Physics 214 UCSD/225a UCSB

Lecture 7

Finish Chapter 2 of H&M

- November revolution, charm and beauty

CP symmetry and violation

- Simple example
- Unitarity matrix for leptons and quarks

Beginning of Neutrino Physics

We’ll skip some stuff in Chapter 2

- Magnetic moment of proton etc.
- November revolution
  - Charm
  - Beauty
  - OZI suppression
- I encourage you to read up on this in chapter 2 of H&M

CP Symmetry

- So far, we talked about charge conjugation as the symmetry between particle and antiparticle.
- Well, that was good enough for QCD, but makes no sense in weak interactions.
- For weak interactions we need a simultaneous flip of Charge and Parity, or CP conjugation.
- We will discuss this in detail next quarter. Today, we simply introduce some basics, and introduce CP violation.

Simple Example

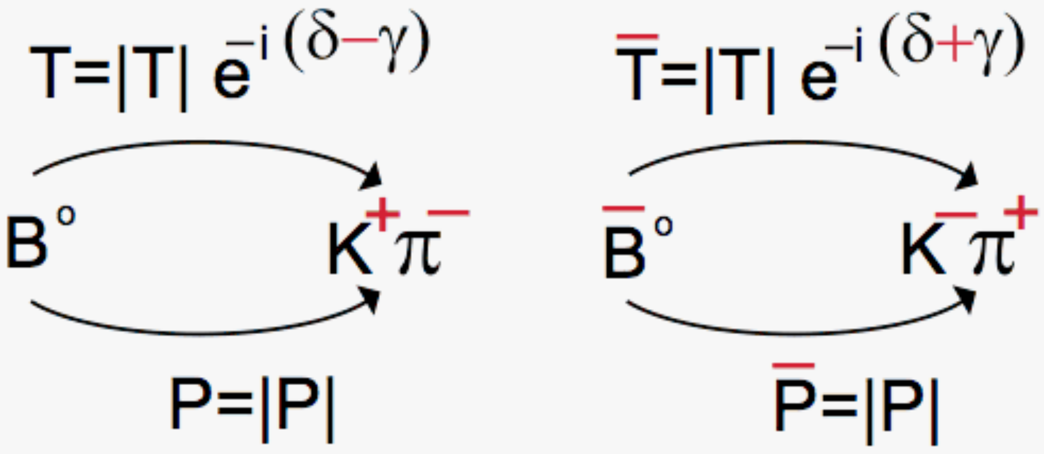
CP ($\overline{B^0}$ -> $K^-\pi^+$) = B^0 -> $K^+\pi^-$

b anti-d Meson　　anti-b d Meson

Theory: If the partial widths for these two decays are not the same, then CP is violated.

Experiment: if the branching fractions for these two decays are not the same then CP is violated.

Breaking CP is easy



⇒*Add complex coupling to Lagrangian.*

⇒*Allow 2 or more channels*

⇒*Add CP symm. Phase, e.g. via dynamics.*

$\delta =$ strong phase shift

$\gamma =$ difference in weak phase

CP $\gamma = -\gamma$ CP $\delta = +\delta$

T,P are real numbers.

$$A_{cp} = \frac{\mathcal{B}(B^0\to K^+\pi^-)-\mathcal{B}(\bar{B}^0\to K^-\pi^+)}{\mathcal{B}(B^0\to K^+\pi^-)+\mathcal{B}(\bar{B}^0\to K^-\pi^+)} = \frac{\left|P+Te^{-i(\delta-\gamma)}\right|-\left|P+Te^{-i(\delta+\gamma)}\right|}{\left|P+Te^{-i(\delta-\gamma)}\right|+\left|P+Te^{-i(\delta+\gamma)}\right|}$$

$$= \frac{-2|TP|\ \sin\gamma\ \ \sin\delta}{|T|^2+|P|^2+2|TP|\cos\gamma\ \ \cos\delta}$$

The rest is simple algebra.

CP Violation in Standard Model

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}$$

$$J^+_\mu = (\bar{\nu}_{eL}\bar{\nu}_{\mu L}\bar{\nu}_{\tau L})\gamma_\mu \begin{pmatrix} e^-_L \\ \mu^-_L \\ \tau^-_L \end{pmatrix} + (\bar{u}_L\bar{c}_L\bar{t}_L)\gamma_\mu \mathbf{V}_{\mathbf{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Note:

-> V_{CKM} is a *3x3 unitary matrix of couplings* .

-> It provides the complex coupling in the Lagrangian to allow for CP violation.

We'll get back to the details next quarter.

Breaking CP in Standard Model

- Where does the CP violating phase come from?
  - 3x3 unitary matrix => 3 angles + 6 phases
    - $2N^2$ parameters, N^2 constraints from unitarity
  - 6 spinors with arbitrary phase convention
    - Only relative phase matters because only $|M|^2$ is physical.

    ⇒Only 5 phases can be used to define a convention.

  ⇒ One phase left in 3x3 matrix that has physical consequences.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_x c_z & s_x c_z & s_z e^{-i\phi} \\ -s_x c_y - c_x s_y s_z e^{i\phi} & c_x c_y - s_x s_y s_z e^{i\phi} & s_y c_z \\ s_x s_y - c_x c_y s_z e^{i\phi} & -c_x s_y - s_x c_y s_z e^{i\phi} & c_y c_z \end{pmatrix}$$

x,y,z are Euler angles. c=cos, s=sin.

Note: sin(z) = 0 <=> NO CP violating phase left !!!

CP violation summary

- CP violation is easy to add in field theory:
  - Complex coupling in Lagrangian
  - Interference of channels with:
    - Different CP violating phase
    - Different CP conserving phase
- Standard Model implements this via:
  - CP violating phase in charged current coupling across 3 families
  - CP conserving phase via:
    - Dynamics, e.g. Breit Wigner resonance lineshape
    - Flavor Mixing & oscillation in neutrino or quark sector
  - The scale of CP violation allowed in the standard model quark sector is well measured, and not sufficient for cosmology.

Let's look at neutrino sector in some detail !

Aside: Scale of CP violation

- Standard model allows for CP violation only in the quark sector.
- The existence of neutrino oscillations requires extending the standard model to include CP violation in the lepton sector.
- In hw4 problem 1c you will calculate what is sometimes called the “Jarlskog Invariant”. You will show that the scale of CP violation allowed depends on the product of 4 sin terms:
  - The three family mixing angles
  - The CP violating phase

Mixing in Standard Model

- Weak eigenstates not equal mass eigenstates.
  - Mass eigenstates responsible for propagation in time.
    - Eigenstates of the Hamiltonian (excluding decay) are the mass eigenstates.
  - Weak eigenstates responsible for production and/or decay.

