# Introduction to Wave Kinetics

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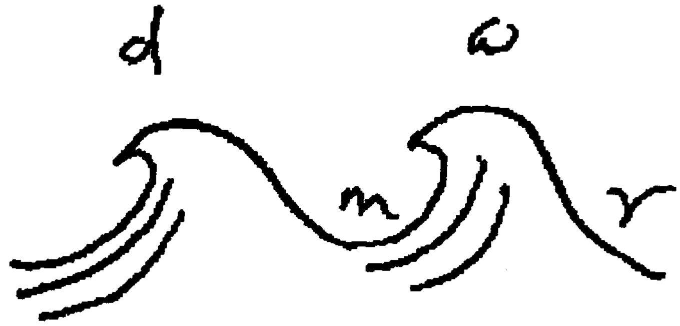
ISS 2007; Chengdu

N.B. I have benefited from discussions with > 50 colleagues on this subject.

# Why Wave "Kinetics"?

" A wave is never found alone, but is mingled with as many other waves as there are uneven places in the object where the said wave is produced. At one and the same time there will be moving over the greatest wave of a sea innumerable other waves proceeding in different directions"

— Leonardo da Vinci
Codice Atlantico, c. 1500.

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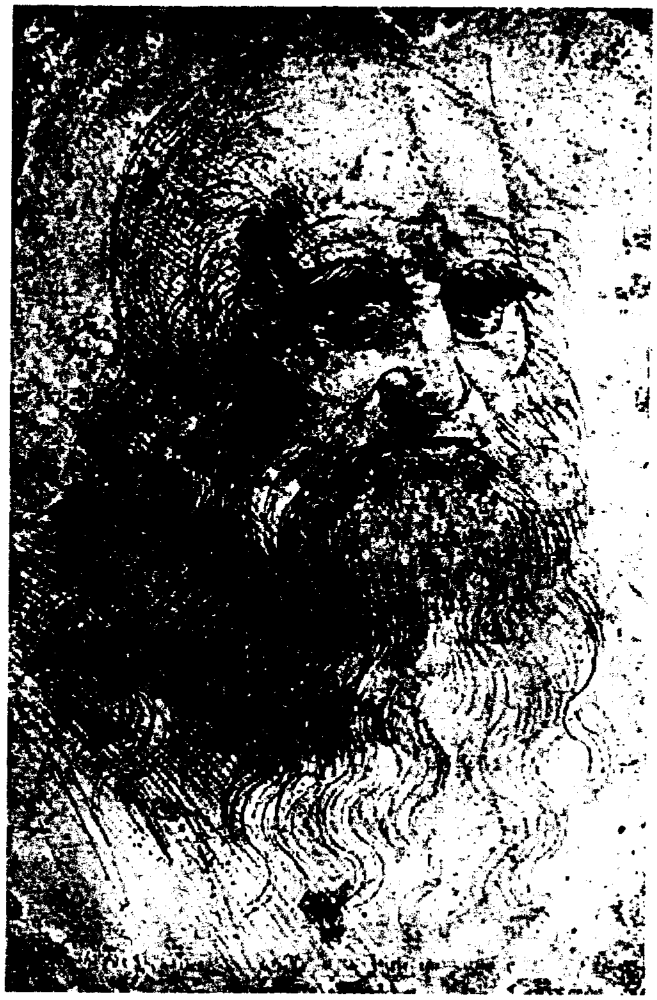
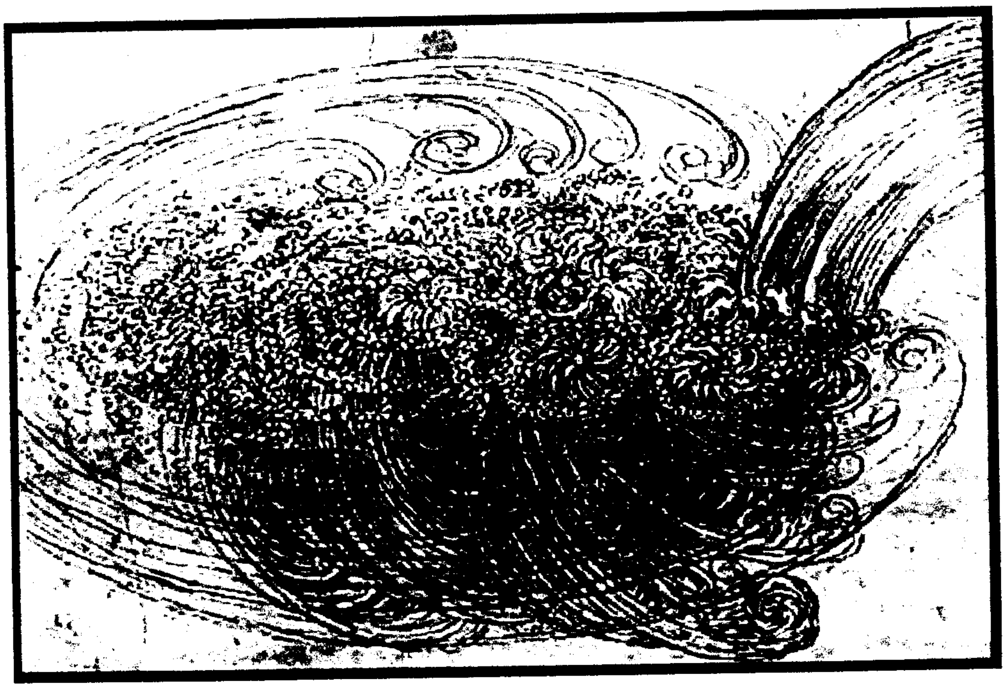
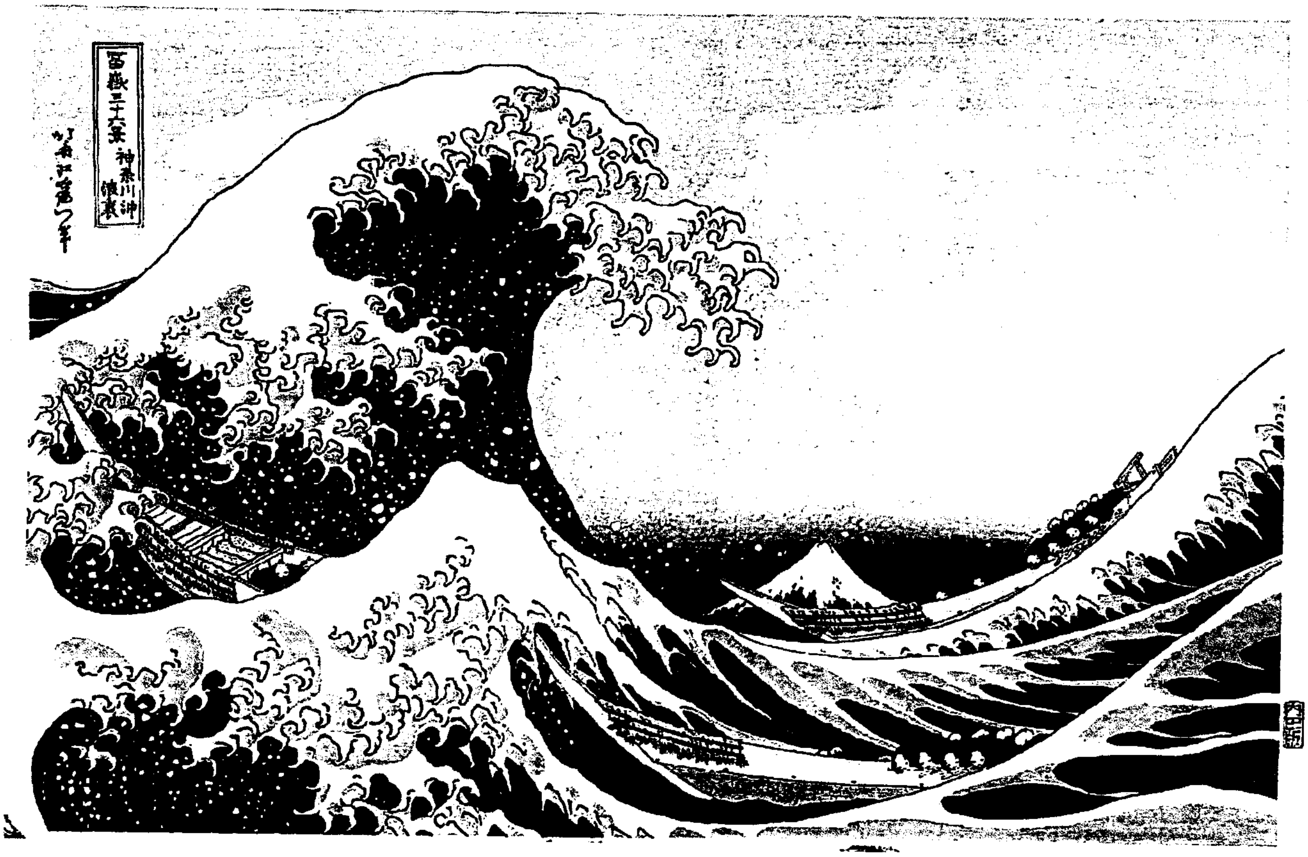


