Physics 214 UCSD/225a UCSB

Lecture 13

- Finish H&M Chapter 6
- Start H&M Chapter 8

Result worthy of discussion

1. $\sigma \propto 1/s$ must be so on dimensional grounds
2. $\sigma \propto \alpha^2$ two vertices!
3. At higher energies, Z-propagator also contributes:

More discussion

4. Calculation of e+e- -> q qbar is identical as long as sqrt(s) >> Mass of quark.

$$\sigma(e^+e^- \to q\bar{q}) = 3 \cdot \sum_{q-flavor} e_q^2 \sigma(e^+e^- \to \mu^+\mu^-)$$

# of flavors

Charge of quark

Sum over quark flavors

Measurement of this cross section was very important !!!

Measurement of R

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \cdot \sum_{q-flavor} e_q^2$$

Below charm threshold: R = 3 [$(2/3)^2$ + $(1/3)^2$ + $(1/3)^2$] =2

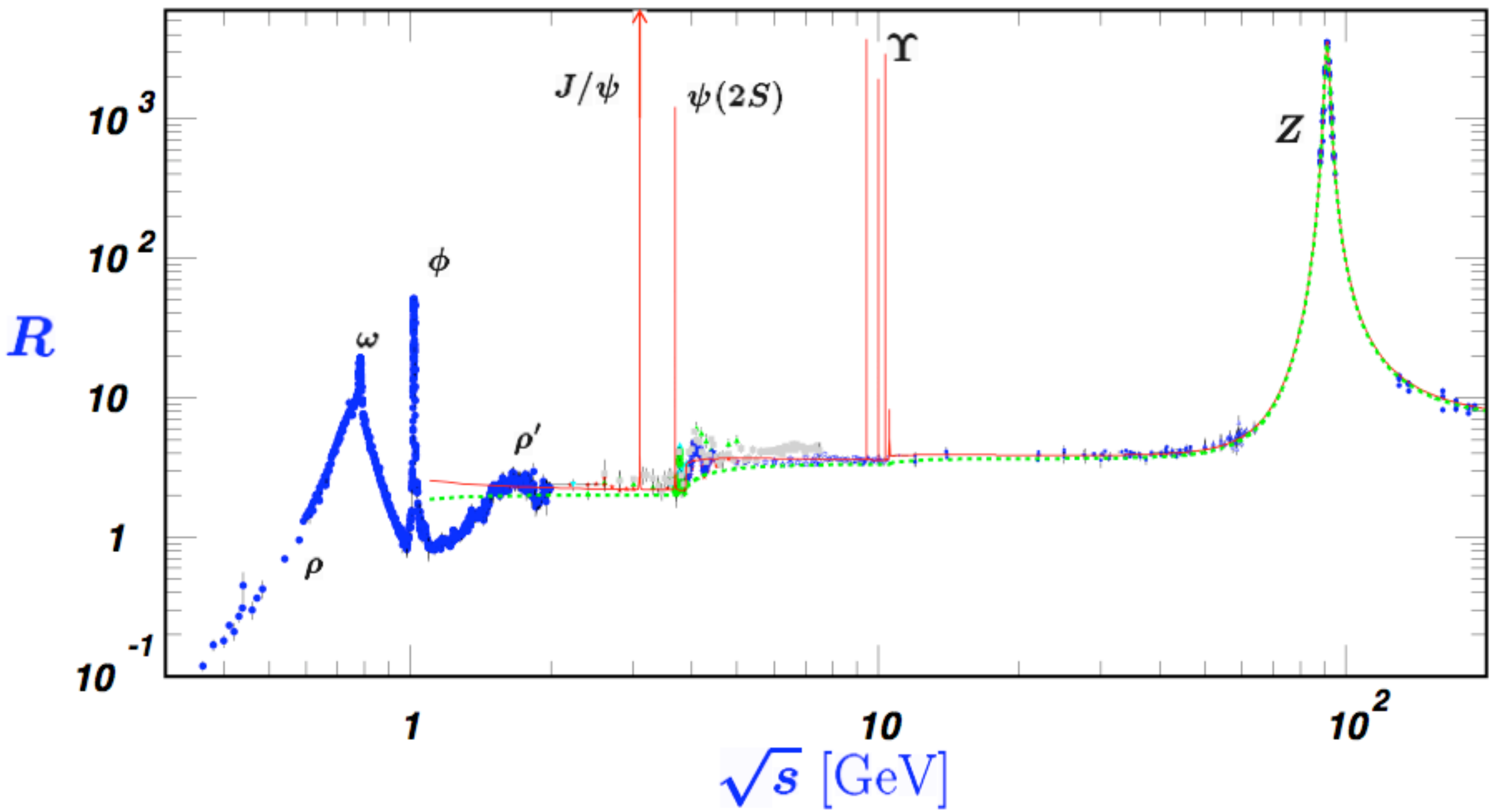
Between charm and bottom: R = 2 + 3(4/9) = 10/3