⇒ Oscillation between weak eigenstates as a function of time.

⇒ Discuss this in detail for Neutrino sector now.

Neutrino mixing in vacuum

- At the W vertex an electron-neutrino is created together with a positron.
- That electron-neutrino is a superposition of mass eigenstates:

$$\left|\nu_e(t)\right\rangle = \sum_{i=1}^{3} U_{ei}^{*} \left|\nu_i(t)\right\rangle$$

- The time evolution of the mass eigenstate can be described either in its rest-frame or in the labframe:

$$\left|\nu_i(t)\right\rangle = e^{-i m_i t_i} \left|\nu_i(0)\right\rangle = e^{-i(E_i t - p_i L)} \left|\nu_i(0)\right\rangle$$

- For interference among the mass eigenstates to be possible, they all have to have the same E because experimentally we average over time.

Interference for same energy states

- A neutrino beam does of course include a spectrum of neutrino energies.
- Each neutrino acquires a phase factor according to its energy: $e^{-iE_j t}$
- Two neutrinos with differing energies will thus have a relative phase factor of: $e^{-i(E_j - E_k)t}$
- As the time between production and detection of the neutrino has a large spread, this phase factor leads to no observable interference.
- As a result, only states of equal energy contribute to the interference effect that is observed.

Oscillation Amplitude

$$Amp(\nu_\mu \rightarrow \nu_\tau) = \left\langle \nu_\tau \middle| e^{-iEt} \sum_{i=1}^{3} e^{ip_i L} U_{\mu i}^* \middle| \nu_i \right\rangle$$

$$Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iEt} \sum_{i,j=1}^{3} e^{ip_i L} U_{\mu i}^* U_{\tau j} \left\langle \nu_j \middle| \nu_i \right\rangle$$

$$Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iEt} \sum_{i=1}^{3} e^{ip_i L} U_{\mu i}^* U_{\tau i}$$

Next we taylor expand p_i using:

$$p_i = \sqrt{E^2 - m_i^2} = E - \frac{m_i^2}{2E} + \ldots$$

Oscillation Probability

$$Amp(\nu_\mu \to \nu_\tau) = e^{-iE(t-L)} \sum_{i=1}^{3} e^{-i\frac{m_i^2}{2E}L} U_{\mu i}^* U_{\tau i}$$

$$\Pr ob(\nu_\mu \to \nu_\tau) = \left| Amp(\nu_\mu \to \nu_\tau) \right|^2 = \left| \sum_{i=1}^{3} e^{-i\frac{m_i^2}{2E}L} U_{\mu i}^* U_{\tau i} \right|^2$$

In homework, you do this for the general case of N flavors.
Here we do it for the simpler case of 2 flavors only.
Note: For 2 flavors the U_{jk} are real, not imaginary.

Simple math aside

$$\left|1-e^{ix}\right|^2 = \left(1-[\cos x + i\sin x]\right)\left(1-[\cos x - i\sin x]\right)$$

$$= [1-\cos x]^2 + \sin^2 x$$

$$= 2(1-\cos x)$$

We’ll need this in a second.

2 flavor oscillation probability

$$\left|U_{11}U_{21}e^{-im_1^2\frac{L}{2E}}+U_{12}U_{22}e^{-im_2^2\frac{L}{2E}}\right|^2=\left|U_{11}U_{21}+U_{12}U_{22}e^{i(m_1^2-m_2^2)\frac{L}{2E}}\right|^2$$

$$=\left|-\cos\theta\sin\theta+\cos\theta\sin\theta e^{i(m_1^2-m_2^2)\frac{L}{2E}}\right|^2=\cos^2\theta\sin^2\theta\left|1-e^{i(m_1^2-m_2^2)\frac{L}{2E}}\right|^2$$

$$=\cos^2\theta\sin^2\theta\left[(1-\cos\Delta)^2+\sin^2\Delta\right]=2\cos^2\theta\sin^2\theta[1-\cos\Delta]$$

$$=\frac{1}{2}\sin^2 2\theta\left[2\sin^2\frac{\Delta}{2}\right]$$

$$\Delta=(m_1^2-m_2^2)\frac{L}{2E}$$

This is a bit simplistic, as it ignores matter effects.
We’ll discuss those in next lecture.

Discussion of Oscillation Equation

$$\Pr ob(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \left[\sin^2 \frac{(m_1^2 - m_2^2)L}{4E} \right]$$

- Depends on difference in mass squared.
  - No mixing if masses are identical
  - Insensitive to mass scale
  - Insensitive to mass hierarchy because $\sin^2 x = \sin^2(-x)$
- Depends on $\sin^2(2\theta)$
  - Need large mixing angle to see large effect
- Depends on L/4E
  - Exp. with unfortunate L/E won't see any effect.
  - Exp. with variable L/E can measure both angle and mass squared difference.
  - Exp. with Δm^2 L/4E >>1 and some energy spread average over $\sin^2$ -> 1/2

Experimental situation

- Sources of electron neutrinos
  - – Sun
  - – Reactors

- Sources of muon neutrinos
  - – From charged pion beams
    - Protons on target gives charged pions.
  - – From charged pion decay in atmosphere

Atmospheric neutrinos

- Expect ν_μ anti-ν_μ in equal numbers
- Expect ν_e half as many as ν_μ or anti-ν_μ
- Can change L as a function of Zenith angle. (L ~ 15km to L ~ 13,000km)
- ν_e Oscillation to ν_μ
  => See excess of ν_μ vs zenith angle
- ν_μ Oscillation to ν_e
  => See excess of ν_e vs zenith angle
- ν_e Oscillation to ν_τ
  => Deficit of ν_e vs zenith angle
- ν_μ Oscillation to ν_τ
  => Deficit of ν_μ vs zenith angle

Let's walk through these arguments one by one!

