Physics 2b Final Review notes

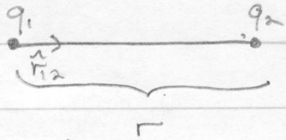
Note to students -

I am going against my better judgement and making these notes available to you online. I am doing this because I don't want to disadvantage anyone who has a genuine conflict with one of the review times. However, having these notes is <u>NOT</u> a substitute for coming to the review sessions. I make <u>NO GUARANTEE</u> that the material contained within is 100% correct. It is your responsibility to check it against other sources. I <u>WILL NOT ANSWER QUESTIONS OR MAKE CLARIFICATIONS ABOUT THESE NOTES OVER E-MAIL</u> so if you want to ask something, come to class. This is not a comprehensive review of the class material, and as always you are responsible for all material presented in the lectures, homeworks and quizzes. All that being said, I hope these are helpful.

Study Hard,
Nick

Physics 2b Final review   Chapter 23   Electric Charge

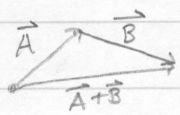
- charge is quantized, $e = 1.6\times10^{-19}\ C$
- two types of charge $+, -$
  - $+,+$ or $-,-$ repel
  - $+,-$ attract
- $\vec{F}_{12} = \dfrac{k q_1 q_2}{r^2}\hat{r}_{12}$



Coulomb's law

"force that one exerts on two"

- superposition principle - more than one force? sum vectorially!



or $\vec{A}+\vec{B} = (A_x+B_x)\hat{\imath} + (A_y+B_y)\hat{\jmath} + (A_z+B_z)\hat{k}$

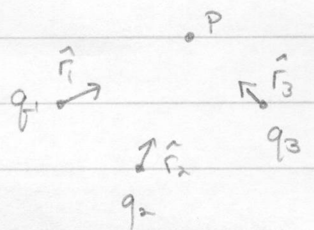
- convenient to introduce $\vec{E} = \dfrac{\vec{F}}{q} = \dfrac{kq}{r^2}\hat{r}$, $\hat{r}$ points from charge to point

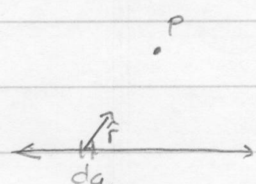
- if there is more than one charge present

$$\vec{E} = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2}\hat{r}_i$$

or

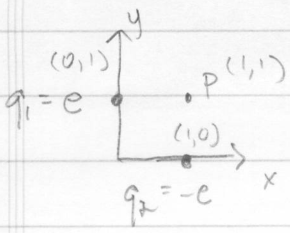
$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$





units: $[q] = C$, $[F] = N$, $[E] = N/C$

example



$$E(P) = k\left[\left(\frac{e}{(1m)^2}\right)(1\hat{\imath})\left(\frac{-e}{(1m)^2}\right)(1\hat{\jmath})\right] = 1.44\times10^{-9}\ N/C\ (\hat{\imath}-\hat{\jmath})$$

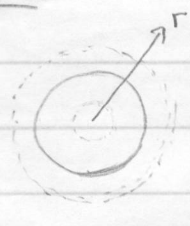
Chapter 24 Gauss's law

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad \varepsilon_0 = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$$

- useful for finding $\vec{E}$ in highly symmetric situations
- useful for finding q_{enc} when you can measure $\vec{E}$

- $\oint$ ← indicates an integral over a closed surface
- this surface is an imaginary mathematical tool, so we are free to choose its shape
- in practice we choose the shape so that $\vec{E}\cdot d\vec{A}$ is constant or zero so that $\oint \vec{E}\cdot d\vec{A} = EA'$ where A' is the part of A where $\vec{E}\cdot d\vec{A} \neq 0$
- we make this choice by symmetry arguments

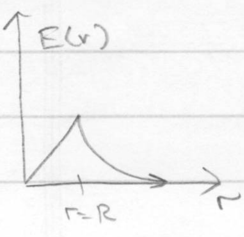
<u>example</u>



uniformly distributed spherically symmetric Q radius R, what is E everywhere?

