Physics 214 UCSD/225a UCSB

Lecture 15

- Finish off from yesterday
- Parton Density Functions
  - What do they look like?
  - Some processes that measure them.
- parton-parton luminosity
  - how to calculate it.
  - some crude scale factors for LHC vs Tevatron, and how they derive themselves from the PDFs.

The parton picture of the proton

- Proton is made up of some set of partons.
  - Some of which are charged
  - Others aren't.
- Each parton carries a fraction, x, of the momentum.

	Proton	Parton
Energy	E	xE
Momentum	p_L	x p_L
	$p_T = 0$	$p_T = 0$
Mass	M	xM

- All fractions add up to 1:

$$\sum_i \int dx \quad x f_i(x) = 1$$

Incoherence assumption (1)

- “Natural” frame of reference for scattering is the center of mass frame of e-p.
- In that frame, the valence quarks are relativistic => time dilation guarantees that gluon exchange between them (i.e. in the proton restframe) are slooooow.
  - Note: This is no different than the fact that an unstable particle lives longer when viewed from a frame in which it is moving with speed close to c.

Incoherence assumption (2)

- dt during which the virtual photon interaction takes place is << than the time for the partons to interact with each other.
  - ⇒We can add probabilities for interacting with each parton, rather than the amplitudes.
  - ⇒This is referred to as the *incoherence assumption*, and implicit in our use of $f_i(x)$:

$$\sum_i \int dx \quad x f_i(x) = 1$$

Recap of parton structure function

- There is only one F(x).
- It is made out of the incoherent sum of *probabilities for finding a given type i of parton at a given x* in the proton:

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x f_i(x)$$

- The experimental problem is thus to *extract $f_i(x)$ from a large variety of measurements*.
- For deep inelastic e-proton, the gluon structure function can be obtained from the requirement that it all adds up. Gluons are the leftovers.

Simple Example for determining structure function for quarks.

- Compare e-proton with e-neutron deep inelastic scattering.

  ⇒This gives us F^{ep} and F^{en} structure function.

$$\frac{1}{x}F^{ep} = \left(\frac{2}{3}\right)^2\left(u^p(x)+\overline{u}^p(x)\right)+\left(\frac{1}{3}\right)^2\left(d^p(x)+\overline{d}^p(x)\right)+\left(\frac{1}{3}\right)^2\left(s^p(x)+\overline{s}^p(x)\right)$$

$$\frac{1}{x}F^{en} = \left(\frac{2}{3}\right)^2\left(u^n(x)+\overline{u}^n(x)\right)+\left(\frac{1}{3}\right)^2\left(d^n(x)+\overline{d}^n(x)\right)+\left(\frac{1}{3}\right)^2\left(s^n(x)+\overline{s}^n(x)\right)$$

We then assume that all sea quark contributions are the same for ep and en. And the valence quark ones are related by isospin.

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$u^p = d^n = u(x)$
$d^p = u^n = d(x)$
$s^p = s^n = s(x)$
$u - \bar{u} = u_v$
$d - \bar{d} = d_v$

=>

$$\frac{1}{x} F_2^{ep}(x) = \frac{1}{9}\left[4u_v(x) + d_v(x)\right] + \frac{12}{9} S(x)$$

$$\frac{1}{x} F_2^{en}(x) = \frac{1}{9}\left[u_v(x) + 4d_v(x)\right] + \frac{12}{9} S(x)$$

Here S(x) refers generically to sea quarks, while 12/9 accounts for the sum of e^2 for u,d,s and their anti-quarks in the sea.

Note: charm and beauty is ignored in this discussion.

Some observations

- Since gluons create the sea q-qbar pairs, one should expect a momentum spectrum at low x similar to bremsstrahlung:

=> $S(x) \to 1/x$ as $x \to 0$ at fixed Q^2 .

=> $F^{en}/F^{ep} \to 1$ as $x \to 0$

=> $F^{en}/F^{ep} \to (u_v + 4d_v)/(4u_v + d_v)$ as $x \to 1$

- Experimentally, we observe:
  $F^{en}/F^{ep} \to 1$ as $x \to 0$ as expected.
  $F^{en}/F^{ep} \to 0.25$ as $x \to 1$ => u_v appears to dominate at high x.

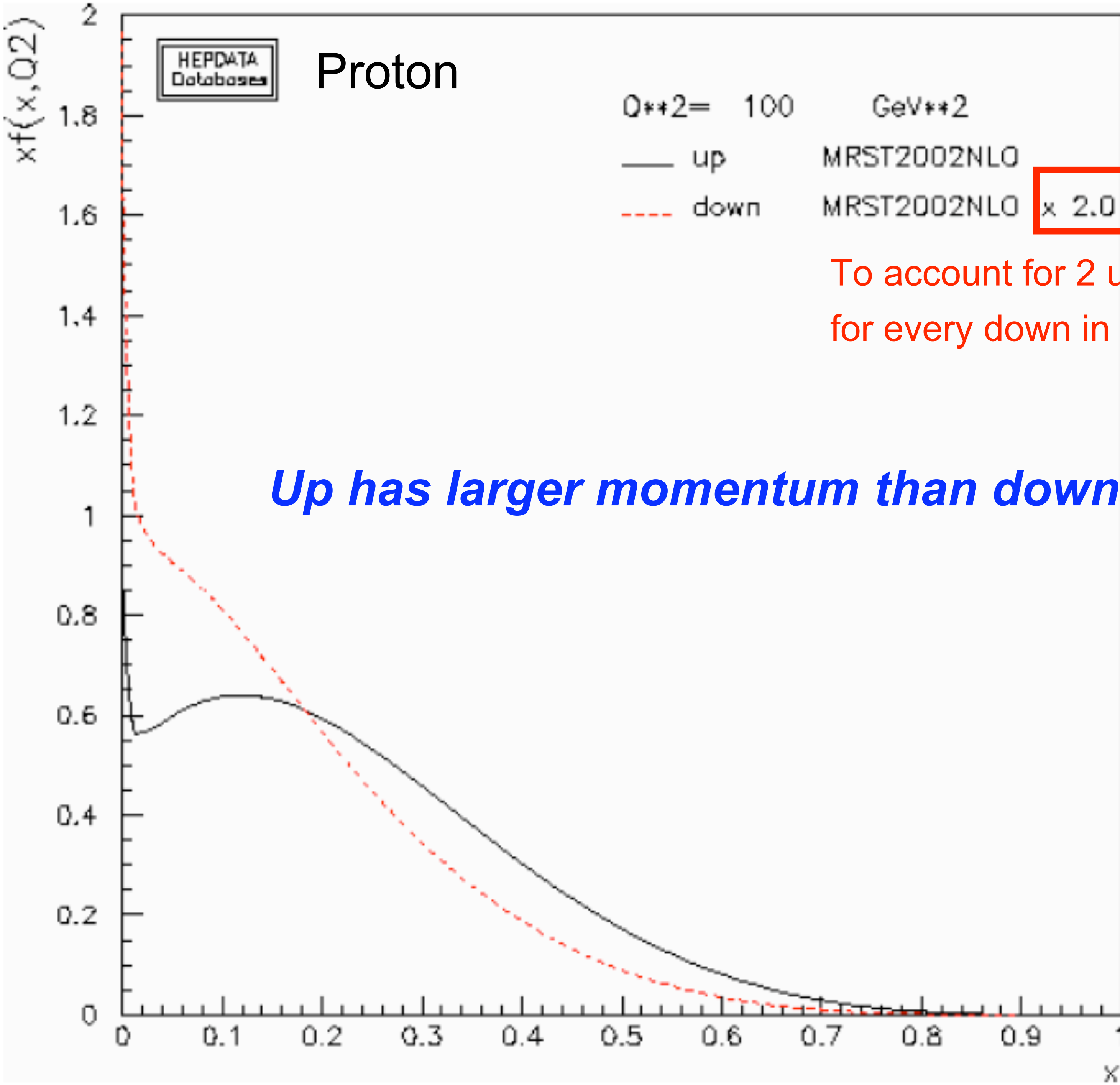
- This means that up quarks dominate in proton while down quarks dominate in neutron at large x.
- The dominant valence quark dominates at large x.