Atmospheric neutrinos

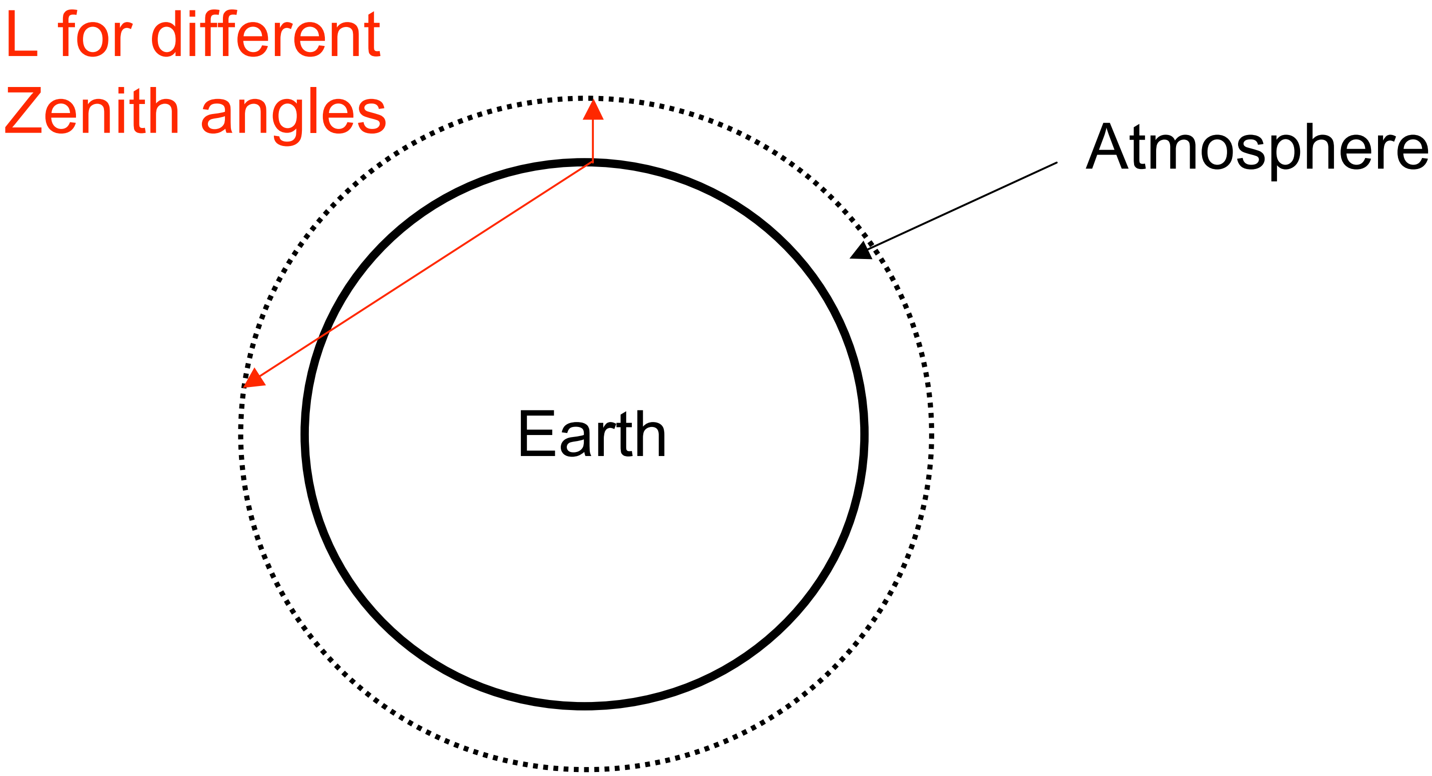
- Expect ν_μ anti-ν_μ in equal numbers
  - There are equal number of π^+ and π^- produced in hadronic collisions in the atmosphere.
    - π^+ decays to $\mu^+ \nu_\mu$
    - π^- decays to μ^- anti-ν_μ
- Expect ν_e half as many as ν_μ or anti-ν_μ
    - π^+ decays to $\mu^+ \nu_\mu$
    - μ^+ decays to anti-ν_μ + e^+ + ν_e
    - And accordingly for CP conjugate

For every ν_e there is one ν_μ and one anti-ν_μ
because of anti-muon decay chain.

For every anti-ν_e there is one ν_μ and one anti-ν_μ
because of muon decay chain.

Atmospheric neutrinos

- Can change L as a function of Zenith angle. (L ~ 15km to L ~ 13,000km)