Above bottom: R = 10/3 + 3(1/9) = 11/3

Measurement of R was crucial for:

1. ***Confirm that quarks have 3 colors***
2. ***Search for additional quarks***
3. ***Search for additional leptons***

Experimental Result



Ever more discussion

5. $d\sigma/d\Omega \propto (1 + \cos^2\theta)$

5.1 θ is defined as the angle between e+ and mu+ in com. $\cos^2\theta$ means that the outgoing muons have no memory of the direction of incoming particle vs antiparticle.

Probably as expected as the e+e- annihilate before the mu+mu- is created.

5.2 Recall, phase space is flat in $\cos\theta$. $\cos^2\theta$ dependence thus implies that the initial state axis matters to the outgoing particles. Why?

Helicity Conservation in relativistic limit

- You showed as homework that u_L and u_R are helicity eigenstates in the relativistic limit, and thus:

$$\overline{u}\gamma^{\mu}u = (\overline{u}_L + \overline{u}_R)\gamma^{\mu}(u_L + u_R)$$

- We'll now show that the cross terms are zero, and helicity is thus conserved at each vertex.
- We then show how angular momentum conservation leads to the cross section we calculated.

Let' do one cross product explicitly:

$$\bar{u}\gamma^{\mu}u = (\bar{u}_L + \bar{u}_R)\gamma^{\mu}(u_L + u_R)$$

$$\bar{u}_L = u_L^{T*}\gamma^0 = u^{T*}\frac{1}{2}(1-\gamma^5)\gamma^0 = \bar{u}\frac{1}{2}(1+\gamma^5)$$

$$u_R = \frac{1}{2}(1+\gamma^5)u$$

$$\bar{u}_L\gamma^{\mu}u_R = \bar{u}\frac{1}{4}(1+\gamma^5)\gamma^{\mu}(1+\gamma^5)u = \bar{u}\gamma^{\mu}\frac{1}{4}(1-\gamma^5)(1+\gamma^5)u = 0$$

Here we have used:

$$\boxed{\begin{aligned} \gamma^5\gamma^{\mu} &= -\gamma^{\mu}\gamma^5 \\ \gamma^5 &= \gamma^{5T*} \\ \gamma^5\gamma^5 &= 1 \end{aligned}}$$

Helicity conservation holds for all vector and axialvector currents as E>>m.

- $e_L^- e_R^+ \rightarrow mu_L^- mu_R^+$ J_z +1 -> +1
- $e_L^- e_R^+ \rightarrow mu_R^- mu_L^+$ J_z +1 -> -1
- $e_R^- e_L^+ \rightarrow mu_L^- mu_R^+$ J_z -1 -> +1
- $e_R^- e_L^+ \rightarrow mu_R^- mu_L^+$ J_z -1 -> -1
- *Next look at the rotation matrices:*

$$d^1_{11}(\theta) = \frac{1}{2}\left(1+\cos\theta\right) \approx \frac{-u}{s}$$

$$d^1_{-1-1}(\theta) = \frac{1}{2}\left(1+\cos\theta\right) \approx \frac{-u}{s}$$

$$d^1_{-11}(\theta) = \frac{1}{2}\left(1-\cos\theta\right) \approx \frac{-t}{s}$$

$$d^1_{1-1}(\theta) = \frac{1}{2}\left(1-\cos\theta\right) \approx \frac{-t}{s}$$