• Fitting structure functions of proton and anti-proton is an industry. There are 3 independent groups doing it, using a large number of independent measurements including ep,en, neutrino-p, neutrino-n, photon cross section, DY, W forward-backward asymmetry etc. etc. etc.

• This is very important “engineering” work for the LHC !!!

We’ll have a seminar on this next week!

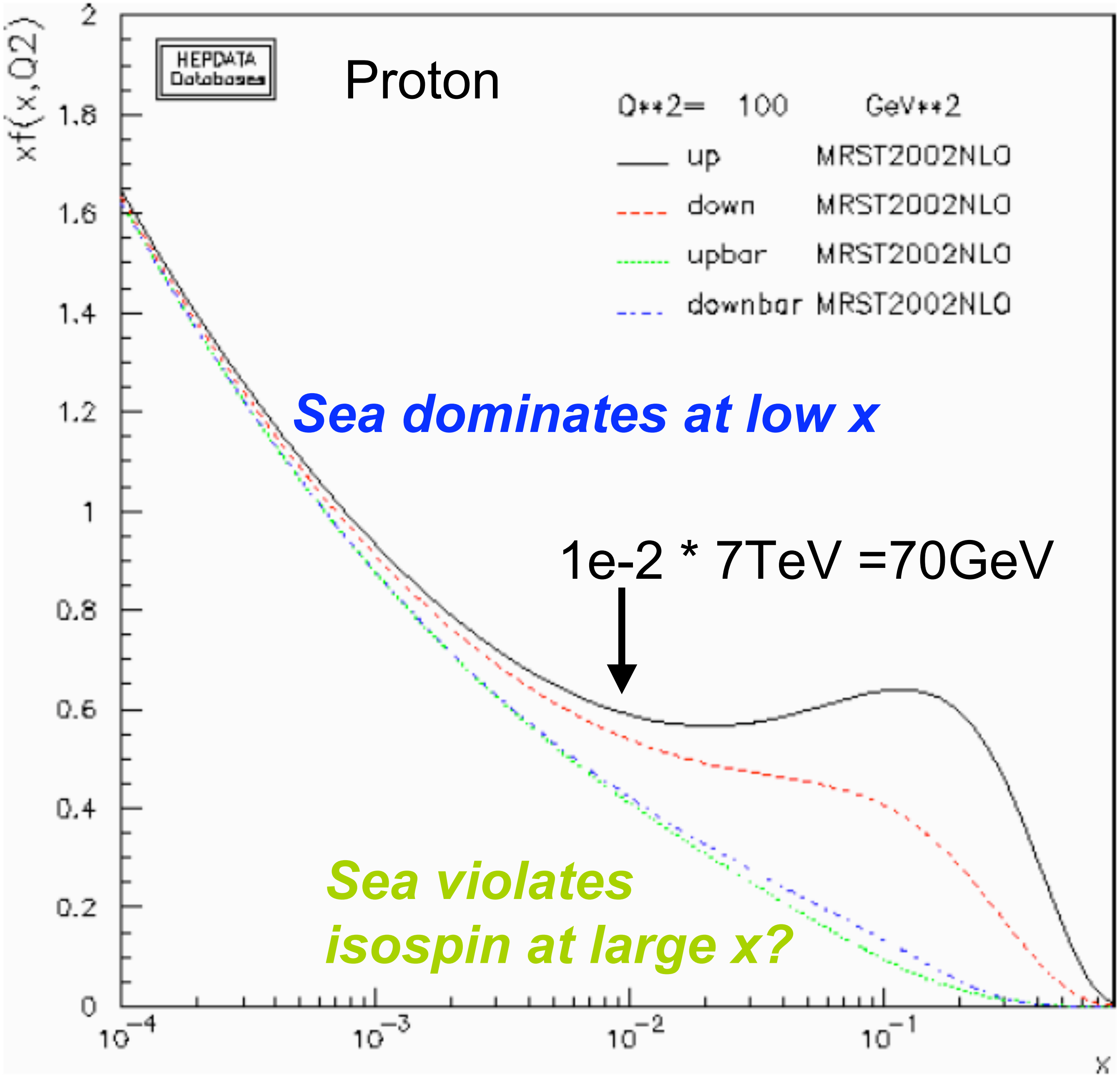
For now, let’s just look at some examples.



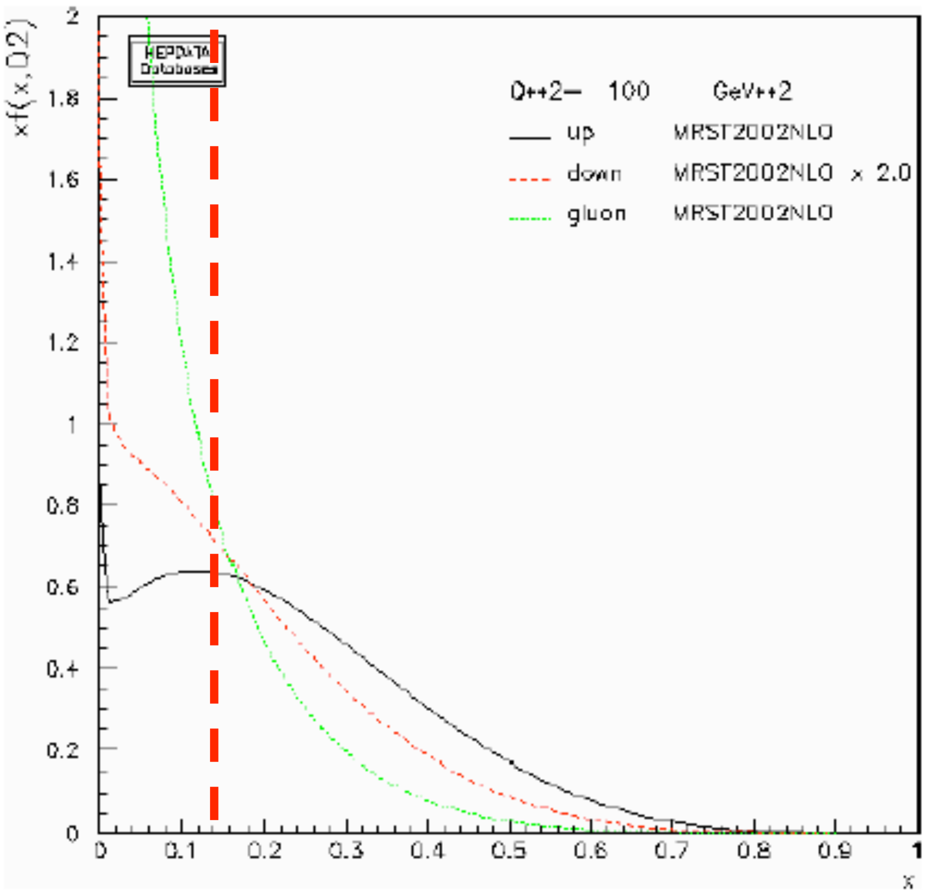
To account for 2 up
for every down in proton !!!

