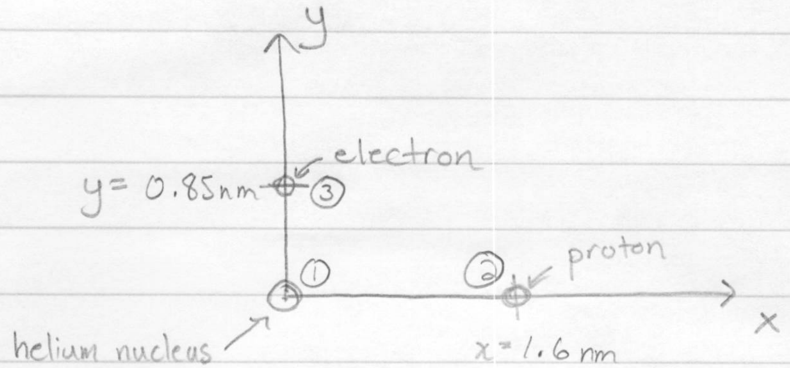

Find the net force on the helium nucleus.

First, number the charges:

① Helium nucleus

② proton

③ electron

Then, from the problem statement, we have:

$q_1 = +2e \qquad \vec{r}_1 = 0\hat{\imath} + 0\hat{\jmath}$

$q_2 = +e \qquad \vec{r}_2 = 1.6\text{nm}\,\hat{\imath} + 0\hat{\jmath}$

$q_3 = -e \qquad \vec{r}_3 = 0\hat{\imath} + 0.85\text{nm}\,\hat{\jmath}$

From the principle of superposition we have:

$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$, which can be read as "The total force on ① is the force that ② exerts on ① plus the force that ③ exerts on ①."

The half arrow above F indicates it is a vector quantity, which are bold in your textbook.

From Coulomb's Law, we have:

$$\vec{F}_{21} = \frac{kq_2q_1}{|\vec{r}_{21}|^2}\hat{r} = \frac{kq_2q_1}{|\vec{r}_{21}|^2}\frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

Here I have used the relation $\hat{r} = \dfrac{\vec{r}_{21}}{|\vec{r}_{21}|}$

where the straight lines indicate a magnitude.

Note that the indices on $\vec{r}_{21}$ are in the same order as $\vec{F}_{21}$, meaning we want the vector from ② to ①. In general, $\vec{r}_{ab}$ is computed:

$$\vec{r}_{ab} = \vec{r}_b - \vec{r}_a \quad \text{NOT} \quad \cancel{\vec{r}_{ab} = \vec{r}_a - \vec{r}_b}.$$

Getting back to the problem, we have

$$\vec{F}_{21} = \frac{k q_2 q_1}{|\vec{r}_{21}|^2} \frac{\vec{r}_{21}}{|\vec{r}_{21}|} = \frac{k q_2 q_1}{|\vec{r}_{21}|^3} \vec{r}_{21}$$

Note that I can multiply $|\vec{r}_{21}|^2 \times |\vec{r}_{21}| = |\vec{r}_{21}|^3$ because these are magnitudes and therefore simply numbers.

Now we can compute $\vec{r}_{21}$ and $|\vec{r}_{21}|$.

$$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2 = (0\hat{\imath} + 0\hat{\jmath}) - (1.6\text{nm}\,\hat{\imath} + 0\hat{\jmath}) = (-1.6\text{nm}\,\hat{\imath} + 0\hat{\jmath})$$

$|\vec{r}_{21}| = \sqrt{(-1.6\text{nm})^2 + (0\text{nm})^2}$ from the Pythogorean theorem.

$$= 1.6\text{nm}$$

Now we know everything and can simply plug in

$$\vec{F}_{21} = \frac{k q_2 q_1}{|\vec{r}_{21}|^3} \vec{r}_{21} = \frac{(9\times10^9\,\text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\text{C})(2\times1.6\times10^{-19}\text{C})(-1.6\times10^{-9}\text{m}\,\hat{\imath})}{(1.6\times10^{-9}\text{m})^3}$$

$$= -1.8\times10^{-10}\,\text{N}\,\hat{\imath}$$

Similarly, for $\vec{F}_{31}$ we have

$$\vec{F}_{31} = \frac{k q_3 q_1}{|\vec{r}_{31}|^3} \vec{r}_{31} = \frac{(9\times10^9\,\text{Nm}^2/\text{C}^2)(-1.6\times10^{-19}\text{C})(2\times1.6\times10^{-19}\text{C})(-0.85\times10^{-9}\text{m}\,\hat{\jmath})}{(0.85\times10^{-9}\text{m})^3}$$

$$= 6.38\times10^{-10}\,\text{N}\,\hat{\jmath}$$

Finally, $\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} = \boxed{-1.8\times10^{-10}\,\text{N}\,\hat{\imath} + 6.38\times10^{-10}\,\text{N}\,\hat{\jmath}}$