Cross products cancel in Spin average:

$$\overline{|M|^2} \propto \left(1+\cos^2\theta\right)$$

$$d^J_{1-1}$$

Initial J_z final J_z

Conclusion on relativistic limit

- Dependence on scattering angle is given entirely by angular momentum conservation !!!
- This is a generic feature for any vector or axialvector current.
- We will thus see the exact same thing also for V-A coupling of Electroweak interactions.

Propagators

Spinless:

$$\frac{i}{p^2 - m^2}$$

Massive Vector Bosons:

$$\frac{i\left(-g^{\mu\nu} + p^{\mu} p^{\nu} / M^2\right)}{p^2 - M^2}$$

See H&M Ch.6.10ff for more details.

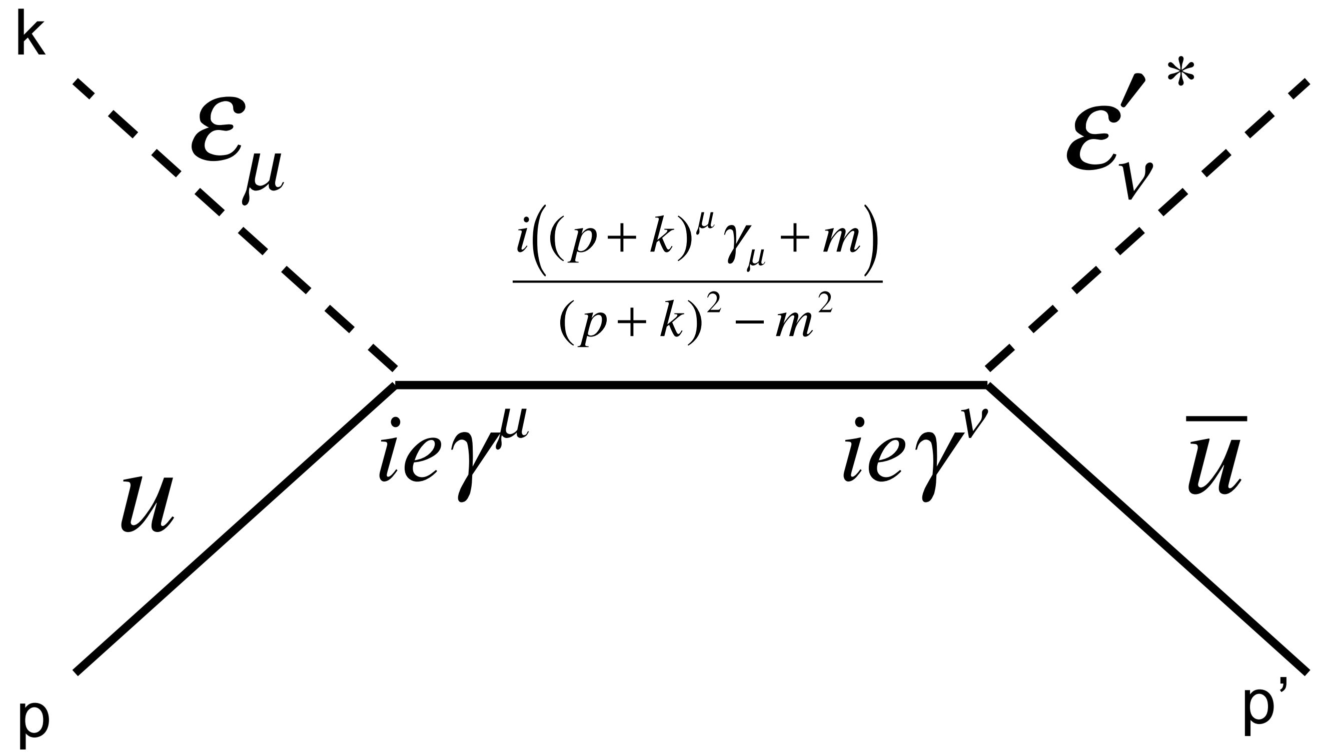
Spin 1/2, e.g. electron:

$$\frac{i\sum_s \bar{u} u}{p^2 - m^2} = \frac{i\left(p^{\mu}\gamma_{\mu} + m\right)}{p^2 - m^2}$$