L for different Zenith angles

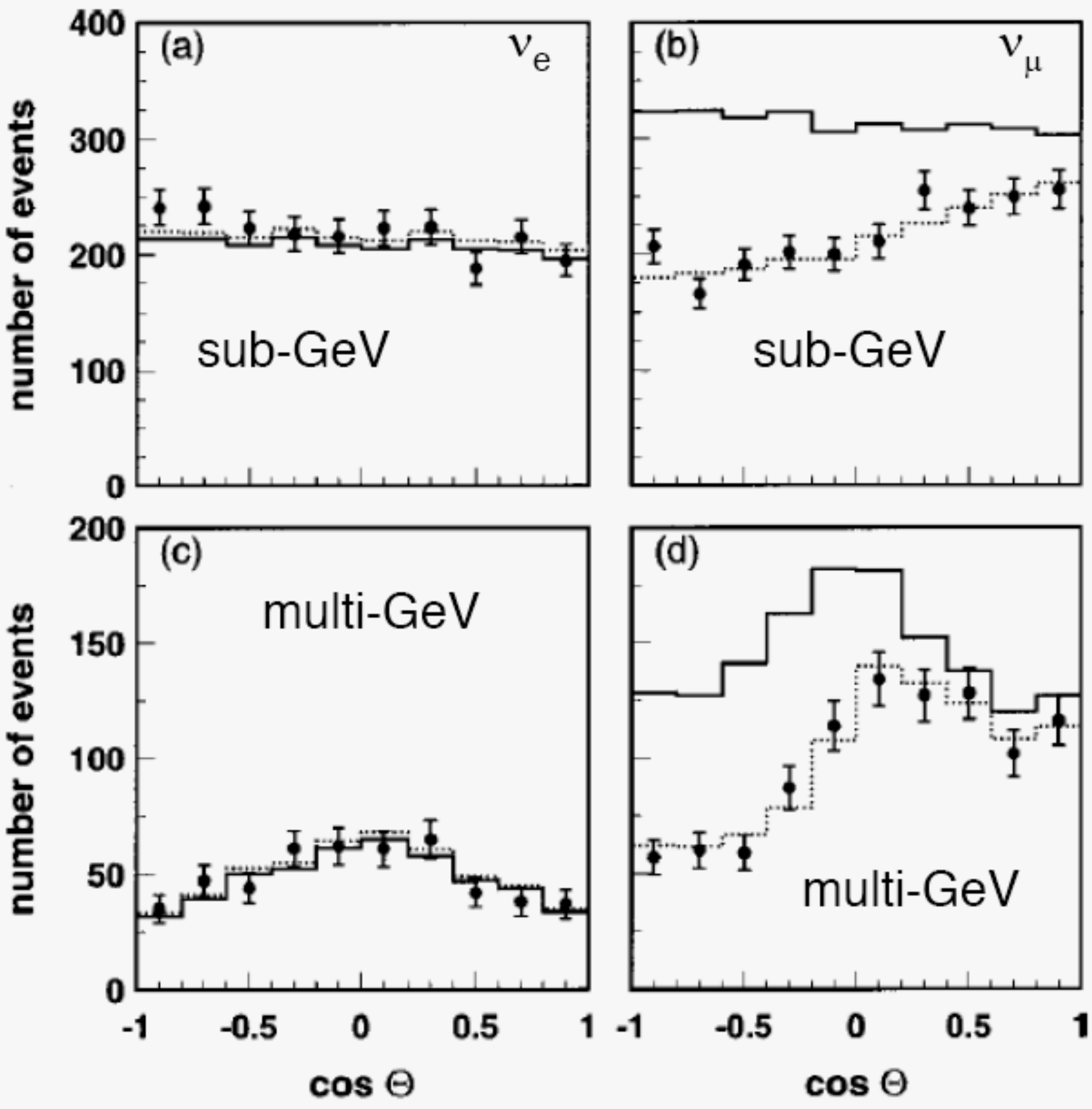
Atmosphere

Earth

Atmospheric neutrinos

- ν_e Oscillation to ν_μ
  - ⇒ See excess of ν_μ vs zenith angle
    - ⇒ L depends on zenith angle.
    - ⇒ only specific L gives you maximal interference effect for a given E
    - ⇒ deficit of ν_e and excess of ν_μ at the appropriate zenith angle.
- ν_μ Oscillation to ν_e
  - => See excess of ν_e vs zenith angle
- ν_e Oscillation to ν_τ
  - => Deficit of ν_e vs zenith angle but no excess of ν_μ
- ν_μ Oscillation to ν_τ
  - => Deficit of ν_μ vs zenith angle but no excess of ν_e

Super Kamiokande Results



cosθ=-1 up-going,
L≈13,000 km

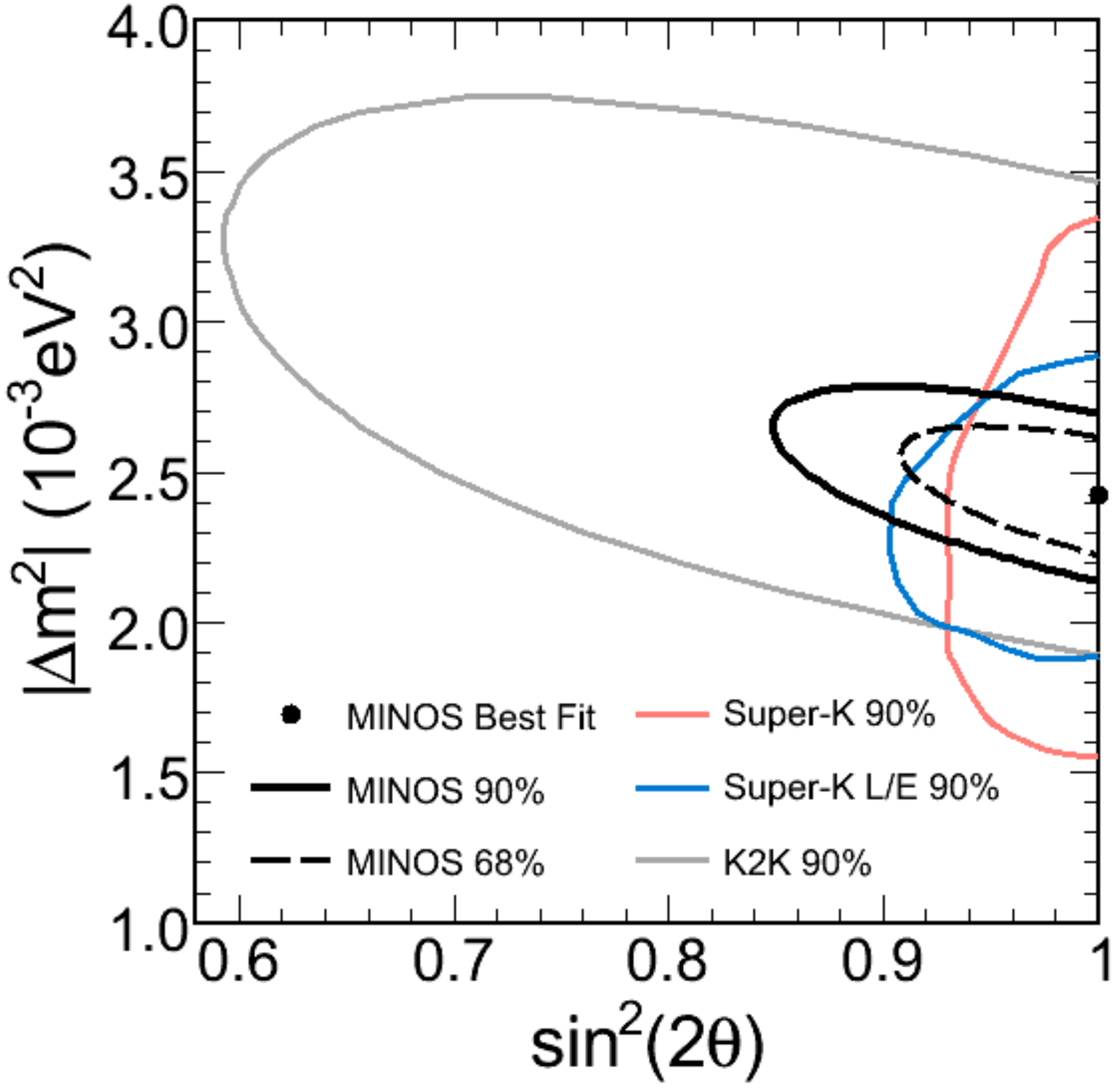
cosθ=1 down-going,
L≈15 km

Left 2 plots:

ν_e as expected

Right 2 plots:

ν_μ deficit largest at large L



Interpreted as
ν_μ -> ν_τ

i.e. 23 mixing.