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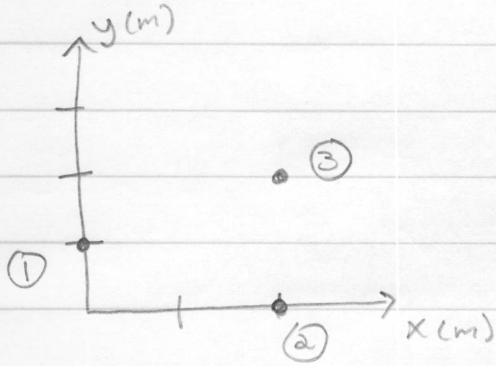


$q_1 = 68\,\mu C \qquad \vec{r}_1 = 1m\,\hat{j}$

$q_2 = -34\,\mu C \qquad \vec{r}_2 = 2m\,\hat{i}$

$q_3 = 15\,\mu C \qquad \vec{r}_3 = 2m\,\hat{i} + 2m\,\hat{j}$

Find the force on 3.

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$\vec{F}_{13} = \frac{k q_1 q_3}{|\vec{r}_{13}|^3}\,\vec{r}_{13} = \frac{(9\times10^9\,Nm^2/C^2)(68\times10^{-6}C)(15\times10^{-6}C)\,(2m\hat{i} + 1m\hat{j})}{\left(\sqrt{(2m)^2+(1m)^2}\right)^3}$$

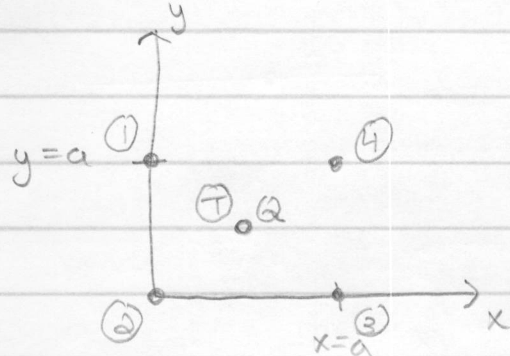
$$= 1.64\,N\hat{i} + 0.82\,N\hat{j}$$

$$\vec{F}_{23} = \frac{k q_2 q_3}{|\vec{r}_{23}|^3}\,\vec{r}_{23} = \frac{(9\times10^9\,Nm^2/C^2)(-34\times10^{-6}C)(15\times10^{-6}C)\,(2m\,\hat{j})}{(2m)^3}$$

$$= -1.15\,N\hat{j}$$

$$\boxed{\vec{F}_3 = 1.64N\hat{i} - 0.33\,N\hat{j}}$$

Setup a coordinate system. Any coordinate system will do, as long as you are consistent.



We are free to choose which charge is negative.

$q_1 = +q \qquad \vec{r}_1 = 0\hat{i} + a\hat{j}$

$q_2 = +q \qquad \vec{r}_2 = 0\hat{i} + 0\hat{j}$

$q_3 = +q \qquad \vec{r}_3 = a\hat{i} + 0\hat{j}$

$q_4 = -q \qquad \vec{r}_4 = a\hat{i} + a\hat{j}$

$\vec{F}_T = \vec{F}_{1T} + \vec{F}_{2T} + \vec{F}_{3T} + \vec{F}_{4T}$

To simplify the calculation, notice that $\vec{F}_{1T}$ and $\vec{F}_{3T}$ will be equal in magnitude and opposite in direction, and will therefore cancel. Likewise, $\vec{F}_{2T}$ and $\vec{F}_{4T}$ will be equal in magnitude AND direction.

We then have $\vec{F}_T = 2\vec{F}_{2T} = 2\left(\frac{k q_2 q_T}{|\vec{r}_{2T}|^2}\right)\hat{r}_{2T}$

$= \frac{2kqQ}{\left(\sqrt{(a/2)^2+(a/2)^2}\right)^2}\hat{r}_{2T}$

$= \boxed{\frac{4kqQ}{a^2}\hat{r}_{2T}}$

The force is either directly towards or away from the negative charge, depending on Q.