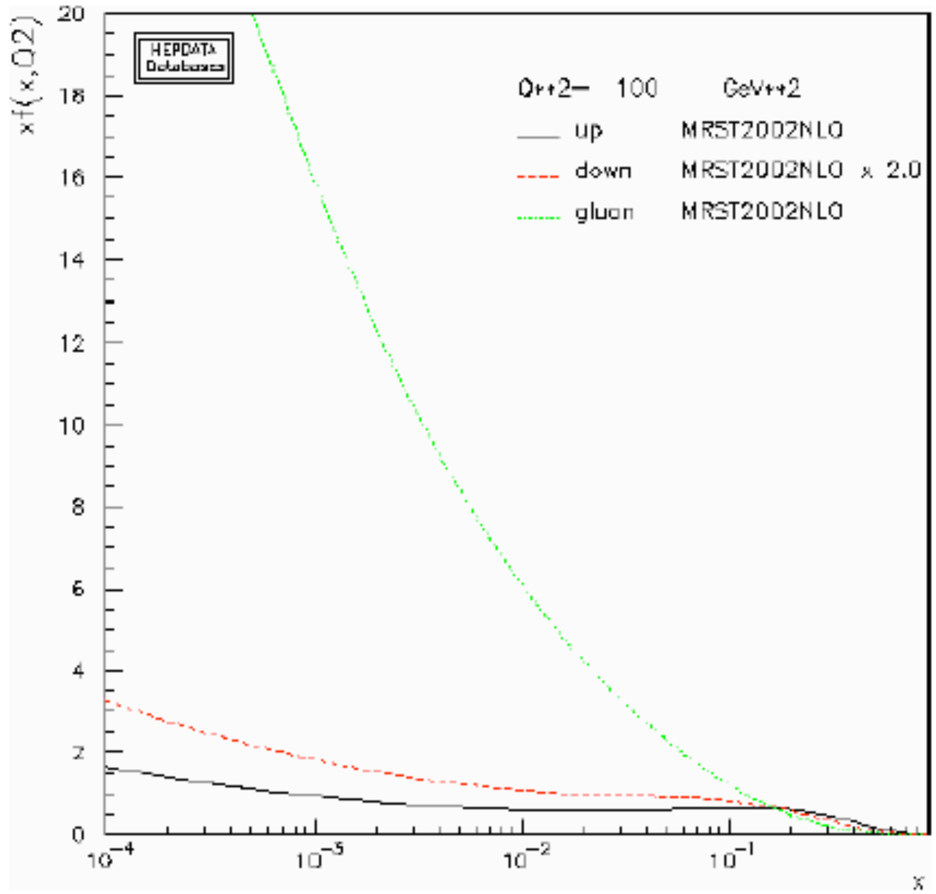
Up has larger momentum than down quark.

PDFs from http://durpdg.dur.ac.uk/hepdata/pdf3.html



A 14TeV collider can be pp instead of ppbar !!!





Gluons dominate at low x .

To set the scale, x = 0.14 at LHC is 0.14 * 7TeV = 1TeV

=> The LHC is a gluon collider !!!

Parton Model and Bjorken Scaling

We introduced two definitions for "x".

One from e-p scattering:

$$x = \frac{-q^2}{2p \cdot q} = \frac{1}{\omega}$$

And one from the parton model momentum fraction.

Section 9.2 in H&M shows that these are actually the same.

$$f_2^{\,i}(\omega) = \delta\left(1 - \frac{1}{x\omega}\right)$$

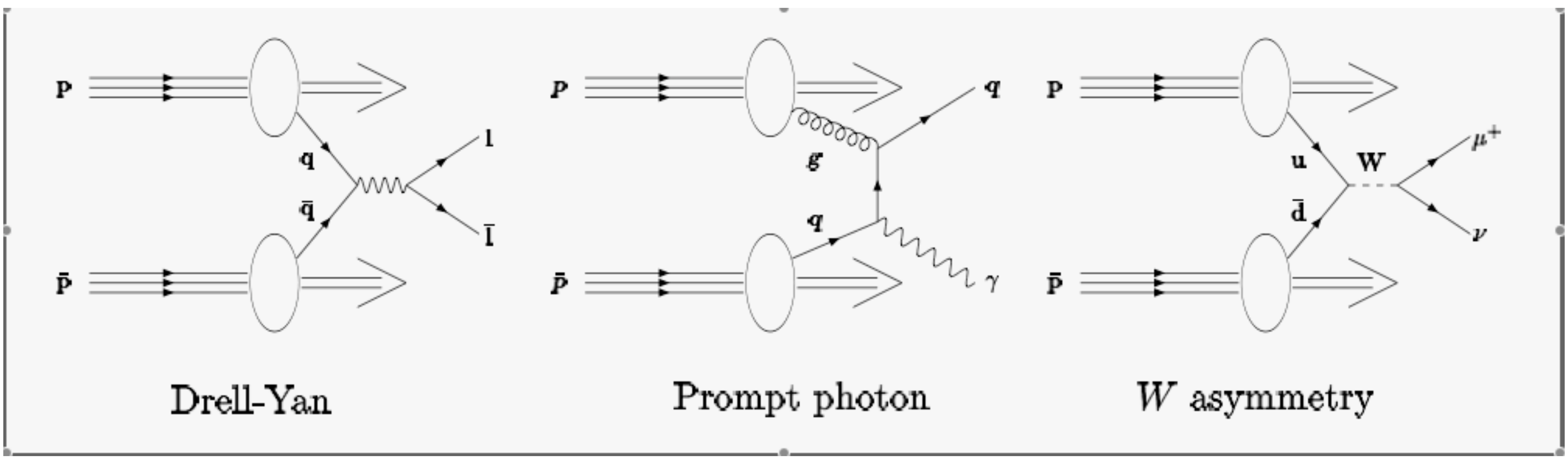
Is the F_2 structure function for the ith parton, that has a momentum fraction x.

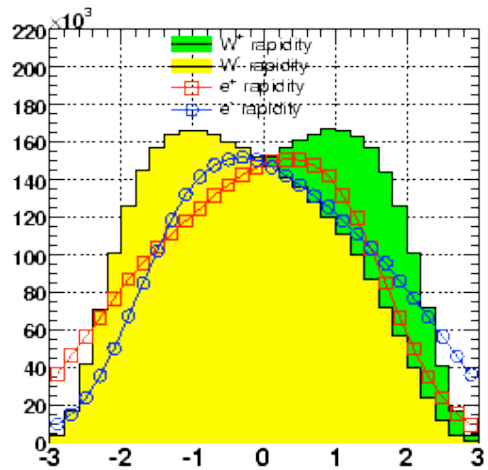
As we sum over all partons: $F_2(\omega) = \sum_i \int dx e_i^2 f_i(x) x \delta\left(x - \frac{1}{\omega}\right)$

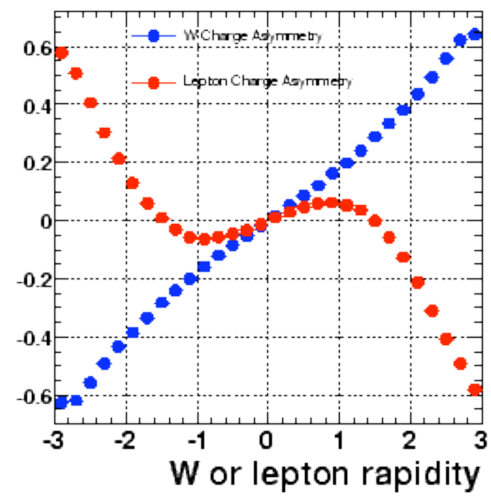
The δ-function here means that the virtual photon must have just the right x to be absorbed by a parton with momentum fraction,x, of the proton.

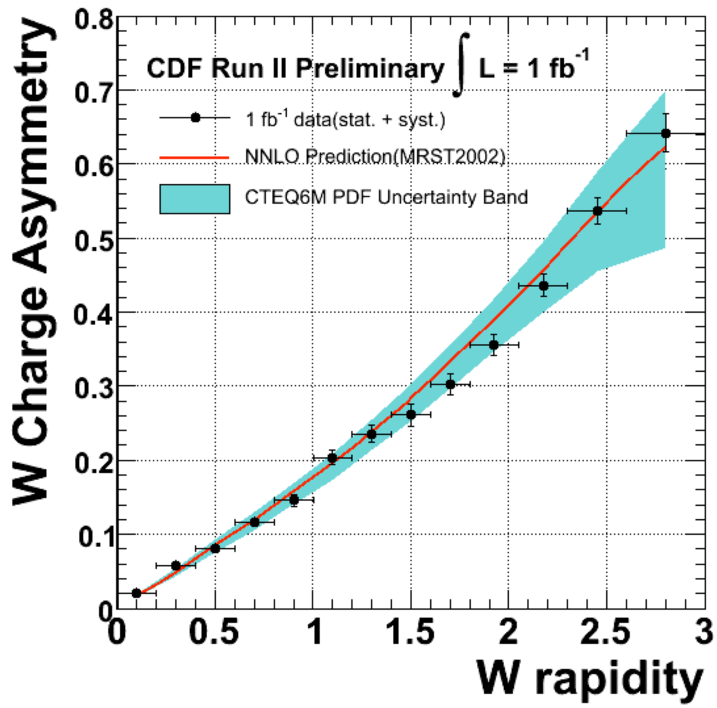
Ways to measure PDFs

- The HERA collider collides electrons on protons. This has produced a wealth of data.
  - Including measurement of the charm content of the proton by reconstructing charmed mesons in the final state.
- In addition, hadron collider data from these processes are used to fit PDFs:



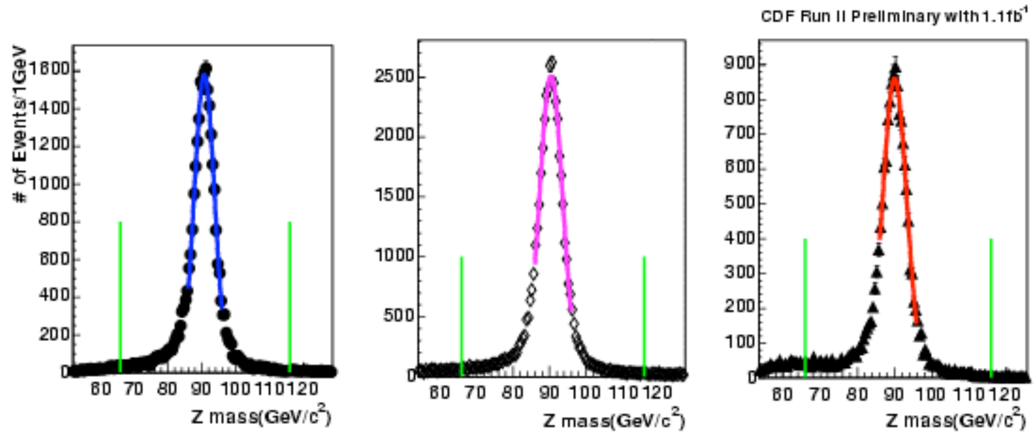


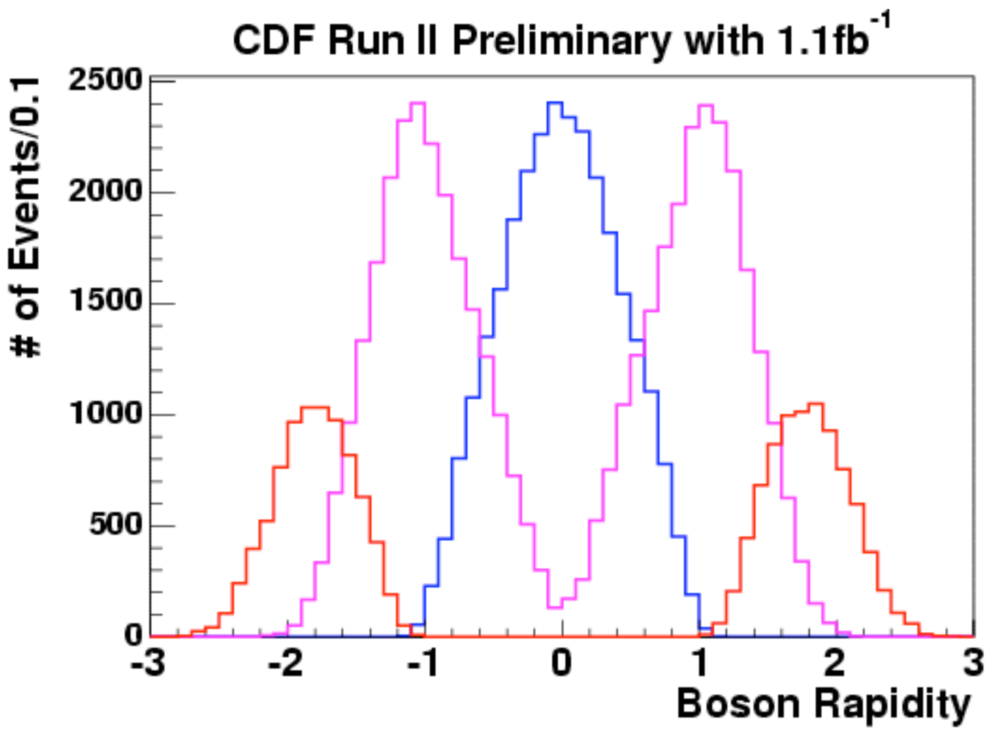




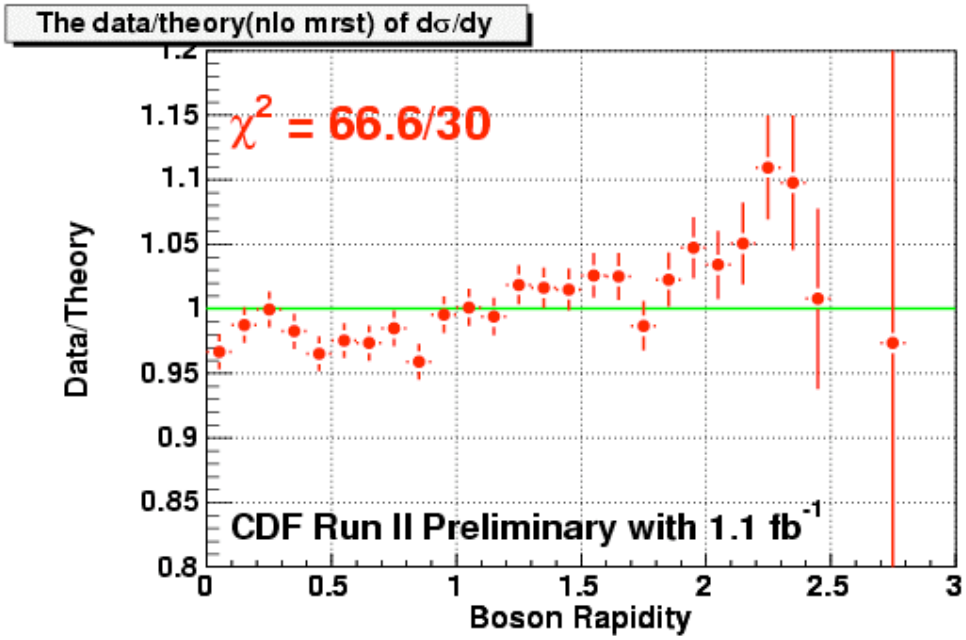
Most sensitive probe of d/u momentum ratio in proton at $Q^2 \sim M^2_W$.

Drell-Yan at Z pole from CDF





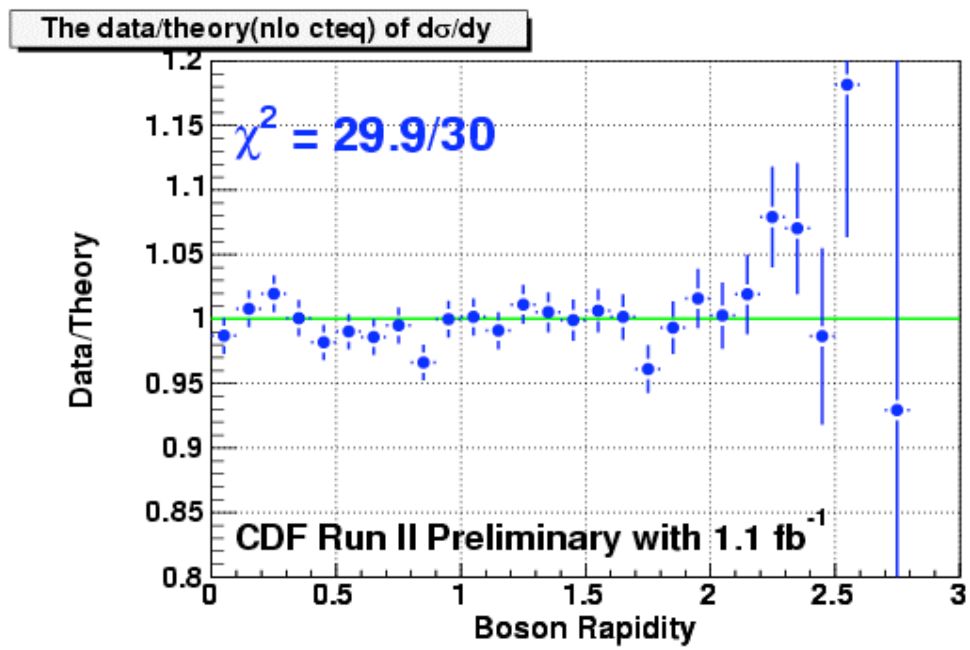
Different detection topologies:
Central-Central,
Central-forward,
Forward-forward



DY vs rapidity from CDF for two different PDF sets.