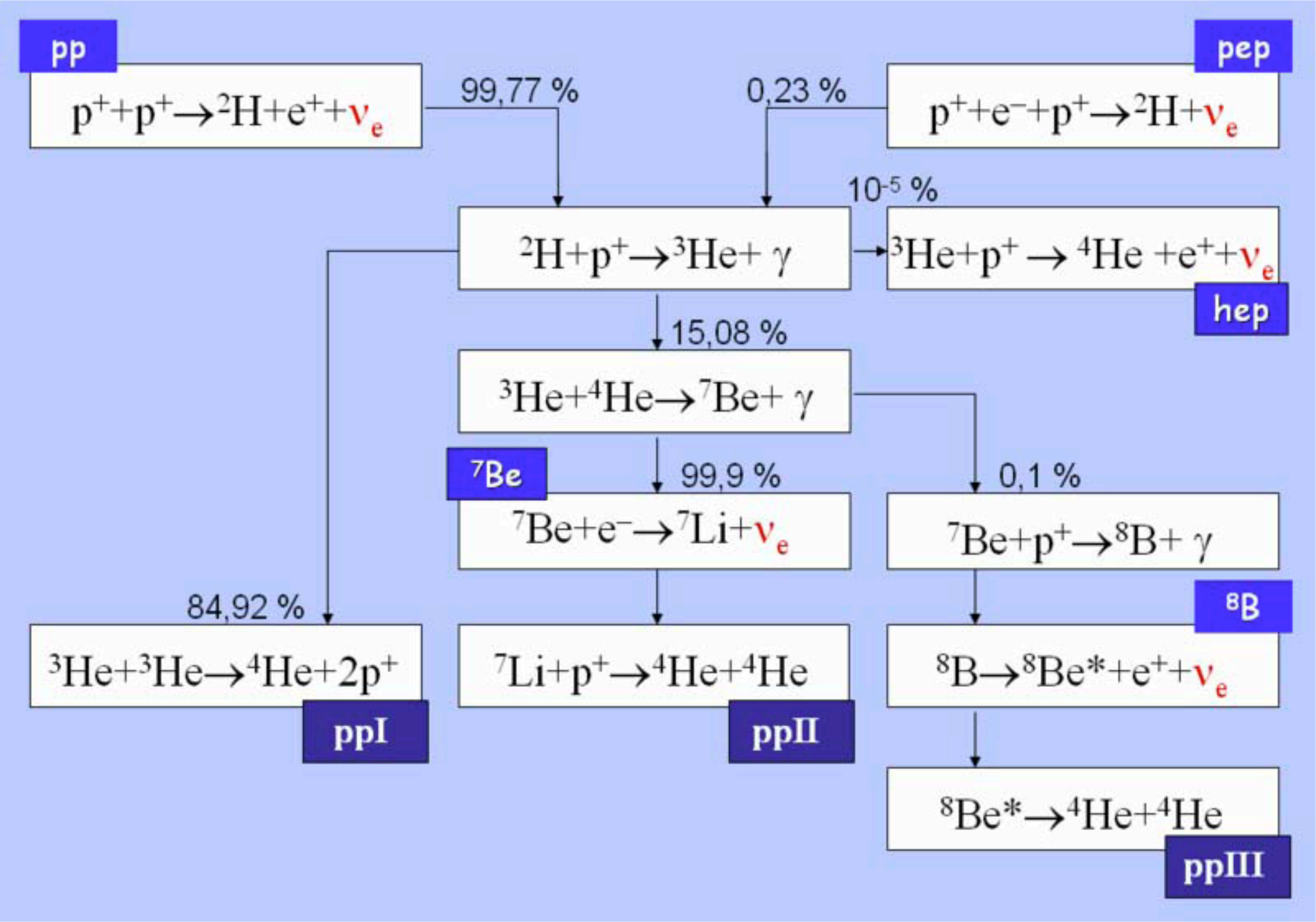
Latest Result from MINOS

P. Adamson *et al*, Phys. Rev. Lett.**101**: 131802 (2008)

Neutrinos from the Sun

- Many mechanisms, all leading to electron neutrinos with varying energies.
  - Expect: 0.5 $\sin^2(2\theta)$ of solar model flux convolved with energy dependent efficiency if electron neutrino oscillations exist.
- Neutrino energy too low to produce muons or taus.
  - For decades, all experiments could measure was a neutrino flux half that of the solar neutrino model prediction.
    - People did not trust the model nor experiments as both are quite complicated!
  - Super Kamiokande was first to have pointing accuracy, and thus show that flux came from sun.
  - SNO showed that total neutrino flux agrees with solar model, and electron neutrino flux is short by factor 2.