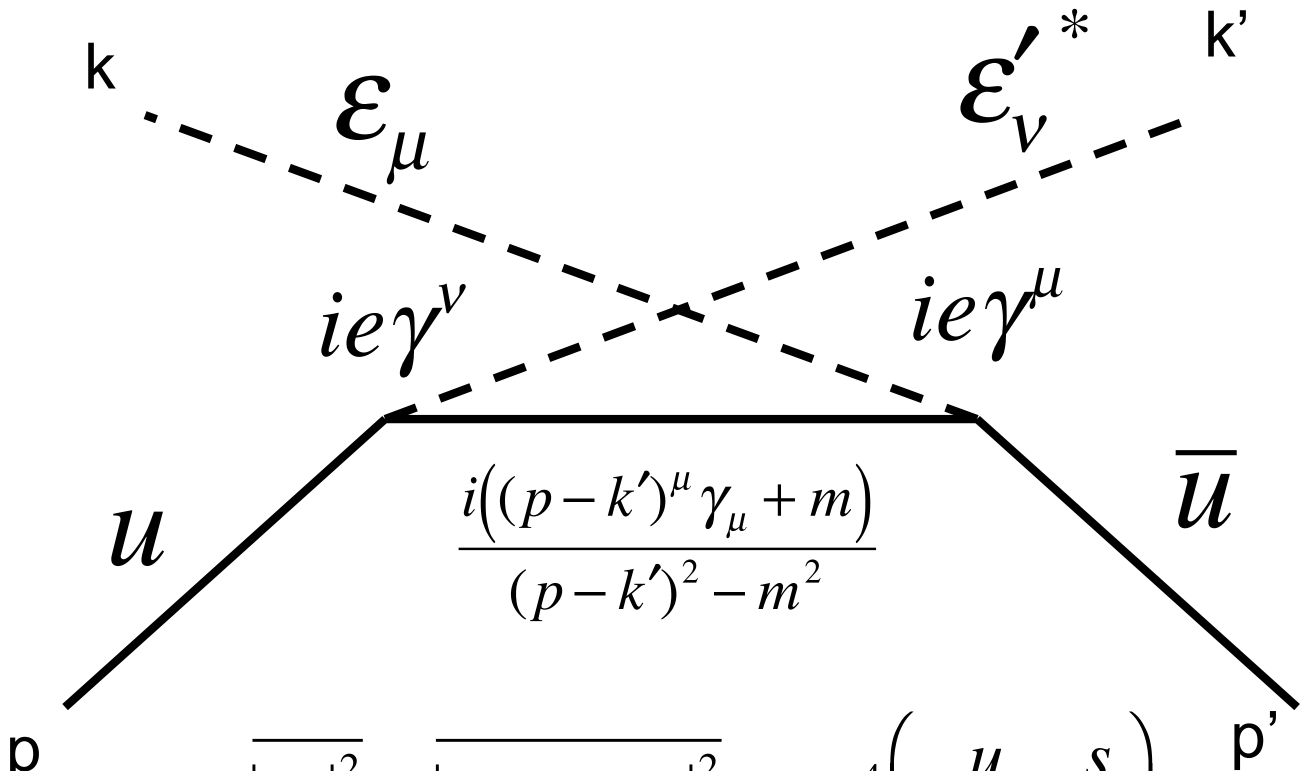
Photon:

$$\frac{-ig_{\mu\nu}}{q^2}$$

Compton Scattering: e- gamma -> e- gamma



Compton Scattering: e- gamma -> e- gamma



$$\overline{|M|^2} = \overline{|M_1 + M_2|^2} = 2e^4\left(-\frac{u}{s} - \frac{s}{u}\right)$$

Where we neglect the electron mass, and refer to H&M Chapter 6.14 for details.