- choose spherical surface b/c $\vec{E}$ must be radially outward by symmetry
- $\oint \vec{E}\cdot d\vec{A} = EA = q_{enc}/\varepsilon_0$, if $r<R$ $q_{enc} = \frac{\frac{4}{3}\pi r^3 Q}{\frac{4}{3}\pi R^3}$

$$E\,4\pi r^2 = \frac{r^3 Q}{\varepsilon_0 R^3} \Rightarrow E = \frac{kQr}{R^3},\quad r<R$$



$r>R$ $\quad E\,4\pi r^2 = Q/\varepsilon_0 \Rightarrow E = \frac{kQ}{r^2},\quad r>R$

Where is $E(r)$ the greatest? At the surface!

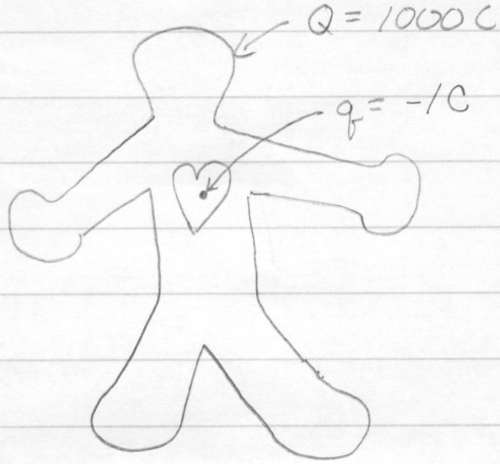
Know how to do all the cases in the book

Write them down, check them, know how to use them

Chapter 24 continued

- $E = 0$ inside a conductor
- all the charge on a conductor is on the surface
- surface charges can charge, but unless you charge or discharge a conductor, the total charge is conserved

example



What will the surfaces charges be?

outer = 999 C

inner = 1C of love

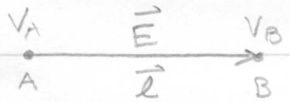
Make sure you know how this is related to Gauss's law.

Chapter 25

Electric potential

- Electric fields arising from static charge distributions are conservative $\Leftrightarrow$ they can be written as ∇V $\Leftrightarrow$ the work required to move a charge from one point to another in such a field is independent of the path
- Therefore, it is convenient to introduce the Electric potential

$$\Delta V_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = -\vec{E} \cdot \vec{l} \text{ for a constant field} \quad [V] = V$$



$\Delta V_{AB} = -El$ For constant field along the field

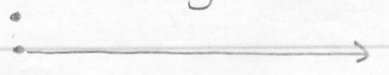
- For discrete and continuous charge distributions, V is calculated by $V = \sum_i kq_i/r_i$, $V = \int \frac{k\,dq}{r}$
- practice some examples from book, I want to do diff. ex.
- if you are given V, calculate $E_l = -dV/dl$ (E along l)

- conductors in electrostatic eq. are equipotentials

Example

- electric potential's relationship to potential energy is too often over looked

e^-, initially at rest



$\Delta V_{AB} = +100V$

$\Delta U_{AB} = qV = (-e)(100V)$

Conservation of energy $\Rightarrow KE = 100eV = \frac{1}{2}mv^2$

What is it's velocity @ B?

- you should know how to do dynamics problems (from 2A)
- notice that $\Delta V_{AB} > 0$ but $\Delta U_{AB} < 0$ for e^- b/c $q < 0$

Chapter 26 Electrostatic Energy and Capacitors

Electrostatic Energy

- any electric field has an energy density

$$u_E = \frac{1}{2}\varepsilon_0 E^2, \quad [u_E] = J/m^3$$

- the total energy stored in the field is

$$U = \int u_E \, dV = \int \frac{1}{2}\varepsilon_0 E^2 dV = \frac{1}{2}\varepsilon_0 E^2 V \text{ for constant field}$$