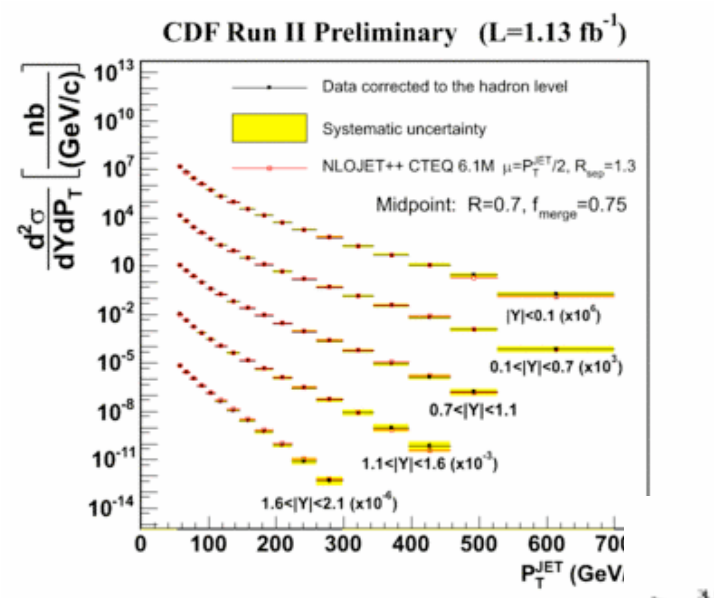
In both cases the total cross section is normalized to what's seen in data.

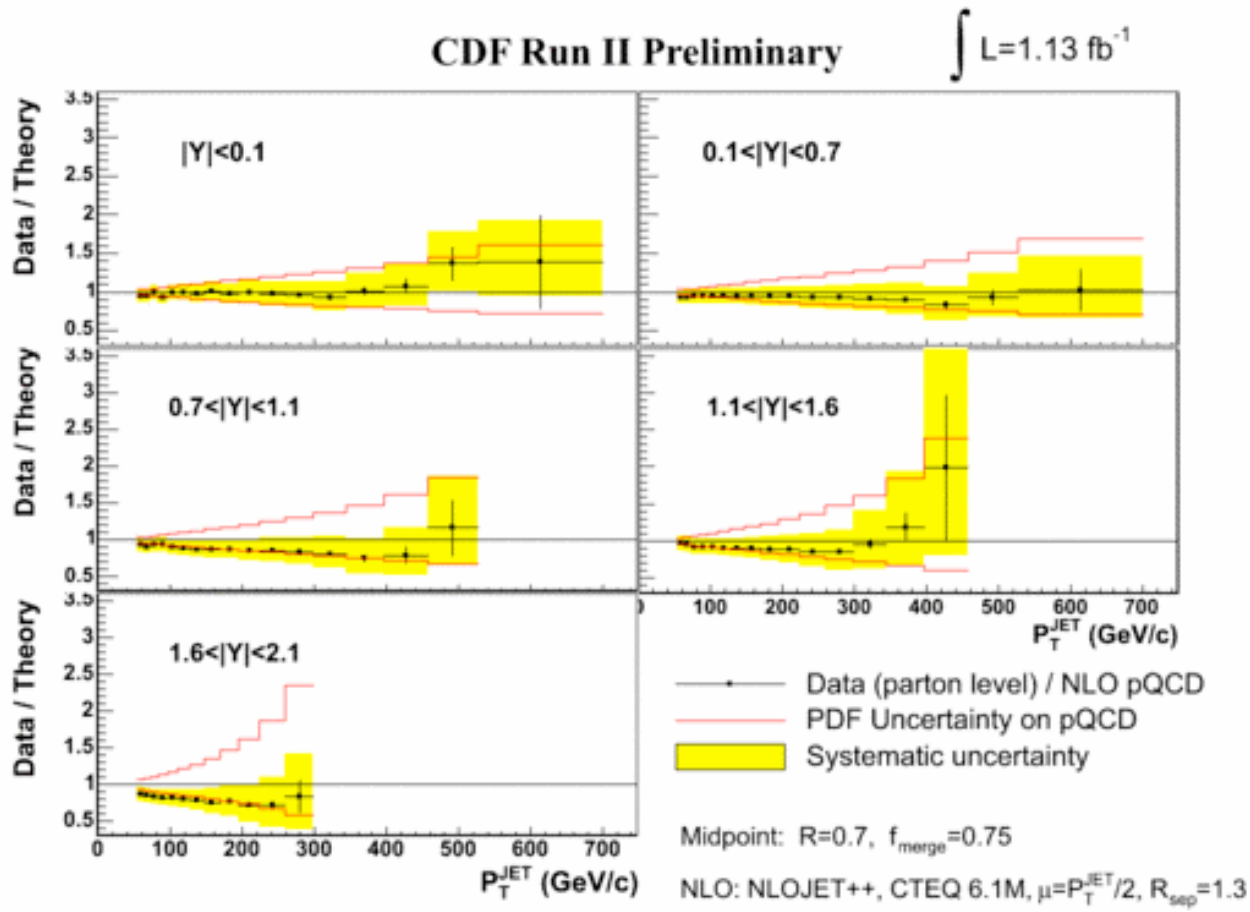
The differences are small but noticeable.



Inclusive Jet production at CDF vs pT for different rapidity ranges.

Yellow = experiment systematics
Black = experiment data & stats
Red = PDF uncertainty