Pair annihilation via crossing

- Like we've done before: s <-> t

$$\overline{|M|^2} = \overline{|M_1 + M_2|^2} = 2e^4\left(\frac{u}{t} + \frac{t}{u}\right)$$

t = -2 kk' = -2 pp'
u = -2 kp' = -2 k'p

Ignoring the electron mass.

First step towards chapter 8

- In chapter 8 we investigate the structure of hadrons by scattering electrons of charge distributions that are at rest in the lab.
- As an initial start to formalism review e- mu- scattering with the initial muon at rest.
- Let's start with what we got last time, neglecting only terms with electron mass:

$$\overline{|M|^2} = \frac{8e^4}{t^2}\left[(k'p')(kp) + (k'p)(kp') - M^2 kk'\right]$$

$$\overline{|M|^2} = \frac{8e^4}{t^2}\left[(k'p')(kp) + (k'p)(kp') - M^2 kk'\right]$$

As we will want frame for which p = (M,0), it's worth rewriting this using q = k - k' .
As we won't care for the muon recoil p', we eliminate p' via:
p' = k - k' + p.
As we ignore the electron mass, we'll drop terms with k^2, or k'^2, and simplify $q^2 = -2kk'$.
We then get:

$$\overline{|M|^2} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2(kp - k'p) + 2(k'p)(kp) + \frac{1}{2}M^2 q^2\right]$$

I'll let you confirm this for yourself.

$$\overline{|M|^2} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2\left(kp - k'p\right) + 2\left(k'p\right)\left(kp\right) + \frac{1}{2}M^2q^2\right]$$

Now got to muon restframe: p = (M,0)
This means kp = EM and k'p = E'M .

$$\overline{|M|^2} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2M\left(E - E'\right) + 2EE'M^2 + \frac{1}{2}M^2q^2\right]$$

$$= \frac{8e^4}{q^4}2EE'M^2\left[-\frac{q^2}{2M^2}\frac{M\left(E - E'\right)}{2EE'} + 1 + \frac{q^2}{4EE'}\right]$$

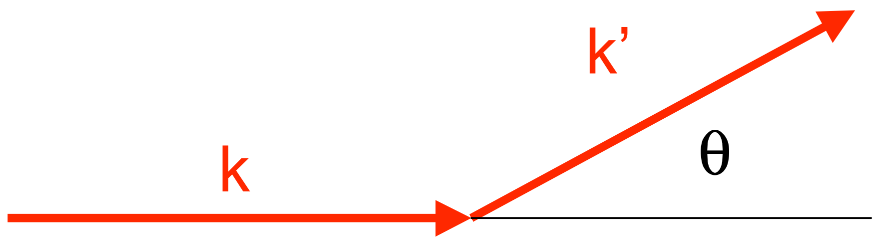
$$q^2 = -2kk' = -4EE'\sin^2\theta/2$$

$$= \frac{8e^4}{q^4}2EE'M^2\left[-\frac{q^2}{2M^2}\frac{M\left(E - E'\right)}{2EE'} + 1 - \sin^2\frac{\theta}{2}\right]$$

$$= \frac{8e^4}{q^4}2EE'M^2\left[-\frac{q^2}{2M^2}\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right]$$

$$q^2 = -2pq = -2M(E-E')$$

Aside on scattering angle



$$-2kk' = -2\,(EE' - kk'\cos\theta) = -2EE'\,(1-\cos\theta) = -4EE'\sin^2\theta/2$$

Recall: We eliminated any need to know p' in favor of a measurement of the labframe angle between k and k'

Aside

$$q^2 = (k-k')^2 = (p-p')^2$$
$$q^2 = -2kk' = -2pp'$$

However, we already know that $p' = q + p$, and thus:

$$q^2 = -2p(q+p) = -2pq + 2m^2 = -2pq$$

Always neglecting terms proportional to the electron mass.

$$\overline{|M|^2} = \frac{8e^4}{q^4} 2EE'M^2 \left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2} \right]$$