Capacitors

- two charged conductors, seperated by some distance
- a way of storing energy in an electric field
- the capacitance, $C = Q/V$ ($[C] = F$), is the ratio of the charge on each conductor to the potential difference that develops across the plates due to the seperation of charge. It depends only on the geometry of the capacitor. An important special case is the parallel plate capacitor, for which $C = \varepsilon_0 A/d$ where A is the area of one plate and d is the seperation.
- the total energy stored in a capacitor is $U = \frac{1}{2}CV^2$

Chapter 26 (continued)

Capacitors (continued)

- remember trick for finding whether two elements are series/parallel

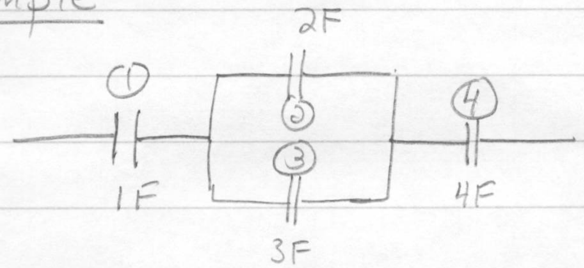
- Capacitors in circuits have the following properties:
  - in parallel, the effective capacitance is given by:

$$C = C_1 + C_2 + C_3 + \ldots$$

  - in series, the inverse of the effective capacitance:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Example



What is C_{eff}?

First combine ② and ③ to get a single 5F capacitor.

This new capacitor is then in series with ① and ④ so

$$\frac{1}{C_{eff}} = \frac{1}{1F} + \frac{1}{5F} + \frac{1}{4F} \Rightarrow C_{eff} = 0.69F$$

Dielectrics in capacitors

- a dielectric is an insulating material that can be inserted into a capacitor to change its properties
- the following table summarizes what happens when you insert a dielectric w/ constant K

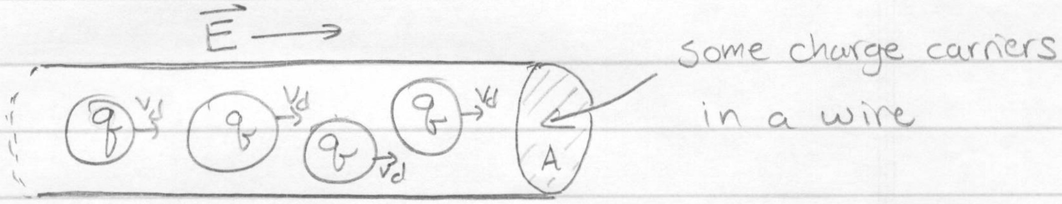
$C_0 \to KC_0$ (always true)

connected to battery	not connected to battery
$V_0 \to V_0$	$Q_0 \to Q_0$
$Q_0 \to Q = (KC_0)V_0 = KQ_0$	$V_0 \to V = Q_0/(KC_0) = V_0/K$
$U_0 \to U = \frac{1}{2}(KC_0)V_0^2 = KU_0$	$U_0 \to U = \frac{1}{2}(KC_0)(V_0/K)^2 = U_0/K$

Here the "o" subscript indicates the value before inserting the dielectric. Can you see why all this is true?

Chapter 27 Electric current

- So far we have been studying electro<u>statics</u> i.e. none of the charges are moving
- but, some of the most interesting things only happen when charges are moving
- a flow of electric charge is called a current and is given the symbol I ($[I] = A$).
- we first developed a microscopic view of current:



$I = n\,q\,A\,v_d$ ← drift velocity of charges

\# density of charge carriers — charge of each carrier — cross sectional area of wire

- if there is no $\vec{E}$ applied, then v_d will be zero and the current will be zero
- if you apply an $\vec{E}$, the charge carriers will bounce around everywhere, but will, on average, drift in the direction given by $\vec{F} = q\vec{E}$ with a speed v_d, and hence there will be a current
- the current density $\vec{J} = nq\vec{v}_d$ gives a measure of the current that is independent of A
- the conductivity of a material is a measure of "how well it turns $\vec{E}$ into $\vec{J}$" $\vec{J} = \sigma\vec{E}$
- the resistivity is simply the inverse $\rho = \dfrac{1}{\sigma}$