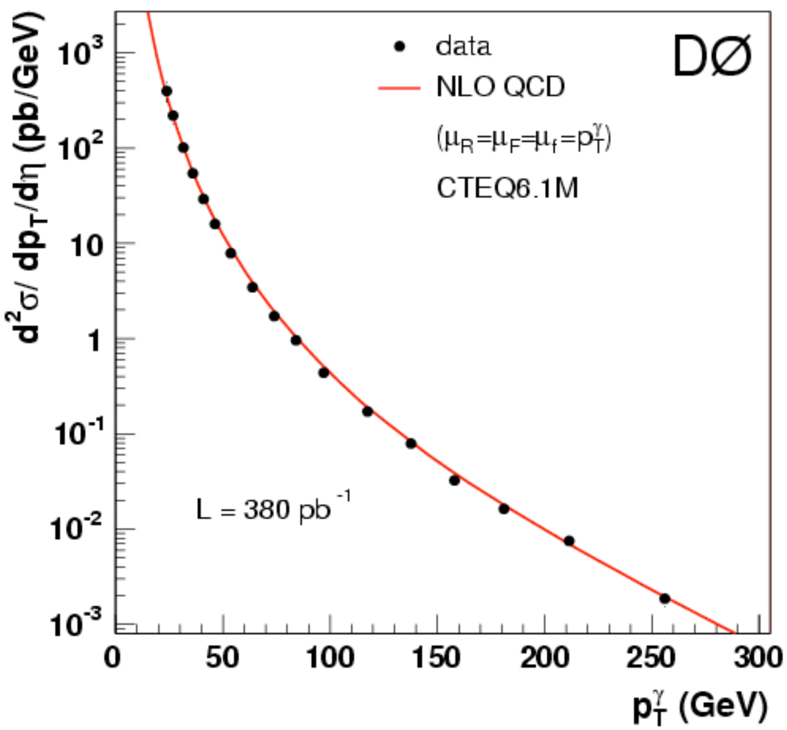


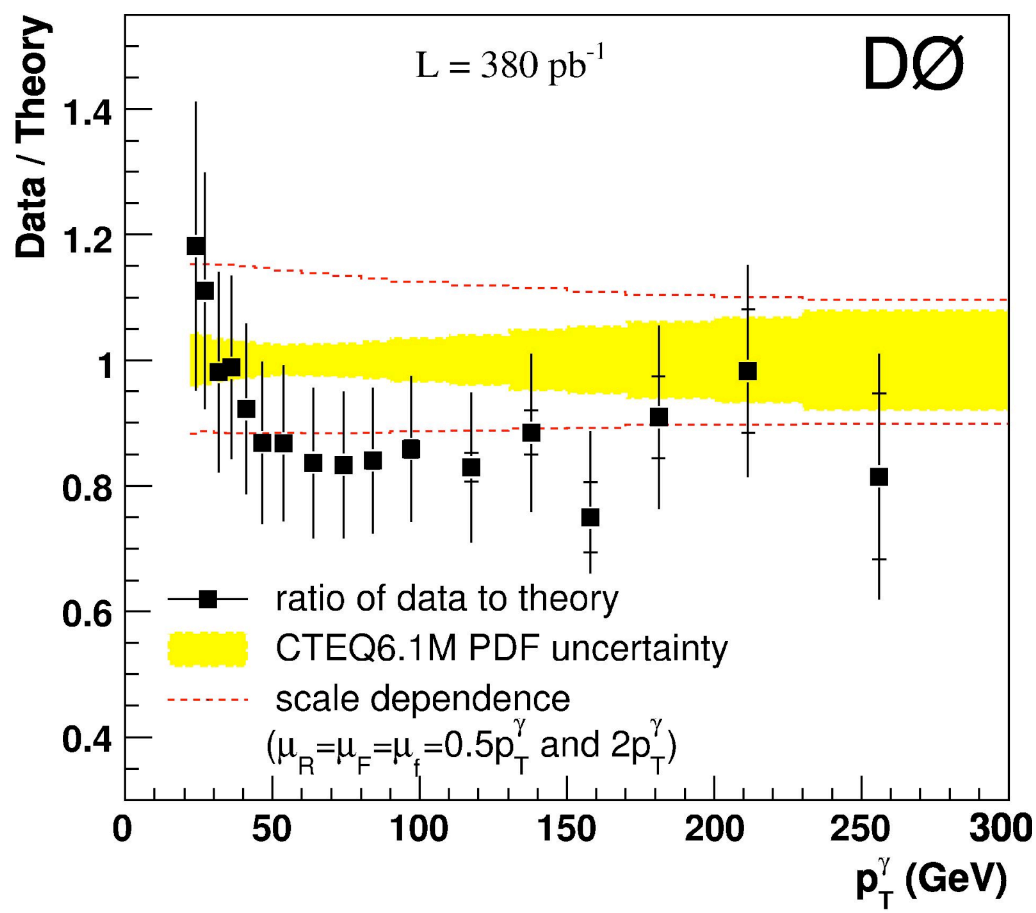


PDF uncertainty significant for large rapidity.

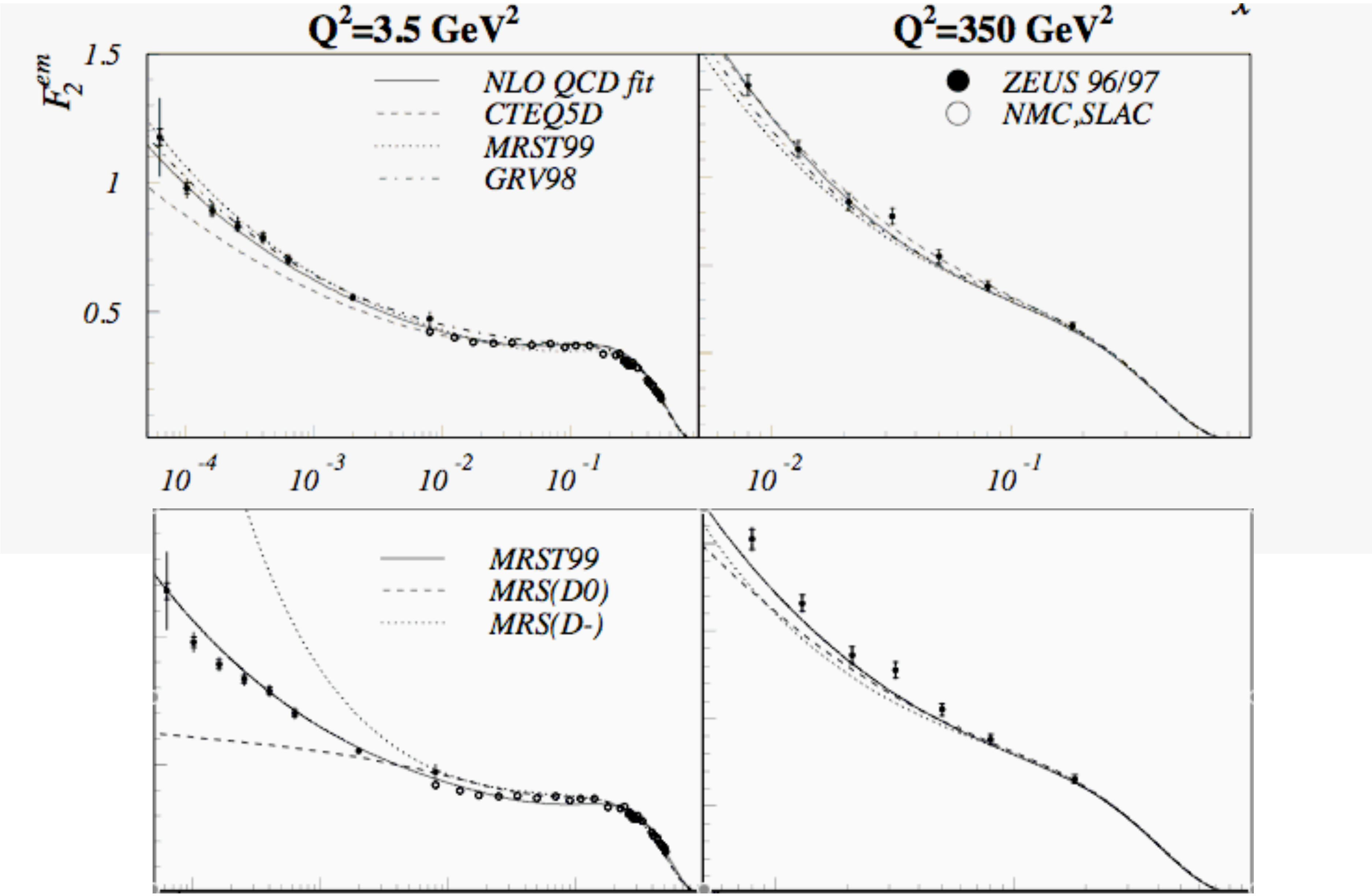
Note:
500GeV => x=0.5

Prompt photons at D0





Comparing e-proton data with PDFs.
Top = state of the art ~2001, includes early HERA data.
Bottom = history for one set of PDFs compared to 2001 data.



pp (or ppbar) collision

- Use Feynman diagrams to calculate σ for collision of partons of type i and j at CM energy E. Call this:

$$\hat{\sigma}_{ij}(\hat{s}) \equiv \hat{\sigma}_{ij}(E^2)$$

- To get the cross section of pp, I then need to integrate over all possible x_i, x_j with: $\hat{s} = x_i x_j s$

- In other words, a pp collider is a "broadband collider" spanning a wide range of CM energies, as well as types of colliding partons (!), with propabilities given by the product of PDF of the types of particles colliding.