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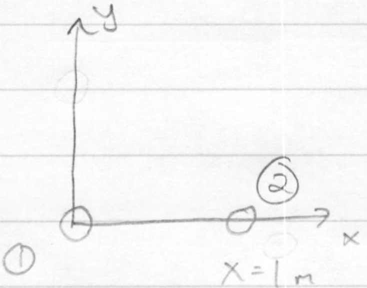
Here "small" means you can treat them as point charges. "metal" means they are conductors and charge can flow between them, and "identical" is to convince you that they would carry the same charge after bringing them together.

Initially, from the problem statement, we have:



$\vec{F}_{12} = \frac{kq_1q_2}{(1m)^2}\hat{i} = -2.5N\hat{i}$ (negative b/c it is attractive)

Then we touch them so that $q' = q_1' = q_2'$.
Also, by conservation of charge we have $2q' = q_1 + q_2$
Note that q' (read "q-prime") is the charge on each sphere after touching them together.
We are told that $\vec{F}_{12}' = \frac{kq'q'}{(1m)^2}\hat{i} = 2.5N\hat{i}$ (repulsive)

Solve for q_1 and q_2 given:

$$\frac{kq_1q_2}{1m^2} = -2.5N \quad \text{and} \quad \frac{k\left(\frac{q_1+q_2}{2}\right)^2}{1m^2} = 2.5N$$

OR $-q_1q_2 = \frac{(q_1+q_2)^2}{4} \Rightarrow q_1^2 + 6q_1q_2 + q_2^2 = 0$

Solving this quadratic equation for q_1 gives:
$q_1 = (-3 \pm \sqrt{8})q_2$ and because $-q_1q_2 = \frac{2.5N(1m)^2}{9\times10^9 Nm^2/C^2}$
gives $\boxed{q_1 = \pm 40.2\mu C \text{ and } q_2 = \mp 6.90\mu C}$
or vice versa.

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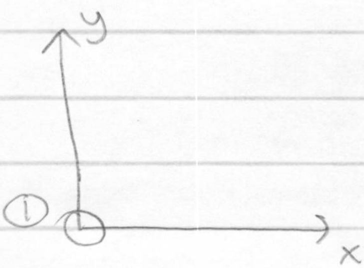

$q_1 = 65\mu C$

The electric field version of Coulomb's law states:

$$\vec{E} = \frac{kq}{|\vec{r}|^2}\hat{r} = \frac{kq}{|\vec{r}|^3}\vec{r}$$

(a) $$\vec{E} = \frac{k(65\times10^{-6}C)}{\left(\sqrt{(50\times10^{-2}m)^2}\right)^3}(50\times10^{-2}m\,\hat{i} + 0\,\hat{j})$$

$$= \boxed{2.34\times10^{6}\,N/C\,\hat{i}}$$

(b) $$\vec{E} = \frac{k(65\times10^{-6}C)(50\times10^{-2}m\,\hat{i} + 50\times10^{-2}m\,\hat{j})}{\left(\sqrt{(50\times10^{-2}m)^2+(50\times10^{-2}m)^2}\right)^3}$$

$$= \boxed{827\,kN/C\,\hat{i} + 827\,kN/C\,\hat{j}}$$

(c) $$\vec{E} = \frac{k(65\times10^{-6}C)(-25\times10^{-2}m\,\hat{i} + 75\times10^{-2}m\,\hat{j})}{\left(\sqrt{(-25\times10^{-2}m)^2+(75\times10^{-2}m)^2}\right)^3}$$

$$= \boxed{-296\,kN/C\,\hat{i} + 888\,kN/C\,\hat{j}}$$

H_2O molecule

dipole moment $p = qd = 6.2 \times 10^{-30}$ Cm

What is d if $q = e$?

$$d = \frac{p}{q} = \frac{6.2 \times 10^{-30}\ \text{Cm}}{1.6 \times 10^{-19}\ \text{C}} = \boxed{3.88 \times 10^{-11}\ \text{m}}$$

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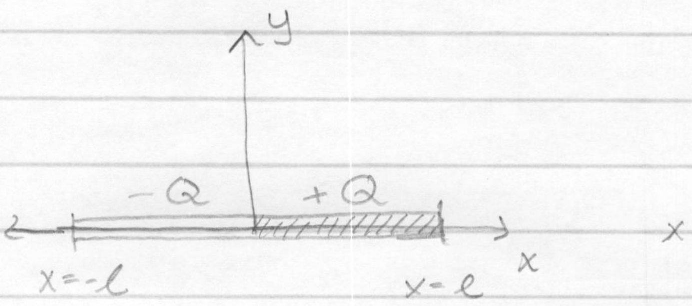


What is E for $x > l$?