Chapter 27 (continued)

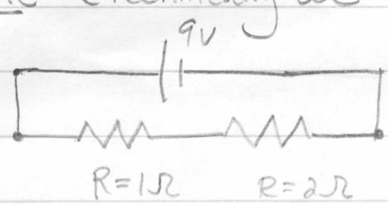
Resistivity and resistance

- resistivity is a property of a material that depends on its microscopic details
- resistance ($[R]=\Omega$) is the property of a certain piece of material that depends on its resistivity and geometry
  - for instance, a cylindrical conductor (wire) with resistivity ρ and length l and area A has a resistance of $R=\rho l/A$
- having the concept of resistance allows us to state the macroscopic (and more familiar version) of Ohm's law $V=IR$, which is given in terms of V instead of $\vec{E}$
- the reason why a material "resists" the flow of current is that the charge carriers are constantly smashing into things, as we mentioned before. This causes heating and we say that energy is "lost to heat". We also say that a resistor is a "dissipative element". This dissipation is quantified by the Power

$$P = IV = I^2R = V^2/R \qquad ([P] = J/s = W)$$

where the different versions come from Ohm's law.

Example (technically we haven't seen resistors yet, but oh well)



b/c not all 9V drops across the 1Ω resistor

What is the power dissipated in the 1Ω resistor?

$I = 9V/3\Omega = 3A$, $P = I^2R = (3A)^2(1\Omega) = 9W$, note: $P \neq (3A)(9V)$!

Chapter 28 Electric Circuits

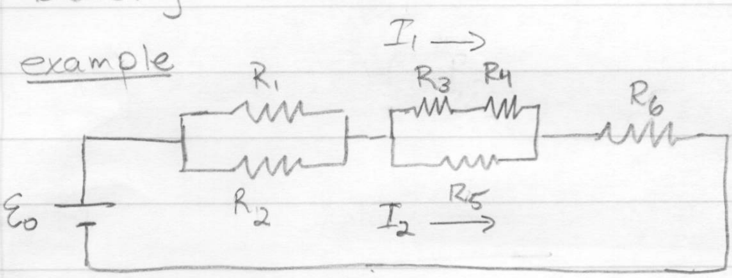
- circuit - multiple circuit elements connected by wires
  - circuit elements - battery, resistor, capacitor, inductor, etc.
- what are possible questions?
  - "What is the current/charge/power/energy/etc. (at time t)?"

Methods for analyzing circuits

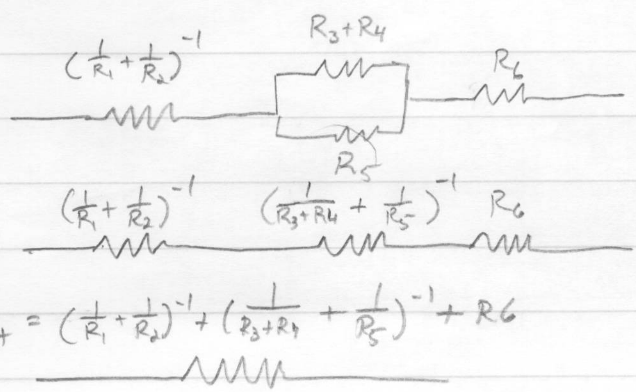
- Simple circuits - Circuit reduction
  - combine elements using combination rules
  - the voltage across any two elements in parallel is the same
  - the current in any two elements in series is the same
  - be careful not to confuse a combined element with an individual one

Circuit element	Combination rules: Parallel	Combination rules: Series	Voltage change (in the direction of I)
Capacitors	$C = \sum_i C_i$	$\frac{1}{C} = \sum_i \frac{1}{C_i}$	$\Delta V = -\frac{Q}{C}$
Resistors	$\frac{1}{R} = \sum_i \frac{1}{R_i}$	$R = \sum_i R_i$	$\Delta V = -IR$
Battery			$\Delta V = +\mathcal{E}_0$

example



What is I in, V across and P dissipated in R_3?