[Leonardo
on waves...]



From Asian art.....

**The great wave at Kanagawa** Hokusai



## Outline

i.) Introduction and Basic Ideas

→ what is wave kinetics?

→ basic structure and limitations?!

ii.) Why Bother? – Utility of Wave Kinetics

→ particle ⊕ quasi-particle picture

applications:

- energetics → conservation laws
- "radiation hydrodynamics" → wave forces
- structure formation
  → Langmuir turbulence
  → zonal flow formation
- non-locality → turbulence spreading

iii.) Basic Theory

→ intuitive arguments

→ outline of Whitham derivation
(see supplement I)

→ full wave kinetics

→ special items: drift waves, 2D
fluids, etc.

iv.) Applications I → Conservation Relations

→ Energetics: simple and
not-so-simple . . . .

→ Parallel Momentum

v.) Applications II → Structure Formation
(Introduction)

→ Langmuir Turbulence
(see supplement II)

→ Zonal Flow Formation

## i.) Introduction and Basic Ideas

→ Wave kinetics or Quasi-Particle Picture

- Boltzmann-like equation for wave population

$$\frac{dN}{dt} = C(N) \; ; \qquad N(\underline{k}, \underline{x}, t) \equiv \left\{ \begin{array}{l} \text{population} \\ \text{density} \end{array} \right.$$

$$\underbrace{\frac{\partial N}{\partial t} + (\underline{v}_{gr} + \underline{V}) \cdot \underline{\nabla} N - \frac{\partial}{\partial \underline{x}} (\omega + \underline{k} \cdot \underline{V}) \cdot \underline{\nabla}_k N}_{LHS}$$

$$= \underbrace{C(N)}_{RHS}$$

a la' Boltzmann

LHS: $\left. \frac{dN}{dt} \right|_{rays}$

characteristics

$$\left. \begin{array}{l} \dfrac{dx}{dt} = \underline{v}_{gr} + \underline{V} \\[2ex] \dfrac{dk}{dt} = -\dfrac{\partial}{\partial \underline{x}} (\omega + \underline{k} \cdot \underline{V}) \end{array} \right\}$$

eikonal equations

⇒ ray trajectories for waves

for LHS (cont'd): short $\lambda$, high $\omega$ wave $\longleftrightarrow$ long $\lambda$ low $\omega$ modulator

eikonal theory $\leftrightarrow$ <u>scale separation</u>

$\tilde{\omega}, \tilde{V} \rightarrow$ modulation field with scales $\rightarrow$ $\begin{cases} \underline{q} \rightarrow \text{wave vector} \\ \Omega \rightarrow \text{frequency} \end{cases}$

$\therefore$ LHS $\downarrow$ disparate scale interaction $\begin{cases} \omega_{\underline{k}} > \Omega, \ \underline{q} \cdot \underline{V}, |\underline{V}|; \text{ etc.} \\ |\underline{k}| > |\underline{q}|; \text{ etc.} \end{cases}$

<u>if</u> $C(N) \rightarrow 0$ and $\omega > \Omega$, etc.
$\rightarrow$ adiabatic invariance....

