

Physics 2b HW 10 Solutions Ch. 33 #42 p.1

RLC circuit, $R = 75 \Omega$, $L = 20 \text{ mH}$, $f_0 = 4 \text{ kHz}$, $\omega_0 = 2\pi f_0$

(a) What is the capacitance?

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{L\omega_0^2} = \frac{1}{20 \text{ mH} (2\pi \times 4 \times 10^3 \text{ rad/s})^2}$$

$$= \boxed{79 \text{ nF}}$$

(b) What is the impedance @ $\omega = \omega_0$?

$$Z = \sqrt{(75 \Omega)^2 + \left((2\pi \times 4 \times 10^3 \text{ rad/s}) 20 \text{ mH} - \frac{1}{(2\pi \times 4 \times 10^3 \text{ rad/s}) 79 \text{ nF}} \right)^2}$$

0 (@ $\omega = \omega_0$, $X_L - X_C = 0$)

$$= \boxed{75 \Omega}$$

(c) What is the impedance @ $f = 3 \text{ kHz}$?

Same as above $\omega / 2\pi \times 4 \times 10^3 \text{ rad/s} \rightarrow 2\pi \times 3 \times 10^3 \text{ rad/s}$

$$Z = \boxed{303 \Omega}$$

#48 $Z(\omega) = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$, $\omega_0 = \frac{1}{\sqrt{LC}}$ ← what we want to get

$Z(\omega) = \sqrt{R^2 + \omega^2 L^2 \left(1 - \omega_0^2 / \omega^2 \right)^2}$ ← Starting with what we are given...

$= \sqrt{R^2 + \omega^2 L^2 \left(1 - \frac{1}{LC \omega^2} \right)^2}$ ← Substituting $\omega_0 = \frac{1}{\sqrt{LC}}$

$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$ ← rearrange, and they are equal

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Note that for an RLC circuit $I_p = \frac{V_p}{Z}$ and at

resonance $I_{res} = \frac{V_p}{R}$, where I_{res} is the current in

the resistor. Therefore $\frac{I_p}{I_{res}} = \frac{R}{Z} = \frac{1}{\sqrt{1 + \frac{(X_L - X_C)^2}{R^2}}}$

$$= \left(1 + \frac{L}{R^2 C} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2 \right)^{-1/2}$$

Since changing ω to $\frac{1}{\omega}$ in this equation does not change the result, then $\omega = \frac{1}{2}\omega_0$ and $\omega = 2\omega_0$ will give the same current, as stated.

(b) you were not asked to learn phasors
If I have time I will cover it in the review.

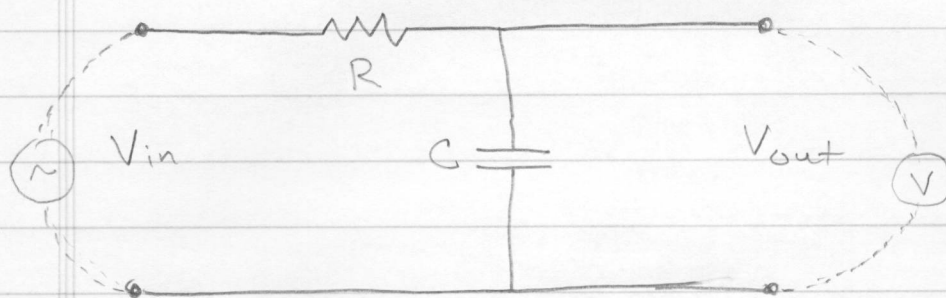
#53

Eqn 33-17 $P_{av} = V_{rms} I_{rms} \cos(\phi) = 120V \times 4.6A \cos(25^\circ) = \boxed{500W}$

#59

Transformers were not covered in lecture. Again, if there is time and interest, I will mention something in the review.

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low pass filter



(a) Show that $V_{out} = \frac{V_{in}}{\sqrt{1+(RC\omega)^2}}$

By looking at what I have sketched into the diagram you can see that we are dealing with a series RC circuit. We can use our analysis of RLC circuits with $L=0$. Note also that $V_{out} = V_{capacitor}$.

$$I_p = \frac{V_p}{Z} = \frac{V_p}{\sqrt{R^2 + (-X_c)^2}}, \text{ note } V_p = V_{in}$$

$$V_{out} = I_p X_c = \frac{V_{in} X_c}{\sqrt{R^2 + X_c^2}} = \frac{V_{in}}{\sqrt{\frac{R^2}{X_c^2} + 1}} = \boxed{\frac{V_{in}}{\sqrt{(R^2 C^2 \omega^2) + 1}}}$$

(b) $\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(R^2 C^2 \omega^2) + 1}} \Rightarrow R^2 C^2 \omega^2 = 1 \Rightarrow \boxed{\omega = \frac{1}{RC}}$