$$R_{tot} = \left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1} + \left(\frac{1}{R_3+R_4}+\frac{1}{R_5}\right)^{-1} + R6$$

$$I_{tot} = \mathcal{E}_0 / R_{tot}$$

$$I_{tot} = I_1 + I_2 \Rightarrow I_2 = I_{tot} - I_1$$

$$I_1 (R_3+R_4) = I_2 R_5$$

$$I_1 (R_3+R_4) = (I_{tot} - I_1) R_5$$

$$I_1 (R_3+R_4-R_5) = I_{tot} R_5$$

$$\boxed{I_1 = \frac{I_{tot} R_5}{(R_3+R_4-R_5)}}$$

$$\boxed{V_{R_3} = I_1 R_3 = \frac{I_{tot} R_5 R_3}{(R_3+R_4-R_5)}}$$

$$\boxed{P_{R_3} = I_1 V_{R_3}}$$

Chapter 28 (continued)

Kirchoff's Circuit laws

- Pg. 722 in your text gives an excellent explanation.
- You should be able to work a problem at the level of Example 28-5
- a note on "Sinha's method"
    - In lecture he did not make it clear, at least to me, how to determine the number of loops (or currents) are necessary to solve an arbitrary problem
    - In discussion/problem session I told you that it was sufficient to "touch" all the elements with at least one current
    - That seemed to work, but I am now convinced that this is not true and I don't actually know what the real answer is
    - If you want to see what I mean, try solving for the total current in the circuit on the previous page using one current through the "top" resistors and another through the bottom resistors and compare your answer to circuit reduction
    - To summarize, use the book's method

RC Circuit

- on page 728 they setup the circuit equation and solve the corresponding differential equation for voltage across a capacitor in an RC circuit
- voltage across a capacitor cannot change instantaneously!

Chapter 29 Magnetic Fields

- Magnetic fields come from <u>moving</u> charges
- The force on a charged particle in a field $\vec{B}$ is

$$\vec{F} = q\vec{v} \times \vec{B}$$

  where $\vec{B}$ is the magnetic field and $[B] = T$.
- The magnitude $|\vec{F}| = qvB\sin\theta$ and the direction of the cross product is given by right hand rule
- We also learned another way to compute the cross product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}(A_yB_z - A_zB_y) - \hat{\jmath}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

  note the position of all the minus signs.
- we know that if a region has both $\vec{E}$ and $\vec{B}$ fields:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = \vec{F}_E + \vec{F}_B$$

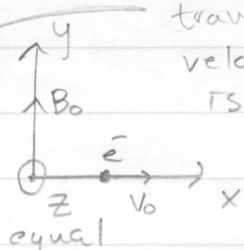
  note that for a positive charge, $\vec{F}_E = q\vec{E}$ is in the direction of $\vec{E}$, but for a negative charge it is in the $-\vec{E}$ direction. The same is true for $\vec{F}_B$ with $\vec{E}$ replaced by $\vec{B}$
- Force on a wire $\vec{F} = I\vec{\ell} \times \vec{B}$, $\vec{\ell}$ is in the direction of I

<u>Example</u>

An electron travels in a straight line along the x-axis at a velocity of $v_0\hat{\imath}$. There is a magnetic field along the y-axis, $\vec{B} = B_0\hat{\jmath}$. What is the direction and magnitude of the electric field?