Solar Model is Quite Complex



pp

$p^+ + p^+ \rightarrow {}^2H + e^+ + \nu_e$

99,77 %

pep

$p^+ + e^- + p^+ \rightarrow {}^2H + \nu_e$

0,23 %

${}^2H + p^+ \rightarrow {}^3He + \gamma$

10^{-5} %

${}^3He + p^+ \rightarrow {}^4He + e^+ + \nu_e$

hep

15,08 %

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

99,9 %

⁷Be

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

0,1 %

${}^7Be + p^+ \rightarrow {}^8B + \gamma$

84,92 %

${}^3He + {}^3He \rightarrow {}^4He + 2p^+$

ppI

${}^7Li + p^+ \rightarrow {}^4He + {}^4He$

ppII

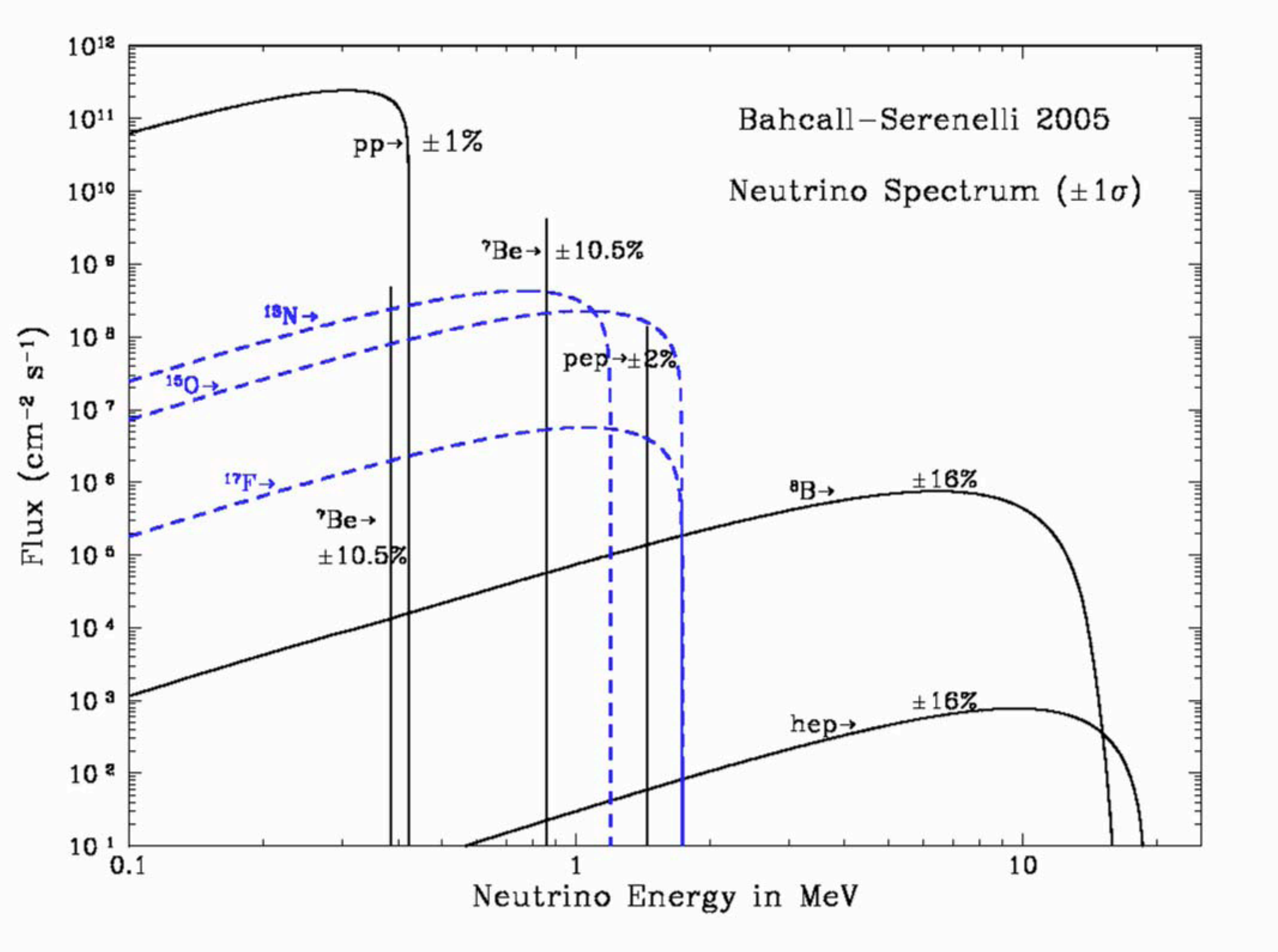
⁸B

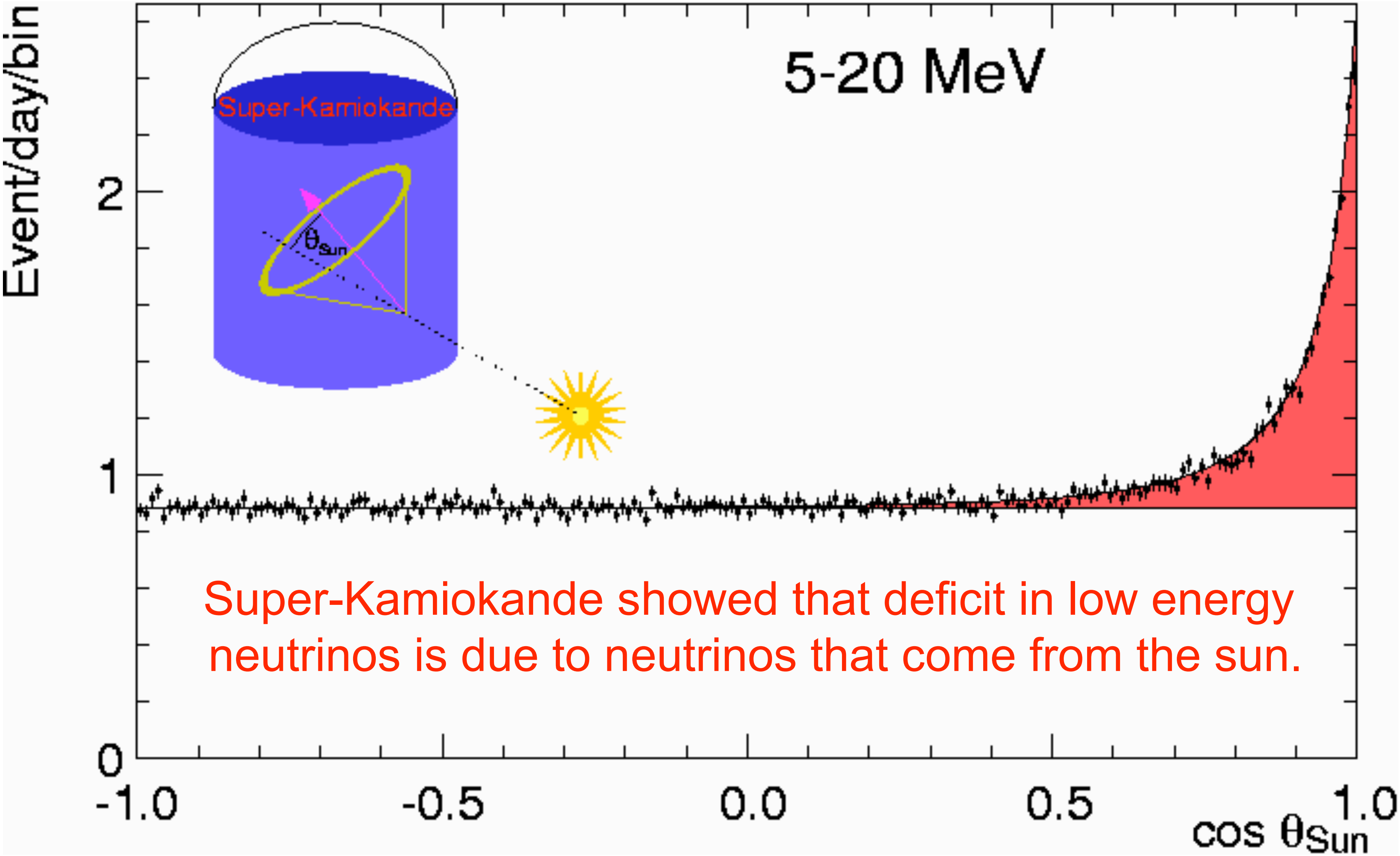
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \rightarrow {}^4He + {}^4He$

ppIII

Neutrino Energies are quite small
Very Challenging Experimentally for many decades