$E = \dfrac{kq}{r^2} \Rightarrow dE = \dfrac{k\,dq}{r^2}$. In this case $dq = \lambda dx$.

$E_{tot} = E_{left} + E_{right}$ by superposition,

where E_{left} is from the −Q bar and E_{right} is from the + Q bar.

integral by u-substitution

$$\int dE_{left} = \int \frac{k\lambda\, dx'}{(x+x')^2} = k\left(\frac{-Q}{l}\right)\int_{x'=-l}^{x'=0} \frac{dx'}{(x-x')^2} = \left(\frac{1}{x} - \frac{1}{x+l}\right)\left(\frac{-kQ}{l}\right)$$

Here the prime notation indicates a "dummy variable" or "variable of integration".

$$\int dE_{right} = \int \frac{k\lambda\, dx'}{(x-x')^2} = k\left(\frac{Q}{l}\right)\int_{x'=0}^{x'=l} \frac{dx'}{(x-x')^2} = \left(\frac{1}{x-l} - \frac{1}{x}\right)\left(\frac{kQ}{l}\right)$$

$$E_{tot} = \frac{-kQ}{x(x+l)} + \frac{kQ}{x(x-l)} = \boxed{\frac{2kQl}{x(x^2-l^2)}}$$

The last step takes a little algebra. Remember (or look up) how to get a common denominator.

From above we have:

$$E = \frac{2kQl}{x(x^2-l^2)} \quad \text{for } x > l$$

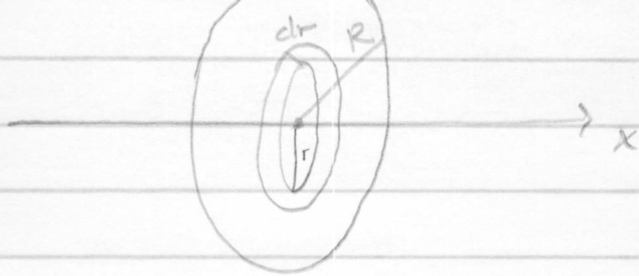
Now assume $x \gg l$ (x much greater than l)

$$\boxed{E \simeq \frac{2kQl}{x^3}} \quad \text{for } x \gg l \text{ b/c } x^2 \gg l^2$$

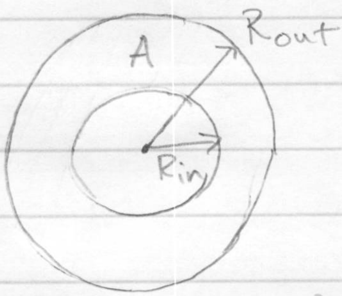
Given $E = \frac{2kp}{x^3}$ (Equation 23-7b) what is p?

We have $\frac{2kQl}{x^3} = \frac{2kp}{x^3}$, solve for p.

$$\boxed{p = Ql}$$



What is the area of a ring with infinitesmal thickness dr?
The area of a ring is obtained by subtracting the area of a smaller circle from the area of a larger circle.



$A = \pi R_{out}^2 - \pi R_{in}^2 = \pi (R_{out}^2 - R_{in}^2)$

For an infinitesmal ring, $R_{in} = r$ and $R_{out} = r + dr$

$dA = \pi[(r+dr)^2 - r^2] = \pi[r^2 + 2r\,dr + dr^2 - r^2]$

The dr^2 term is small, and can be ignored.

$\boxed{dA = 2\pi r\,dr}$

You might be tempted to say the area is the circumference of a circle times dr, which is true, but the above argument is slightly more subtle and correct.

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Given $dA = 2\pi r\,dr$ from above, we can write $dq = \sigma\, dA = \boxed{\sigma\, 2\pi r\, dr}$.