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = 0$$

traveling w/ constant velocity $\Rightarrow$ net force is zero



$\vec{v} \times \vec{B} = V_0 B_0 \hat{k}$

$\Rightarrow (-e)V_0B_0 = (-e)E$ ← magnitudes are equal

$\Rightarrow \boxed{\vec{E} = -V_0B_0\hat{k}}$ convince yourself that it is <u>not</u> $+V_0B_0\hat{k}$

Chapter 30 Sources of magnetic field

- magnetic fields come from moving charges
- steady currents ($\frac{dI}{dt}=0$) will setup static $\vec{B}$ fields
- how to calculate $\vec{B}$ given I?
  - first, the "difficult way": Biot-Savart Law

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I \, d\vec{l} \times \hat{r}}{r^2}$$

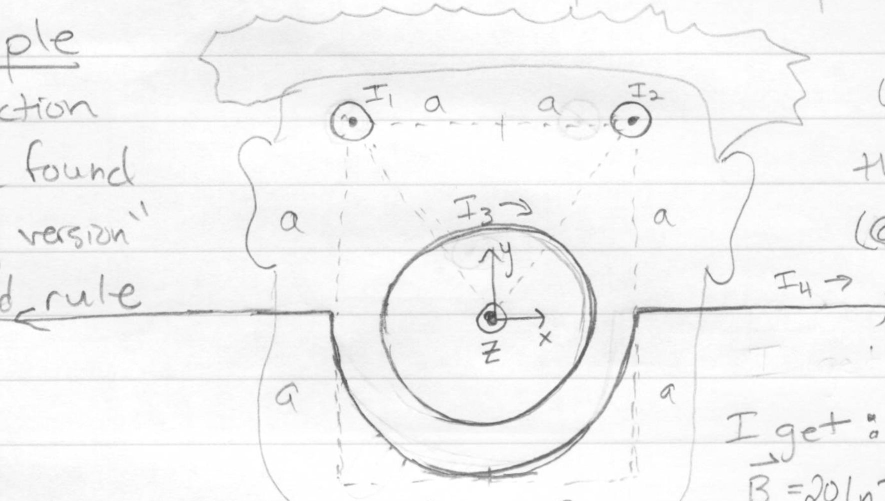
  - in general this is difficult, but it was done in the book for a straight wire, circle of wire on axis and in the HW for a semicircle
  - second, the "easy way": Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}, \quad \mu_0 = 1.26 \times 10^{-6} \, N/A^2$$

    - useful for computing $\vec{B}$ for highly symmetric I, such as infinite wire, charge sheet and solenoid
    - here we want to choose an Amperian loop over which $\vec{B} \cdot d\vec{l}$ is constant or zero on all pieces of the loop
    - this is very much analagous to Gauss's law
- How to calculate $\vec{B}$ for multiple I? Superposition!

example

note: the direction of $\vec{B}$ can be found using "curly version" of right hand rule



What is B at the "tip of the nose"? (@ the origin)

$I_1 = I_2 = I_3 = I_4 = 1A$

$a = 1m$

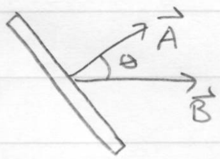
I get:

$\vec{B} = 201 nT \, \hat{i} - 315 nT \, \hat{k}$

Try it!

Chapter 31 Electromagnetic Induction

- time changing magnetic fields give rise to electric fields
- when wires are in the vicinity, these $\vec{E}$ fields in turn give rise to an EMF and current
- How to quantify this?
  - First, define magnetic flux: $\phi_B = \int \vec{B} \cdot d\vec{A}$
    - notice that this is not $\oint \vec{B} \cdot d\vec{A}$ because it is one of the fundamental properties of $\vec{B}$ fields that $\oint \vec{B} \cdot d\vec{A} = 0$ for any closed surface. In other words, there are no isolated magnetic charges (monopoles).
  - if $\vec{B}$ is uniform and A is flat then $\phi_B = BA\cos\theta$, where θ is the angle between $\vec{B}$ and the <u>normal</u> to the surface



if $\theta = 0 \Rightarrow \phi_B = BA$

  - Second, Faraday's Law $\mathcal{E} = -\dfrac{d\phi_B}{dt}$

    which says that ϕ_B must be changing in time to get an $\mathcal{E}$. Note that this could be because B is changing, A is changing, or θ is changing, as we saw in the HW and quizzes.
  - the minus sign is a reminder of Lenz's law, which says that the induced EMF is always setup such that the induced current opposes the <u>change in flux</u>
- use the right hand rule to find the direction of $\mathcal{E}$