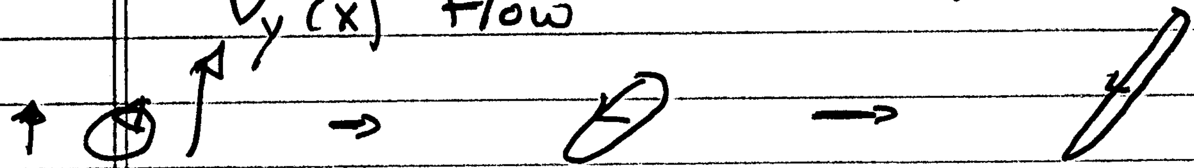
$\frac{dN}{dt} = 0 \Rightarrow N \equiv$ wave action density $= \mathcal{E}_{\underline{k}} / \omega_{\underline{k}}$

$\mathcal{E}_{\underline{k}}$ $\rightarrow$ energy density

$\frac{dN}{dt} = 0$ $\rightarrow$ $\begin{cases} \text{wave population} \\ \text{density conserved} \\ \text{along rays.} \end{cases}$

$\Rightarrow$ tells how wave population responds to modulation field.

i.e. classic example $\rightarrow$ shearing (coherent) $V_y(x)$ flow



$k_x$ increases $\Rightarrow$ $\dfrac{dk_x}{dt} = -\dfrac{\partial}{\partial x}\left(\omega + V_y(x)k_y\right)$
(refraction)

$$\cong -k_y \frac{\partial V_y(x)}{\partial x}$$

how does energy change?

$N = \mathcal{E}/\omega_{\underline{k}}$ , $V_y' < \omega_{\underline{k}}$

$\Rightarrow N \cong N_0$ const
adiabatic invariant

$\Rightarrow$

$$\frac{d\mathcal{E}}{dt} = \frac{d}{dt}\left(N\omega_{\underline{k}}\right) = N_0 \frac{\partial \omega}{\partial \underline{k}} \cdot \frac{d\underline{k}}{dt}$$

$$= N_0 V_{gr_x} \frac{dk_x}{dt}$$

$$= N_0 V_{gr_x}\left(-k_y \frac{\partial V_y(x)}{\partial x}\right)$$

$\dfrac{d\mathcal{E}}{dt} \gtrless 0 \rightarrow$ sgn: $-V_{gr}k_y V_y'$

$$\sim \frac{k_y^2 k_x V_y' \rho_s^2 V_*}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

stochastic $\Rightarrow$ Zonal flows (later)

RHS → <u>non</u> Action-conserving wave interactions

<u>i.e.</u>

$$C(N) = \sum_{\substack{\underline{k}', \underline{k}'' \\ \oplus = k}} \left\{ C_1(\underline{k}', \underline{k}'') N_{\underline{k}'} N_{\underline{k}''} - C_2(\underline{k}, \underline{k}') N_{\underline{k}'} N_{\underline{k}} \right\} * \delta(\omega_{\underline{k}''} - \omega_{\underline{k}} - \omega_{\underline{k}'})$$

effective collision operator
(non-adiabatic wave interactions) → topic for another lecture

so obvious analogy:

$$\frac{\partial N}{\partial t} + (\underline{v}_{gr} + \underline{V}) \cdot \underline{\nabla} N - \frac{\partial}{\partial \underline{x}}(\omega + \underline{k} \cdot \underline{V}) \cdot \nabla_k N = C(N)$$

LHS → N conserving

wave-wave interaction

↕

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \underline{\nabla} f + \frac{q}{m} \underline{E} \cdot \frac{\partial f}{\partial \underline{v}} = C(f)$$

LHS → f conserving

particle collisions

$$\therefore \boxed{N \equiv \text{quasi-particle density}}$$

so: Wave kinetics:

→ Boltzmann-like equation for wave population density (action density → usually) $N$

→ LHS → wave-mean interactions (slow)
RHS → wave-wave interaction (break adiabatic invariance)

→ useful when: scale separation between mean, waves

→ useful because: can apply tricks from particle kinetics to wave population.
i.e. if $C(N) = -\nu_{eff}(N - \langle N \rangle)$

how calculate flux of wave momentum if $\langle N \rangle = \langle N(x) \rangle$ ?
(radiation pressure)
⇒ "radiation hydrodynamics":

see: { Mihalas and Mihalas
Zeldovich and Raizer

→ useful because: natural way to formulate conservation laws of quasi-linear theory

→ useful because: natural way to fomulate structure formation →

- Langmuir turbulence
- zonal flow formation