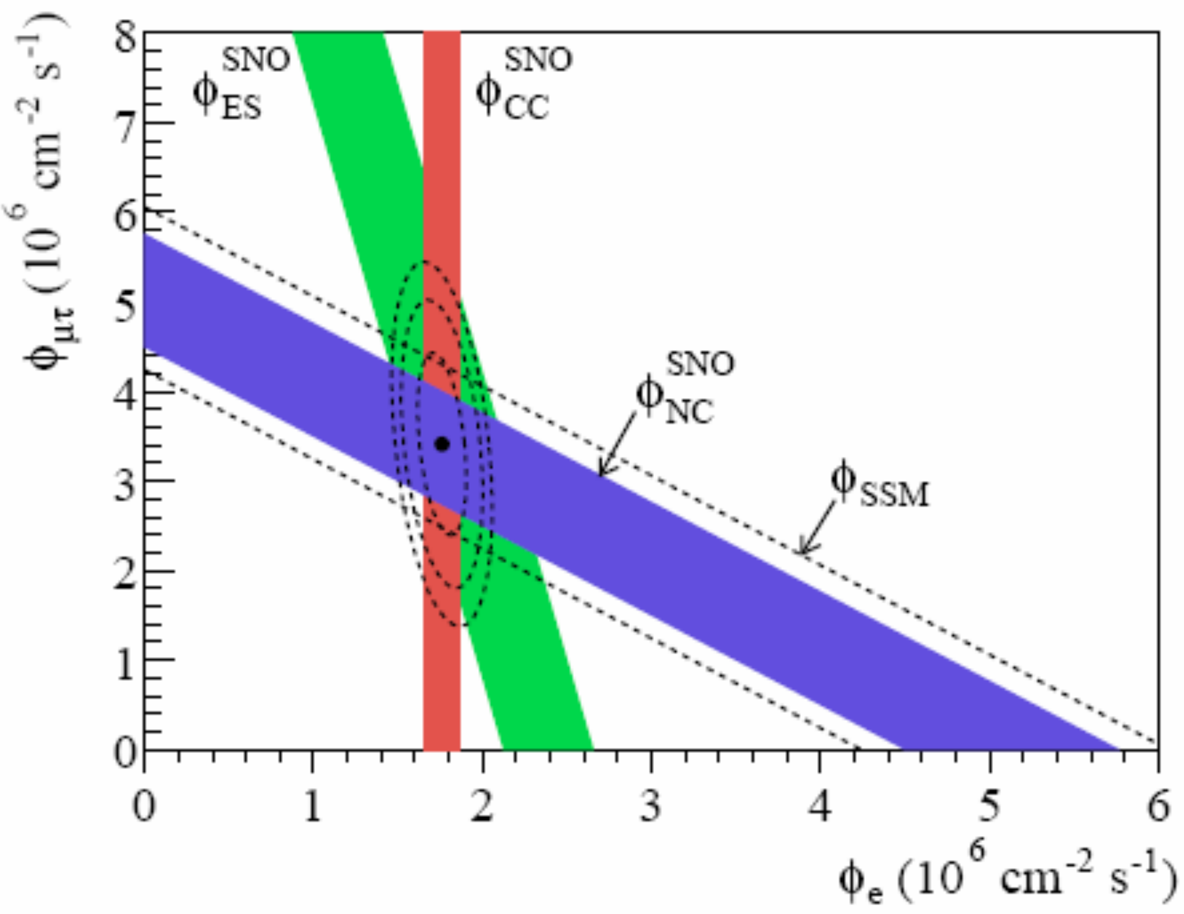
Super-Kamiokande showed that deficit in low energy neutrinos is due to neutrinos that come from the sun.

$$0.465 \pm 0.005^{+0.016}_{-0.015} \text{ of expectation}$$

SNO allowed CC and NC, and was thus sensitive to all neutrino flavors => measures solar flux and electron neutrino flux.

$$
\begin{array}{llll}
\nu_e + \mathrm{d} & \rightarrow & \mathrm{p} + \mathrm{p} + \mathrm{e}^- & \mathrm{(CC)}, \\
\nu_x + \mathrm{d} & \rightarrow & \mathrm{p} + \mathrm{n} + \nu_x & \mathrm{(NC)}, \\
\nu_x + \mathrm{e}^- & \rightarrow & \nu_x + e^- & \mathrm{(ES)}.
\end{array}
$$