Chapter 32 Inductance and Magnetic Energy

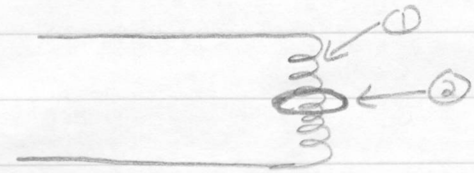
Inductor - loops of coil where a time changing ϕ_B can induce an EMF (and hence a current)

- a common type of inductor is a solenoid

Mutual inductance - two inductors that are close together → a current (I_1) in one of them causes a flux (ϕ_2) in the other

$M \equiv \frac{\phi_2}{I_1}$, Faraday's law ⇒ $\boxed{\mathcal{E}_2 = -M \frac{dI_1}{dt}}$

in words, an EMF is induced in 2 if the current is changing in 1



Look over quiz problem solution and HW.

Self inductance

$L = \frac{\phi_B}{I}$, the flux divided by the current flowing in a single solenoid

note: $[M] = [L] = H$

Inductor EMF - when current is changing in a solenoid, the ϕ_B is also changing so a "back EMF" is setup that opposes the charge in flux

$\boxed{\mathcal{E}_L = -L \frac{dI}{dt}}$

Inductors in circuits

- the current through an inductor cannot change instantaneously! See quiz 8 solutions

Magnetic energy

- Energy stored in a solenoid $U = \frac{1}{2} L I^2$ (J)
- divide by volume to get energy density $u_B = \frac{B^2}{2\mu_0}$ $\left(\frac{J}{m^3}\right)$

Chapter 33 AC Circuits

- So far we have only seen DC Voltages where the magnitude is constant and the sign never changes.
- In many cases, the voltage input to a circuit is a sinusoidal function $V(t) = V_p \sin(\omega t)$ and we must learn to deal with this
- components (resistors, capacitors, and inductors) behave differently under AC voltages

RMS Voltages

- For an AC source we want to define an average voltage, but the true time average is zero $\left(\int_0^{2\pi} \sin(t)\,dt = 0 \right)$
- The "root-mean-square" voltage is defined:

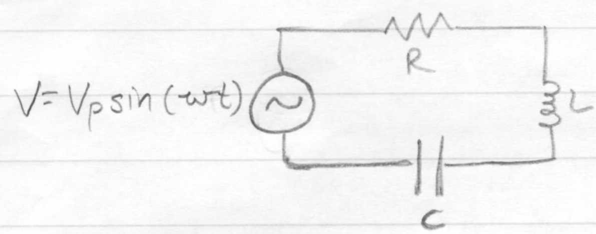
$$V_{rms} = \sqrt{\int_0^{\tau} V^2(t)\,dt / \tau}$$

- plugging in $V(t) = V_p \sin(\omega t)$ yields $V_{rms} = V_p/\sqrt{2}$

(quick note: $[\omega] = \text{rad/s}$, $[F] = \text{Hz}$ and $\omega = 2\pi F$)

RMS current

- first we need to define the circuit



- this circuit has an "effective resistance" $Z = \sqrt{R^2 + (X_L - X_C)^2}$ where $X_L = \omega L$, $X_C = \frac{1}{\omega C}$ and $[X_L] = [X_C] = \Omega$
- the RMS current (in any element, since they are in series) is

