac{4}{3}\pi\left(R^3 - a^3\right)}{\varepsilon_0}$.

Solve for E $\Rightarrow$ $\boxed{E = \frac{\rho\left(R - \frac{a^3}{R^2}\right)}{3\varepsilon_0}}$

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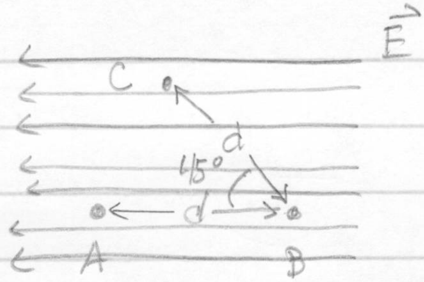
uniform $\vec{E}$



What is ΔV_{AB}? $\Delta V_{AB} = -\vec{E}\cdot\vec{\ell}$ (Eq. 25-2b)

$= \boxed{Ed}$ (b/c we are going against $\vec{E}$)

What is ΔV_{BC}? $\Delta V_{BC} = -\vec{E}\cdot\vec{\ell} = -E\ell\cos\theta$, $\theta = 45°$

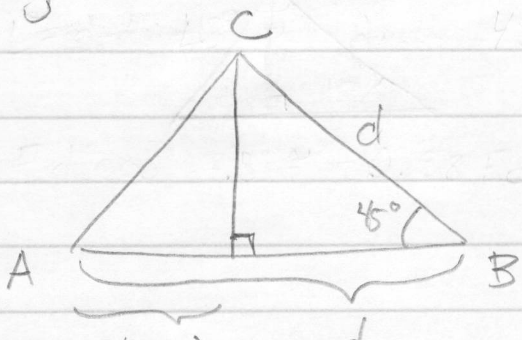
$= \boxed{-Ed/\sqrt{2}}$

What is ΔV_{AC}? Remember, ΔV is independent of path.

Therefore, given ΔV_{AB} and ΔV_{BC}, ΔV_{AC} is easy to get.

$\Delta V_{AC} = \Delta V_{AB} + \Delta V_{BC} = Ed - Ed/\sqrt{2} = \boxed{(1-\frac{1}{\sqrt{2}})Ed}$

Check by completing the triangle and computing explicitly:



$d - d\sin(45°)$

$d(1-\frac{1}{\sqrt{2}})$

$\Delta V_{AC} = Ed(1-\frac{1}{\sqrt{2}})$ It works!

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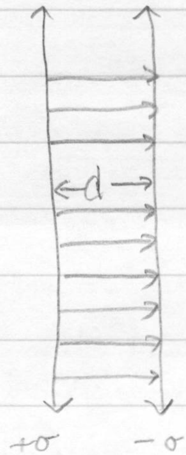
two large flat plates seperated by small d



the arrows indicate the direction of $\vec{E}$

Consider the plates to be infinite (good approximation for small d):

$E = \sigma/\varepsilon_0$ from left to right, as I have drawn it.

$V = -E\ell$ for a uniform $\vec{E}$

$V_{+-} = -Ed$

$V_{-+} = Ed$

They ask for magnitude, so: $|V_{+-}| = |V_{-+}| = \boxed{\frac{\sigma d}{\varepsilon_0}}$

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The important thing to note here is that the total energy is conserved, but potential energy is being converted into kinetic energy.

The release of potential energy from the charged body being in an electric field goes to increasing the kinetic energy:

Body 1: $q_1 V = \frac{1}{2} m_1 v^2$

Body 2: $q_2 V = \frac{1}{2} m_2 (2v)^2 = 2 m_2 v^2$

Divide the two equations: $\frac{q_1}{q_2} = \frac{m_1}{4 m_2}$

Solve for $q_2 = \frac{4 q_1 m_2}{m_1} = \frac{2(3.8\,\mu C)(2g)}{5g} = \boxed{6.08\,\mu C}$

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From Eq. 25-5 we have

$$\Delta V_{AB} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_A}{r_B}\right) = \frac{0.56\,nC}{2\pi\varepsilon_0} \ln\left(\frac{2mm}{1.6cm}\right)$$

$$\boxed{= -20.96\,\text{Volts}}$$

Be careful with the units in this problem. The sign of your answer will depend on how you choose r_A and r_B. Strictly speaking they ask for a magnitude, so the answer should be positive.

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By Gauss's law, you know that outside the surface of the shell, this system will act like a point charge of net charge $+3Q - Q = 2Q$.

For a point charge we have (Eq. 25-3)

$$\Delta V_{AB} = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Let $r_A \to \infty$ and $r_b = c$ so

$$\Delta V_{\infty,\text{shell}} = \frac{k2Q}{c}$$

Within the conducting shell $\vec{E} = 0$ (true for all conductors), therefore $\Delta V_{\text{through the shell}} = 0$.

Inside the inner surface of the shell, but above the surface of the sphere, Gauss's law tells you that the electric field is like that of a point charge $q = -Q$.

$$\Delta V_{\text{shell, sphere}} = k(-Q)\left(\frac{1}{a} - \frac{1}{b}\right)$$

In total we have $\Delta V_{\infty,\text{sphere}} = \Delta V_{\infty,\text{shell}} + \Delta V_{\text{shell,sphere}}$

$$= \frac{2kQ}{c} - kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \boxed{kQ\left(\frac{2}{c} - \frac{1}{a} + \frac{1}{b}\right)}$$

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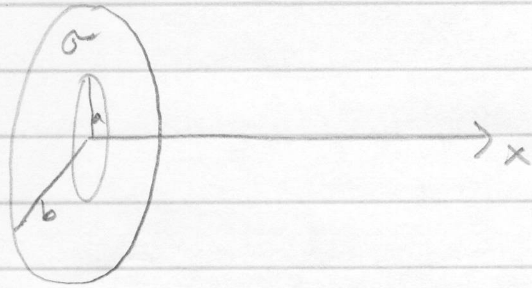
annulus w/ inner radius a, outer radius b, surface charge σ
Find V along the axis.

$$V = \int \frac{k\,dq}{r}$$



This is similar to your last homework where we found $\vec{E}$ along the axis of a disk. First, break the annulus into infinitesmal rings. Then add up all the contributions from the rings for r=a to r=b.
Example 25-7 does most of the work for you; you just need to change the limits of integration and slightly rework dq:

$$V = \int_{r=a}^{r=b} \frac{k(\sigma 2\pi r\,dr)}{\sqrt{x^2+r^2}} = 2\pi k\sigma \int_{r=a}^{r=b} \frac{r\,dr}{\sqrt{x^2+r^2}}$$

$$= \left[2\pi k\sigma \sqrt{x^2+r^2} \right]_{r=a}^{r=b} = \boxed{2\pi k\sigma \left(\sqrt{x^2+b^2} - \sqrt{x^2+a^2} \right)}$$