Let's explore this formally

$$\frac{d\sigma(pp\to f)}{d\hat{s}} = \sum_{ij}\hat{\sigma}_{ij}(\hat{s})\int_0^1\int_0^1 dx_i dx_j f_i(x_i) f_j(x_j)\delta(\hat{s}-x_i x_j s)$$

$$=\sum_{ij}\frac{\hat{\sigma}_{ij}(\hat{s})}{\hat{s}}\int_0^1\int_0^1 dx_i dx_j f_i(x_i) f_j(x_j)\delta\left(1-x_i x_j\frac{s}{\hat{s}}\right)$$

$$\tau=\frac{\hat{s}}{s}$$

<- *to save some writing.*

$$\frac{d\sigma(pp\to f)}{d\tau} = \sum_{ij}\frac{\hat{\sigma}_{ij}(\hat{s})}{\tau}\int_0^1\int_0^1 dx_i dx_j f_i(x_i) f_j(x_j)\delta\left(1-\frac{x_i x_j}{\tau}\right)$$

$$\frac{d\sigma(pp\to f)}{d\tau} = \sum_{ij}\frac{\hat{\sigma}_{ij}(\hat{s})}{\tau}\int_\tau^1 dx_i\frac{\tau}{x_i} f_i(x_i) f_j\left(\frac{\tau}{x_i}\right)$$

Cross section as a function of parton-parton Luminosity

$$\frac{d\sigma(pp \to f)}{d\tau} = \sum_{ij} \frac{\hat{\sigma}_{ij}(\hat{s})}{\tau} \int_{\tau}^{1} dx_i \frac{\tau}{x_i}_{i} f_i(x_i) f_j\left(\frac{\tau}{x_i}\right)$$

$$\boxed{\frac{d\sigma(pp \to f)}{d\tau} = \sum_{ij} \frac{dL_{ij}}{d\tau} \hat{\sigma}_{ij}(\hat{s})}$$

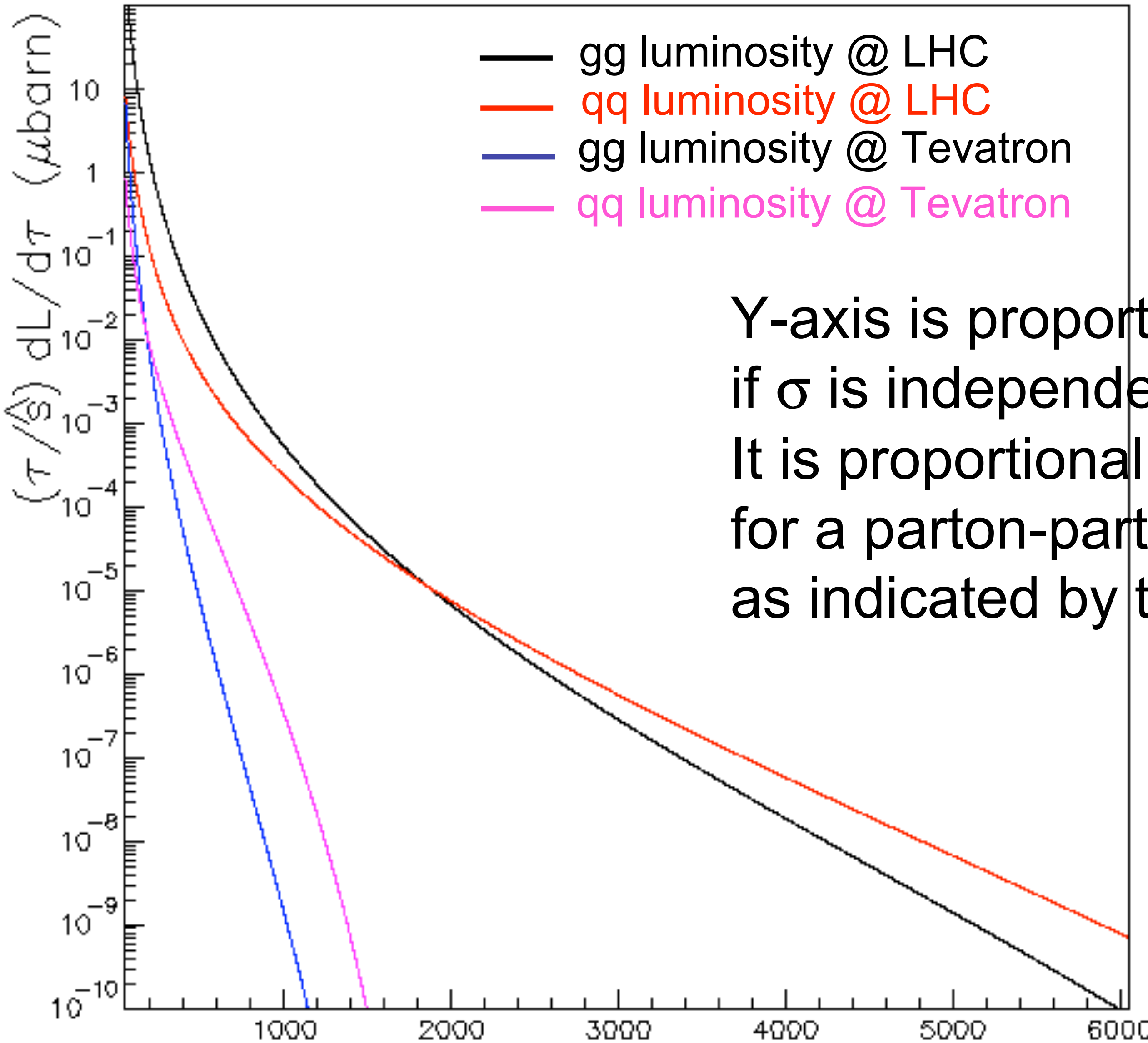
$$\frac{dL_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{dx}{x} \left[f_i(x) f_j\left(\frac{\tau}{x}\right) + f_i\left(\frac{\tau}{x}\right) f_j(x) \right]$$

Discussion of parton-parton Luminosity

$$\frac{dL_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{dx}{x} \left[f_i(x) f_j\left(\frac{\tau}{x}\right) + f_i\left(\frac{\tau}{x}\right) f_j(x) \right]$$