$$I_{rms} = V_p / Z$$

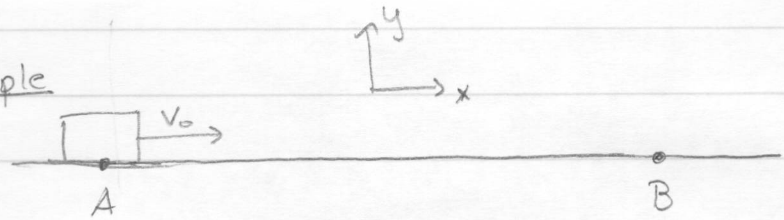
RMS Voltages - given by Ohm's law

$$V_{rms,R} = I_{rms} R$$

$$V_{rms,C} = I_{rms} X_C, \quad V_{rms,L} = I_{rms,L}$$

Dynamics review

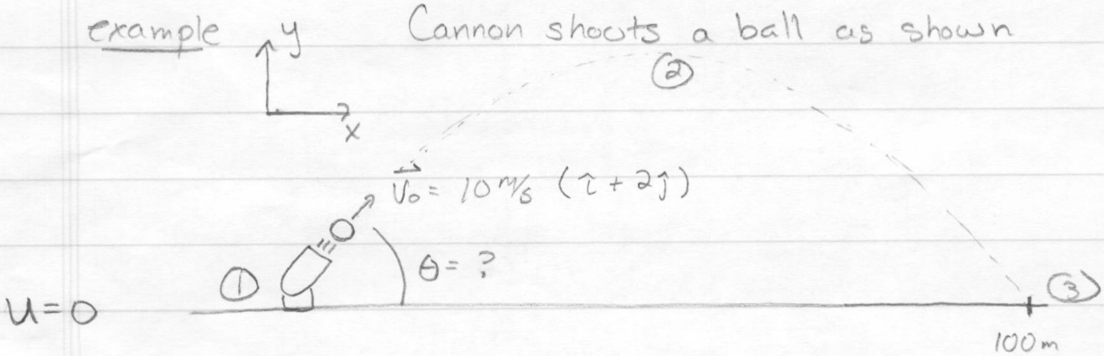
Example



Block sliding across a frictionless surface at a constant speed. How long does it take to get from A to B? ~~$t = V_o/(X(B) - X(A))$~~

example

Cannon shoots a ball as shown



What is the initial angle? $\tan\theta = \frac{y}{x} = \frac{2}{1} \Rightarrow \theta = \tan^{-1}(2)$

What is the initial x-velocity? $|\vec{V}_{0,x}| = |\vec{V}_0|\cos\theta$

What direction is the force on the ball after it leaves the cannon? $F_g = mg\,(-\hat{\jmath})$

What happens to the x-component of the velocity while the ball is in the air? constant

If it lands 100m away from the cannon, how much time did it spend in the air?

$t = 100m / V_{0,x}$

What happens to its total energy while it is flying?

Conserved

Think about the types and relative amounts of energy that the ball has at ①, ② and ③.

All the versions of the right hand rule

"Sweeping fingers" version

- used to compute cross products $\vec{A} \times \vec{B} = \vec{C}$
  1. put fingers in direction of $\vec{A}$
  2. rotate palm until it is possible to "sweep" your fingers in the direction of $\vec{B}$
  3. your thumb, sticking straight out, points in the direction of $\vec{C}$
- if you are using this to compute $\vec{F}_B = q\vec{v} \times \vec{B}$, make sure to flip the result if $q < 0$ to get $\vec{F}_B$

"Curly" versions

I. Direction of $\vec{B}$ field from I

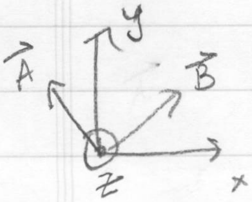
1. place your thumb in the direction of I
2. your fingers naturally "curl" in the direction of $\vec{B}$

II. Direction of I from $\vec{B}$ field

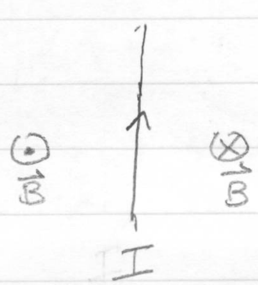
- used when you have a time changing ϕ_B to find the direction of the induced current

1. determine which way you want to "add" $\vec{B}$ field using Lenz's law
2. put your thumb in that direction
3. your fingers will "curl" in the direction of the induced current

Examples



direction of $\vec{C}$ is $(-\hat{k})$



$\frac{dB}{dt} < 0$

$\Rightarrow I_{induced}$ is $I_{induced}$ clockwise