Now recall how to turn this into dσ in the labframe:

$$d\sigma = \frac{1}{64\pi^2} \frac{\delta^{(4)}(p_D + p_C - p_A - p_B)}{v_A E_A E_B} |M|^2 \frac{dp_c^3 dp_D^3}{E_c E_D}$$

=1

(slide 15 lecture 10)

$$d\sigma = \frac{1}{64\pi^2} \frac{\delta^{(4)}(p + k - p' - k')}{EM} |M|^2 \frac{d^3k' d^3p'}{E' p_0'}$$

$$= \frac{|M|^2}{4\pi^2} \frac{\delta^{(4)}(p + k - p' - k')}{4EM} \frac{E'dE'd\Omega}{2} \frac{d^3p'}{2p_0'}$$

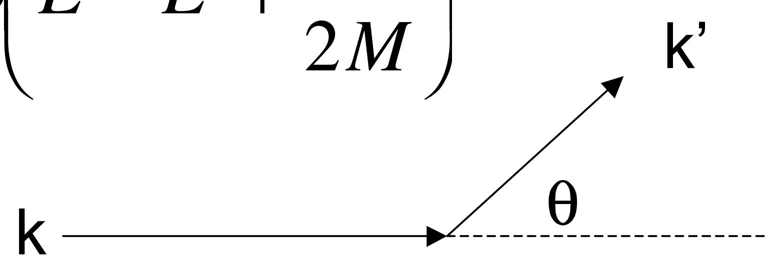
$$d\sigma = \frac{|M|^2}{4\pi^2}\frac{\delta^{(4)}(p+k-p'-k')}{4EM}\frac{E'd^3E'd\Omega}{2}\frac{d^3p'}{2p'_0}$$

What do we do about this ?

Recall, we are heading towards collissions between electron and hadron. The hadronic mess in the final state is not something we care to integrate over !!!

Exercise 6.7 in H&M:

$$\int \delta^{(4)}(p+q-p')\frac{d^3p'}{2p'_0} = \frac{1}{2M}\delta\left(E - E' + \frac{q^2}{2M}\right)$$



Putting it all together:

$$\frac{d\sigma}{dE'd\Omega} = \frac{|M|^2}{4\pi^2}\frac{E'}{8EM}\int \delta^{(4)}(p+q-p')\frac{d^3p'}{2p'_0}$$

$$\overline{|M|^2} = \frac{8e^4}{q^4}2EE'M^2\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

$$\frac{d\sigma}{dE'd\Omega} = 4\alpha^2\frac{2ME'^2}{q^4}[\ldots]\int \delta^{(4)}(p+q-p')\frac{d^3p'}{2p'_0}$$

$$\frac{d\sigma}{dE'd\Omega} = \frac{4E'^2\alpha^2}{q^4}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]\delta\left(E - E' + \frac{q^2}{2M}\right)$$

Now we perform the E' integration by noticing:

$$\frac{1}{2M}\delta\left(E - E' + \frac{q^2}{2M}\right) = \frac{\delta\left(E' - {}^{E}\!/_{A}\right)}{2MA}$$

$$A \equiv 1 + \frac{2E}{M}\sin^2\frac{\theta}{2}$$

Exercise 6.7 H&M

We then finally get:

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]}$$

This is completely independent of the target's final state !!!

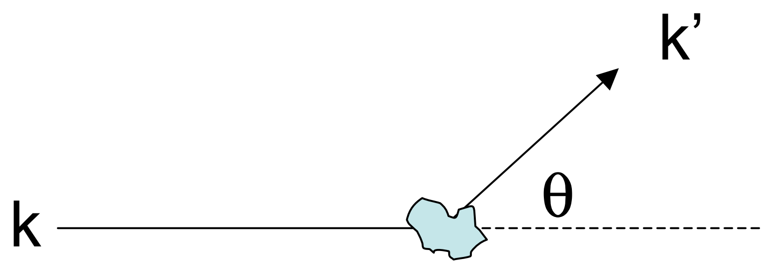
Experimental importance

- You measure only initial and final state electron.
- You have a prediction for electron scattering off a spin 1/2 point particle with charge = -1.
- Any deviation between measurement and prediction indicates substructure of your supposed point particle !!!