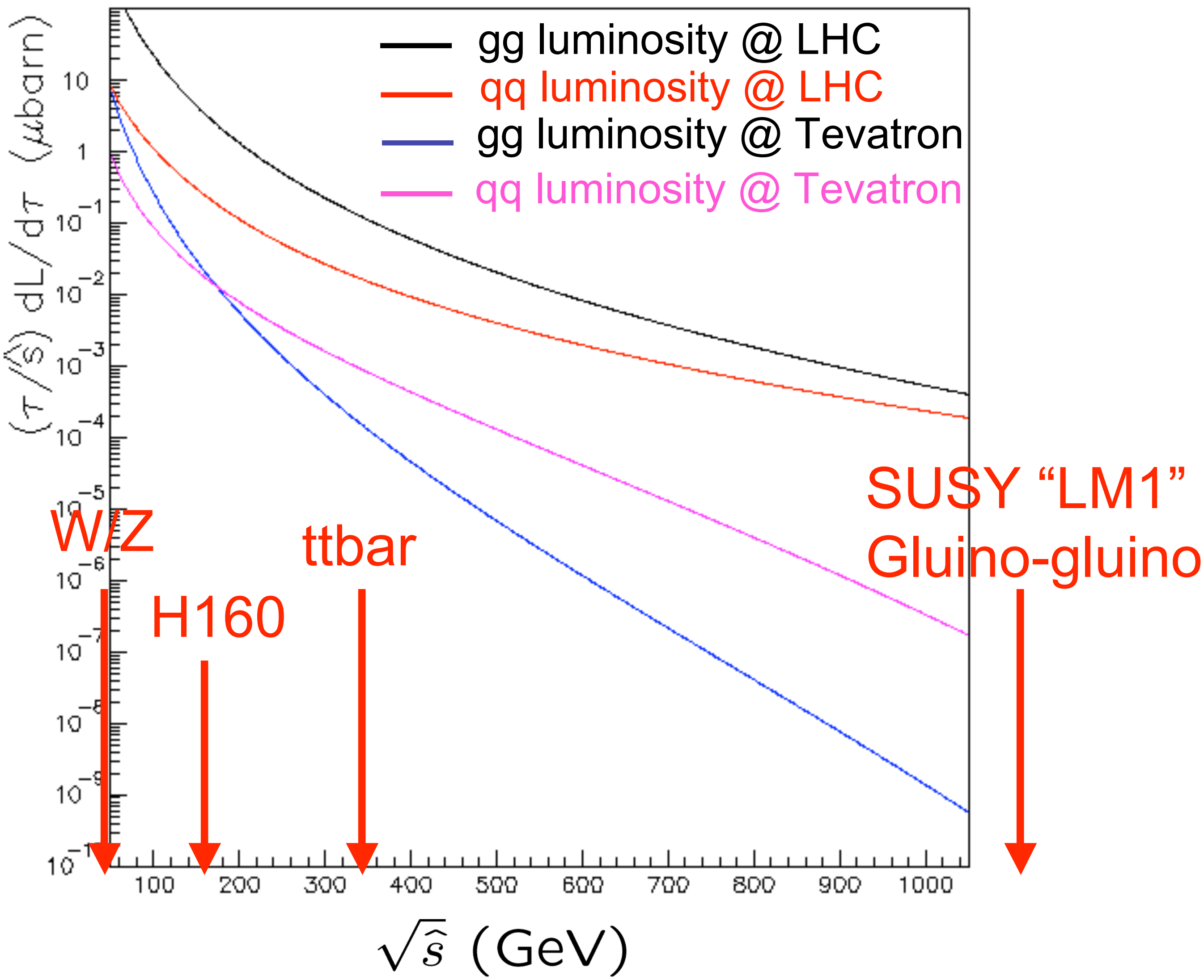
- Function of dimensionless quantity:
  - Scaling => independent of CM energy of proton proton collisions.
- However, $\hat{\sigma}_{ij}(\hat{s}) \equiv \hat{\sigma}_{ij}(E^2)$ depends on E. The collider characteristics only help us understand the energy scale E^2 accessible given an S for proton-proton collisions.

Adding in the Scale

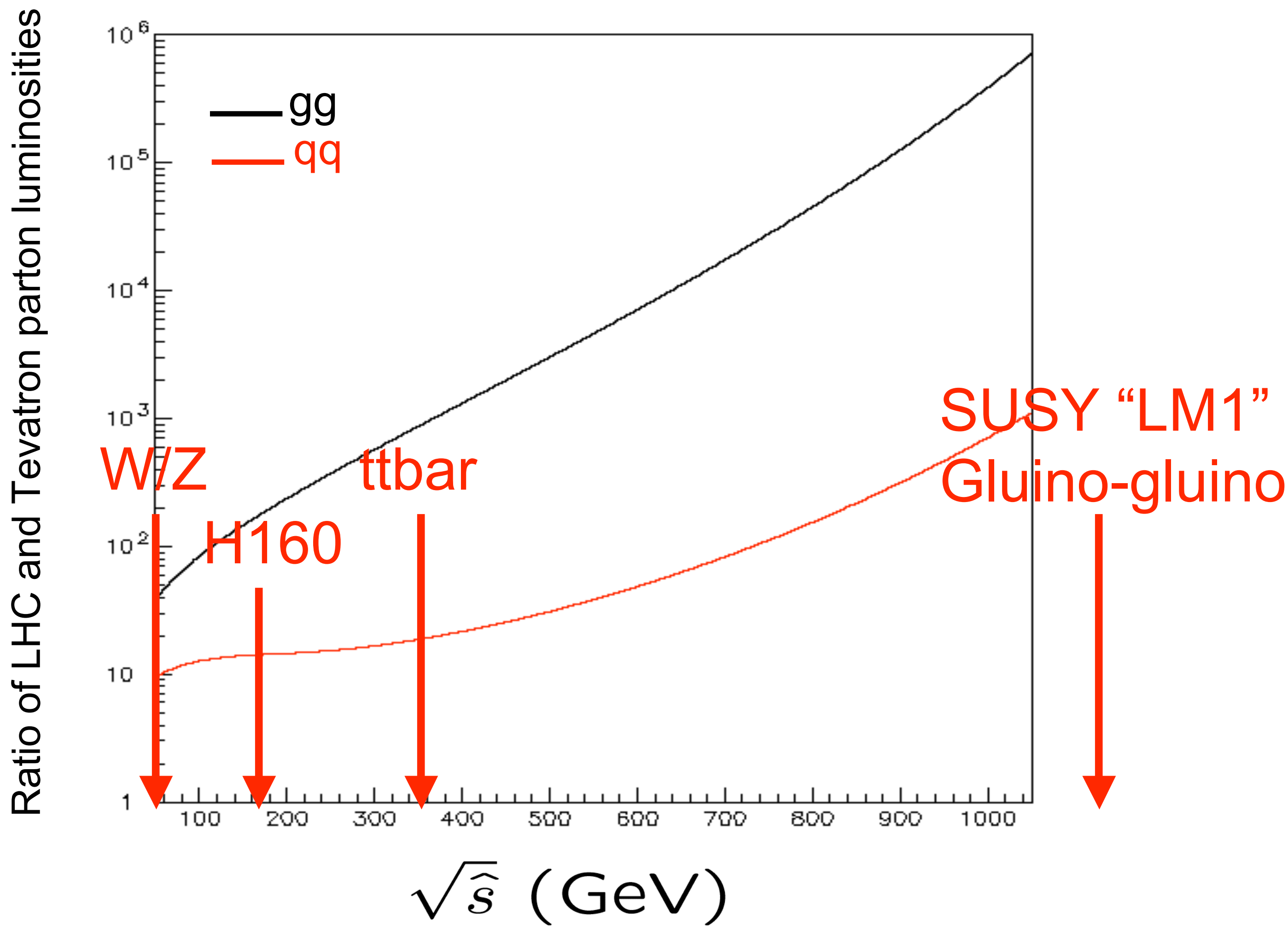


Y-axis is proportional to σ
if σ is independent of $\hat{s}$
It is proportional to probability
for a parton-parton collision with $\hat{s}$
as indicated by the x-axis.

Zooming-in on the < 1 TeV region



LHC vs Tevatron



1st (simplistic) rule of thumb:

- For 1 TeV gg processes, 1 fb^{-1} at FNAL is like 1 nb^{-1} at LHC
- For 1 TeV qq processes, 1 fb^{-1} at FNAL is like 1 pb^{-1} at LHC

Cross sections at 1.96TeV versus 14TeV Tevatron vs LHC

	Cross section		Ratio
$Z \to \mu\mu$	260pb	1750pb	6.7
WW	10pb	100pb	10
H_{160GeV}	0.2pb	25pb	125
$G\text{-}g_{LM1}$	0.0006pb	50pb	1 Million

At $10^{32}cm^{-2}s^{-1}$ CMS might accumulate $10pb^{-1}$ in one day!

… and SUSY might not exist in nature.

Closer Look at SUSY LM1

	Interactions	ng	ns	nn	ll	sb	ss	tb	bb	gg	sg
14TeV	NLO (pb)	0.57234	1.48646	2.41941	0.81955	8.47	7.85	2.206	0.981	8.6	27.9
7TeV	NLO (pb)	0.087455	0.215975	0.765887	0.262451	0.752	1.57	0.1944	0.0676	0.42	2.45
2TeV		1.4e-3	3.4e-4	0.14	0.05	7e-4	1e-5	6e-4	4e-5	9e-6	1e-4

n = neutralino ~ susy Z
g = gluino ~ susy gluon
s = squark ~ susy u,d,s,c quarks

l = slepton ~ susy leptons
b = sbottom ~ susy b quark
t = stop ~ susy t quark

The Tevatron is down by only O(10) for nn against LHC.
Neutralino mass only 100GeV in this model.
The Tevatron is down by O(1e6) for gg against LHC.
Gluino mass is 600GeV in this model.

Tevatron is generally sensitive to different production mechanism for the same mSugra model parameters !!!