The general expression for dE is:

$dE = \frac{k\,dq}{r^2}\hat{r}$, Note that this r and the above r are different. Also, by symmetry, the electric field will only have an x compoment.

$$dE_x = dE\cos\theta = \frac{k\sigma 2\pi r\,dr}{(\sqrt{x^2+r^2})^2}\left(\frac{x}{\sqrt{x^2+r^2}}\right)$$

$$= \boxed{\frac{2k\sigma\pi r x}{(x^2+r^2)^{3/2}}\,dr}$$

Finally, we need to add up all the rings.

$$E_x = \int_{r=0}^{r=R} \frac{2k\sigma\pi r x\,dr}{(x^2+r^2)^{3/2}} = 2k\sigma\pi x\left[\frac{-1}{\sqrt{x^2+r^2}}\right]_{r=0}^{r=R}$$

$$= 2k\sigma\pi x\left(\frac{-1}{\sqrt{x^2+R^2}} + \frac{1}{x}\right)$$

$$= \boxed{2k\sigma\pi\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right) \quad \text{for } x > 0}$$

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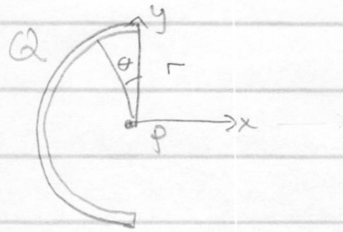

Find the electric field at P.

By symmetry, the field will only have an x component.

$dE = \frac{k\,dq}{r^2}$, $dE_x = \frac{k\,dq}{r^2}\sin\theta$ sin θ in this case because θ is measured from the y-axis

$dq = \lambda\, ds$, $\lambda = Q/(\pi r)$, $ds = r\,d\theta \Rightarrow dq = \frac{Q}{\pi r} r\,d\theta$

$dE_x = \frac{kQ}{r^2\pi} d\theta \sin\theta$

$$E_x = \int_{\theta=0}^{\theta=\pi} \frac{kQ\sin\theta\, d\theta}{r^2\pi} = \frac{kQ}{\pi r^2}\Big[-\cos\theta\Big]_{\theta=0}^{\theta=\pi} = \boxed{\frac{2kQ}{\pi r^2}}$$

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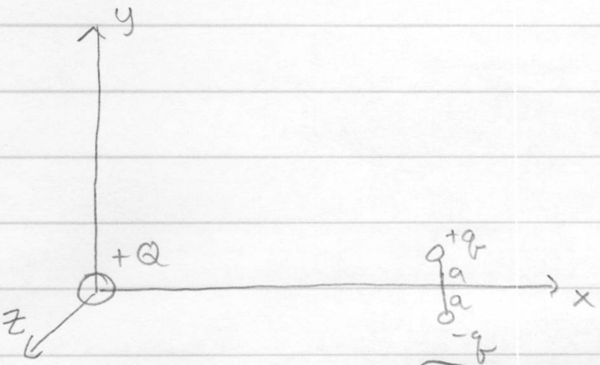

cross products

For $x \gg a$, $\tau = \vec{p} \times \vec{E}_Q = 2qa\hat{j} \times \frac{kQ}{x^2}\hat{\imath}$

$= \boxed{-\frac{2qakQ}{x^2}\hat{k}}$ b/c they are perpendicular and the right hand rule.

By Newton's laws, $\vec{F}_{Qdi} = -\vec{F}_{diQ}$.

For $x \gg a$ we know that $\vec{E}_{di} = -\frac{2kqa}{x^3}\hat{j}$

so $\vec{F}_{diQ} = Q\left(-\frac{2kqa}{x^3}\right)\hat{j}$

and $\vec{F}_{Qdi} = \boxed{\frac{2Qkqa}{x^3}\hat{j}}$ (magnitude and direction given)

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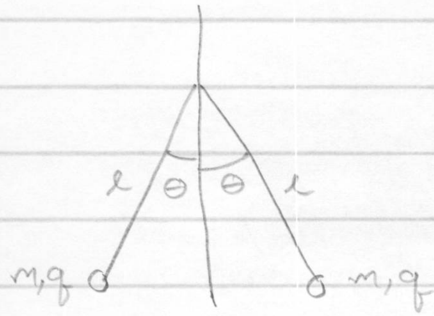
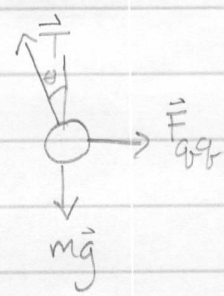


Solve for q in terms of $\ell, \theta, m, g,$ and k.

Draw a free body diagram for one sphere.