Total flux



Interpreted as

ν_e -> ν_μ

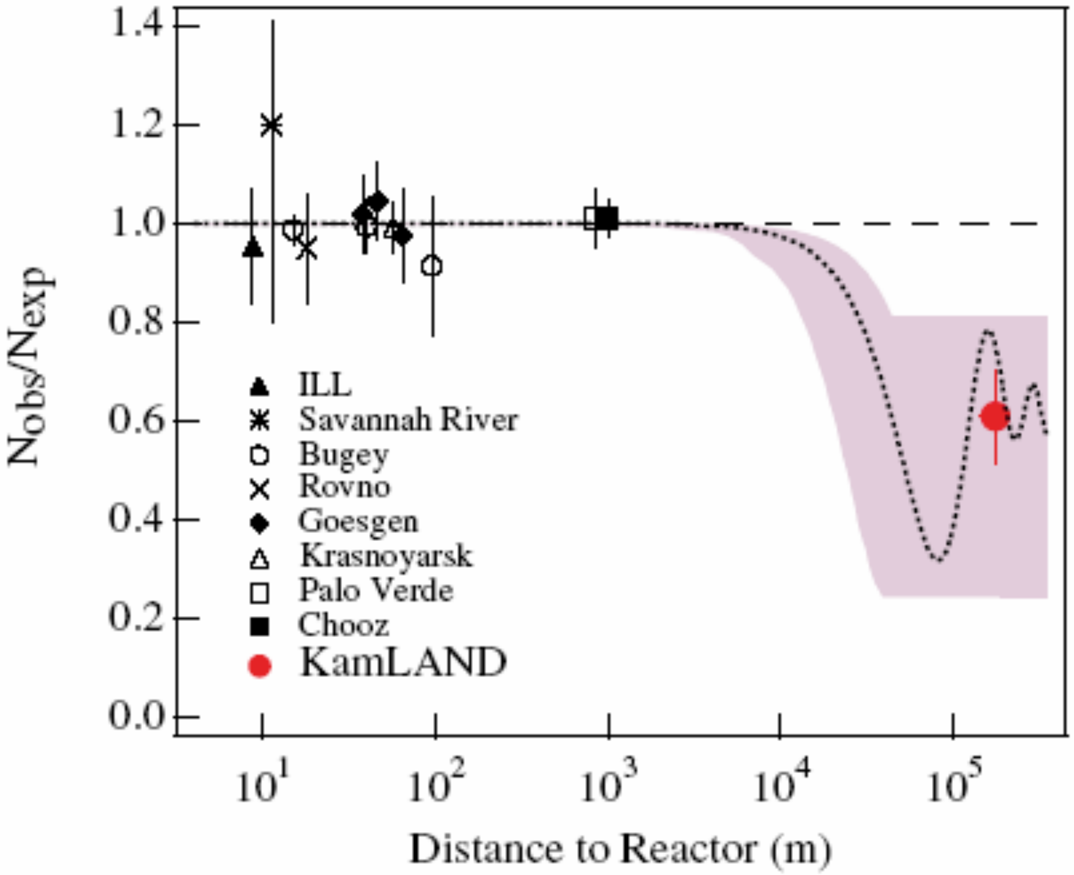
i.e. 12 mixing

Reactor Experiments

All except KamLAND had L that is too small!

⇒ Only KamLAND saw oscillations !!!

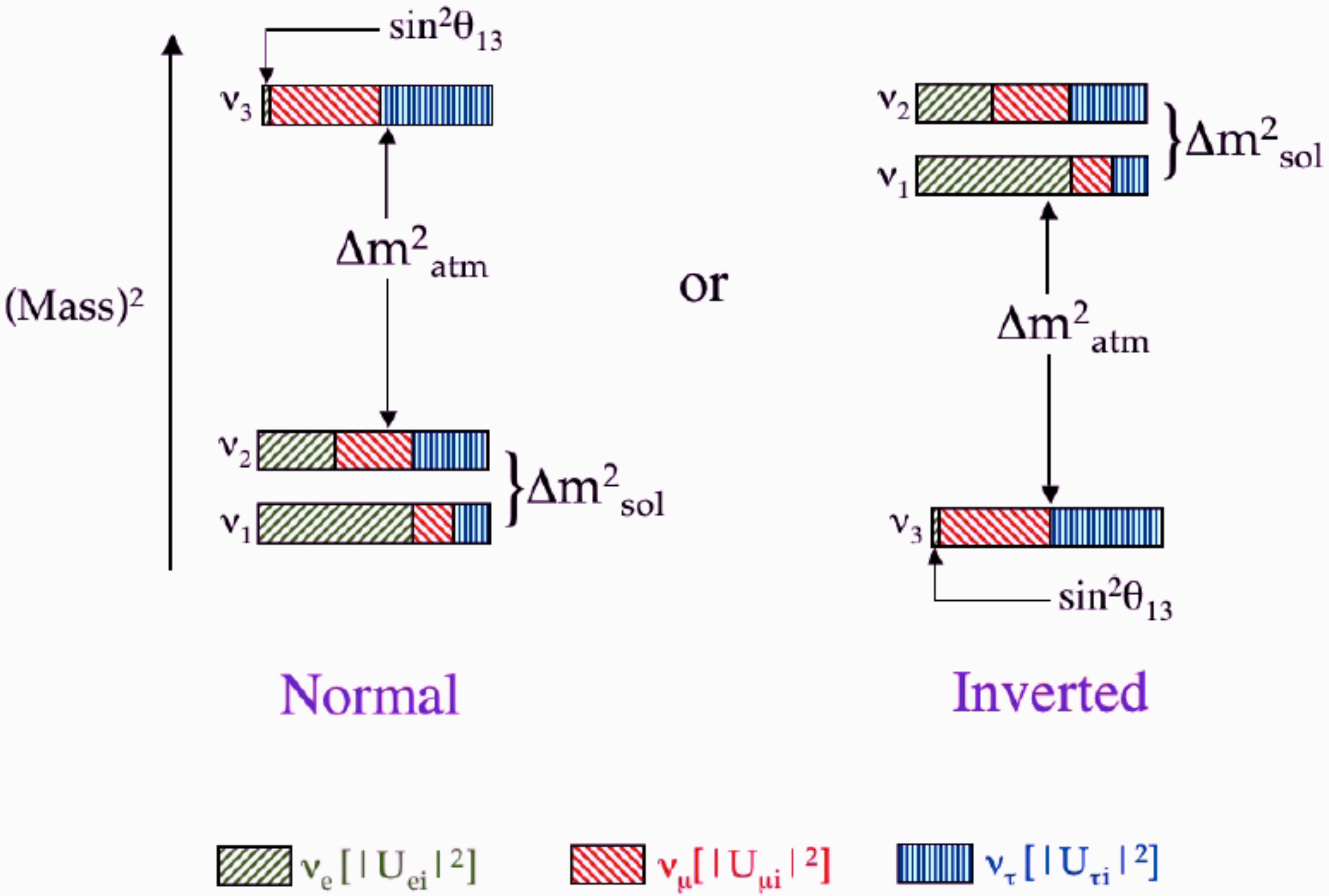
⇒ KamLAND established ν_e disappearance



Interpretation

- Atmospheric must be ν_μ -> ν_τ
  - Though tau appearance has never been seen.
    - New experiment called Opera at Gran Sasso designed to observe tau appearance.
  - However, electron appearance is ruled out.
  - The state that is far in mass from the other two must have very little electron neutrino content!

Two Possible Mass Hierarchies



Things we have not discussed yet.

- Majorana Neutrinos -> see homework
- "Size of CP violation" -> see homework
- Getting well collimated E via off-axis -> see homework
- Reactor neutrinos and sintheta13 -> see homework
- Resolving the mass hierarchy -> Next lecture.