- Next: How does one describe scattering off a charge distribution, rather than a point particle?

=> Beginning of chapter 8 !

Probing the Structure of Hadrons with electron scattering



- All you measure is the incoming and outgoing electron 3 momentum.
- If you had a static target then you can show that this gives you directly the fourier transform of the charge distribution of your target:

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{point} \bullet \left|F(q)\right|^2$$

$$F(q) = \int \rho(x) e^{iqx} d^3x$$

Once the target is not static, we're best of using e-mu scattering as our starting point.

e-proton vs e-muon scattering

- What's different?
- If proton was a spin 1/2 point particle with magnetic moment e/2M then all one needs to do is plug in the proton mass instead of muon mass into:

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right]$$

- However, magnetic moment differs, and we don't have a point particle !!!

e-proton vs e-muon scattering

- Let's go back to where we started:
  - What's the transition current for the proton ?

$$J^\mu = -e\overline{u}(k')\gamma^\mu u(k) e^{i(k'-k)x}$$ electron

$$J^\mu = -e\overline{u}(p')[???]u(p) e^{i(p'-p)x}$$ proton

We need to find the most general parametrization for [???], and then measure its parameters.

What is [???]

- This is a two part problem:
  - What are the allowed 4-vectors in the current ?
  - What are the independent scalars that the dynamics can depend on ?
- Let’s answer the second first:

  $$M^2 = p'^2 = (p+q)^2 = M^2 + 2pq + q^2$$

- I can thus pick either pq or q^2 as my scalar variable to express dependence on kinematics.

What are the 4-vectors allowed?

- Most general form of the current:

$$J^{\mu} = -e\overline{u}(p')[???]u(p)e^{i(p'-p)x}$$

$$[???] = \gamma^{\mu}K_1 + i\sigma^{\mu\nu}(p'-p)_{\nu}K_2 + i\sigma^{\mu\nu}(p'+p)_{\nu}K_3 +$$

$$+(p'-p)^{\mu}K_4 + (p'+p)^{\mu}K_5$$

- Gordon Decomposition of the current: any term with (p+p') can be expressed as linear sum of components with γ^{μ} and $\sigma^{\mu\nu}$ (p'-p).
- K_4 must be zero because of current conservation.

Gordon Decomposition

- Exercise 6.1 in H&M:

$$\bar{u}(p')\gamma^{\mu}u(p) = \bar{u}(p')\left[(p'+p) + i\sigma^{\mu\nu}(p'-p)_{\nu}\right]u(p)$$

- I leave it as a future homework to show this.

Current Conservation

$$q_\mu J^\mu = 0$$

$$0 = q_\mu \overline{u}(p')\left[\gamma^\mu K_1 + i\sigma^{\mu\nu}(p' - p)_\nu K_2 + (p' + p)^\mu K_5\right]u(p)$$

$$q_\mu \gamma^\mu \psi = m\psi \approx 0$$ because of relativistic limit.

$$q_\mu \sigma^{\mu\nu} q_\nu = 0$$ because sigma is anti-symmetric.

As a result, K_5 must be zero.

$$J^\mu = -e\overline{u}(p')\left[\gamma^\mu F_1\left(q^2\right) + i\sigma^{\mu\nu} q_\nu \frac{\kappa}{2M} F_2\left(q^2\right)\right]u(p)e^{i(p'-p)x}$$

Proton Current

$$J^{\mu} = -e\overline{u}(p')\left[\gamma^{\mu}F_1\left(q^2\right) + i\sigma^{\mu\nu}q_{\nu}\frac{\kappa}{2M}F_2\left(q^2\right)\right]u(p)e^{i(p'-p)x}$$

We determine F1, F2, and kappa experimentally, with the constraint that F1(0)=1=F2(0) in order for kappa to have the meaning of the anomalous magnetic moment.

The two form factors F1 and F2 parametrize our ignorance regarding the detailed structure of the proton.