$T\cos\theta = mg$; $F_{qq} = \dfrac{kq^2}{(2\ell\sin\theta)^2}$; $T\sin\theta = F_{qq}$

Combine $\dfrac{\sin\theta}{\cos\theta} = \dfrac{kq^2}{(2\ell\sin\theta)^2}\dfrac{1}{mg} = \tan\theta$

Solve for $\boxed{q = \pm 2\ell\sin\theta\sqrt{\dfrac{mg\tan\theta}{k}}}$

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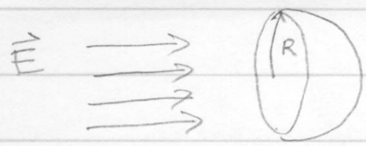

The flux through the closed surface is zero, because it encloses no charge. ($\phi_{tot} = 0$)

The flux through the flat surface is simply $\phi_{Flat} = EA = E\pi R^2$ because the uniform electric field is everywhere perpendicular to the flat surface.

Finally, solve for ϕ_{curved}:

$$\phi_{tot} = -\phi_{Flat} + \phi_{curved} \Rightarrow \boxed{\phi_{curved} = E\pi R^2}$$

The negative sign comes because flux into a closed surface is negative by convention.

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$$\phi = \oint \vec{E} \cdot d\vec{A} = \int_{y=0}^{y=a} E_0 \frac{y}{a} \hat{k} \cdot a\,dy\,\hat{k}$$

$$= \int_{y=0}^{y=a} E_0\, y\, dy$$

$$= \left[\frac{E_0}{2} y^2 \right]_{y=0}^{y=a}$$

$$= \boxed{\frac{E_0 a^2}{2}}$$ Here I have assumed $\phi > 0$.

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Spherical symmetry should tell you that Gauss's law will be useful in finding $\vec{E}$.

Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

A "uniform volume charge density" means that ρ can be found by dividing the total charge by the total volume, or, equivalently, the charge enclosed can be found by multiplying ρ by the enclosed volume.

$$\oint \vec{E} \cdot d\vec{A} = E\,4\pi r^2$$

$$\frac{q_{enc}}{\varepsilon_0} = \frac{\rho V_{enc}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\left(\tfrac{4}{3}\pi r^3 - \tfrac{4}{3}\pi a^3\right), \; a<r<b$$

Combine those:

$$E\,4\pi r^2 = \frac{\rho\,\tfrac{4}{3}\pi\,(r^3 - a^3)}{\varepsilon_0}$$

Solve for E:

$$\boxed{E = \frac{\rho\,(r - a^3/r^2)}{3\varepsilon_0}}$$

Equation 24-7 is $E = \frac{\rho r}{3\varepsilon_0}$, which is consistent with our answer at $a=0$.

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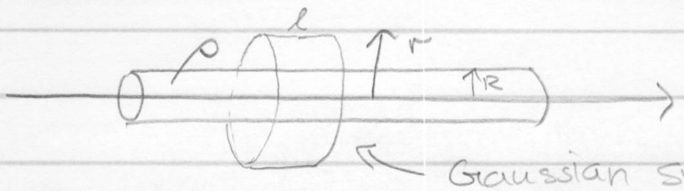

Gaussian surface, cylinder of radius r

Apply Gauss's law.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Outside ($r > R$): $E\, 2\pi r l = \frac{\rho \pi R^2 l}{\varepsilon_0}$

$$\boxed{E = \frac{\rho R^2}{2 \varepsilon_0 r}}$$

Inside ($r < R$): $E\, 2\pi r l = \frac{\rho \pi r^2 l}{\varepsilon_0}$

$$\boxed{E = \frac{\rho r}{2 \varepsilon_0}}$$

In the two different cases, the surface area of the Gaussian surface does not change, but the charge enclosed does.

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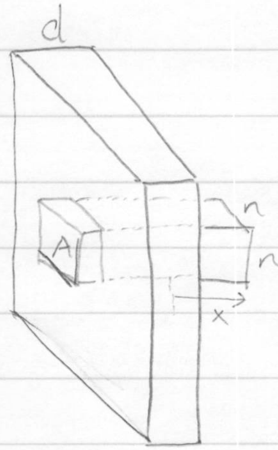

Apply Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

Outside $(x > \frac{d}{2})$: $E\, 2n^2 = \frac{\rho n^2 d}{\varepsilon_0}$

$$\boxed{E = \frac{\rho d}{2\varepsilon_0}}$$

Inside $(x < d/2)$: $E\, 2n^2 = \frac{\rho n^2 2x}{\varepsilon_0}$

$$\boxed{E = \frac{\rho x}{\varepsilon_0}}$$

Your result should be independent of the Gaussian surface that you choose. Convince yourself by computing E with a cylinder.