Homework solution #1

① (a) $M_W = 80.425 \pm 0.038 \ \text{GeV}/c^2$

$M_Z = 91.1876 \pm 0.0021 \ \text{GeV}/c^2$

$\Delta E \cdot \Delta t \sim \hbar \Rightarrow R \sim c\Delta t \sim \frac{\hbar c}{\Delta E}$

$\Rightarrow R_W \sim 2.5 \, 10^{-3}$ fm

$R_Z \sim 2.2 \, 10^{-3}$ fm

charge radius of proton $R_p = 0.870 \pm 0.08$ fm

$$\Rightarrow \boxed{\frac{R_{W/Z}}{R_p} \sim \frac{1}{400}}$$

(b) In the Lab, time dilation is responsible for $\tau \to \gamma\tau$

$L = \beta \cdot \gamma\tau$ with $\beta = \frac{p}{\gamma m}$

$\Rightarrow L = \frac{p}{m} \cdot \tau$. To get from natural units to meters: $\boxed{L = c \frac{p}{m} \tau}$ Use $p = 10$ GeV for all.

μ^+: $L = 63$ km ; τ^+: $L = 480 \ \mu$m ; K^+: $L = 72$ m

π^+: 560 m ; D^+: $L = 1.8$ mm ; B^+: $L = 0.9$ mm

all of the above decay weakly.

1b) continued:

π^0: $L = 2\,\mu m$ Electromagnetic decay

ρ^0: $L = 17\,fm$; J/ψ : $L = 735\,fm$

strong decay both strong and EM decay.

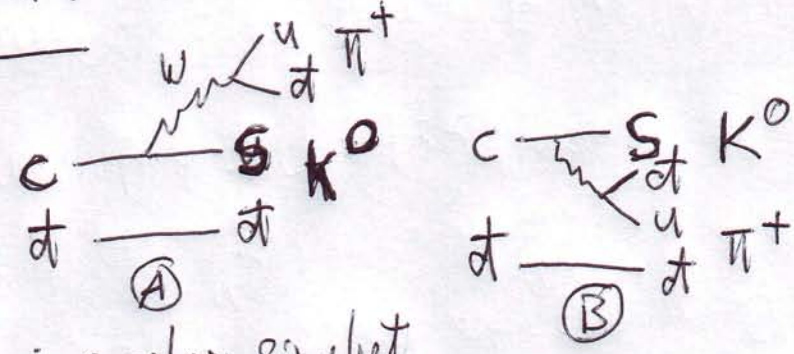
J/ψ is special. Its strong decay is suppressed to the part that it's comparable to its EM decay.

1c) Decay of muon into hadrons is kinematically forbidden due to $m_\mu < m_\pi$.
The same is not true for τ.
Accordingly, a large fraction of τ decays to hadrons, while μ can only decay to $e \bar{\nu}_e \nu_\mu$.

d) $\pi^+ \to \mu^+ \nu_\mu$
$\hookrightarrow e^+ \nu_e \bar{\nu}_\mu$ $\Rightarrow$ $\dfrac{\#\text{ of } \overset{(-)}{\nu}_\mu}{\#\text{ of } \nu_e} = \dfrac{2}{1}$

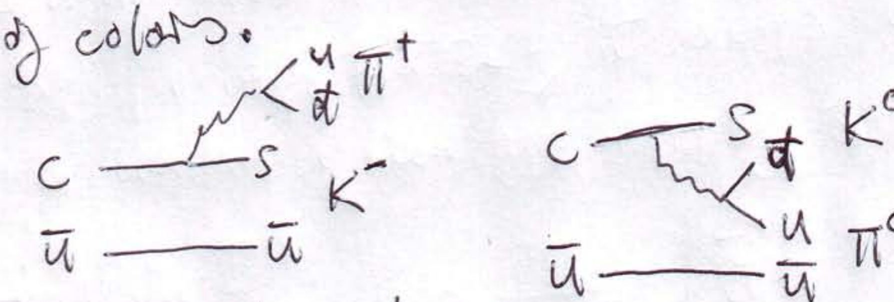
1 e) continued:

D^+



The W is a color singlet.
There is thus no color correlation between the u and d̄ that form the π^+ in B while there is such a color correlation in A. The amplitude ratio $\frac{A}{B} \sim \frac{3}{1}$ where the 3 comes from the # of colors.

D^0



The D^+ thus has two interfering amplitudes while the D^0 does not. If the amplitudes A + B destructively interfer then:

$$A_{D^+} \sim A(1 - 1/3) \qquad A_{D^0} \sim A \quad \Rightarrow \quad \frac{\Gamma_{D^+}}{\Gamma_{D^0}} = \frac{\tau_{D^0}}{\tau_{D^+}} \sim \frac{4/9}{1}$$

1e) continued

measured values:

$$c\tau_{D^+} = 312\,\mu m \qquad\qquad c\tau_{D^0} = 123\,\mu m \quad \rightarrow \quad \frac{\tau_{D^0}}{\tau_{D^+}} = 0.4$$

This compares astonishingly well with the simple argument leading to $4/9 = 0.44$

To be fair, this agreement is a bit of an accident!!!

(2)

(a)

$$\overset{m_1}{\underset{p_1}{\longleftarrow}} M \overset{m_2}{\underset{p_2}{\longrightarrow}} \qquad P_M = (M, \vec{0})$$

$$p_1 = (E_1, \vec{p}) \qquad p_2 = (E_2, -\vec{p})$$

$$P_M = p_1 + p_2 \rightarrow p_1^2 = (P_M - p_2)^2$$

$$m_1^2 = M^2 + m_2^2 - 2P_M p_2$$

$$= M^2 + m_2^2 - 2ME_2$$

$$\Rightarrow \boxed{E_2 = \frac{M^2 + m_2^2 - m_1^2}{2M}}$$

$$\boxed{E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}}$$

HW1 p.5

(2) continued

(a) $$T_1 = E_1 - m_1 = \frac{(M - m_1 - m_2)\cdot (M + m_2 - m_1)}{2M}$$

$m_1 > m_2$ for $\mu = m_1 + m_2 > 0$

$\delta m = m_1 - m_2 > 0$

$$\boxed{E_1 > E_2} \quad \boxed{T_2 > T_1}$$

$$\boxed{T_1 = \frac{(M-\mu)\cdot(M-\delta m)}{2M}}$$

$$\boxed{T_2 = \frac{(M-\mu)\cdot(M+\delta m)}{2M}}$$

Next, calculate momentum:

$$p^2 = E_1^2 - m_1^2 = E_2^2 - m_2^2$$

$$4M^2 p^2 = \underbrace{M^4 + m_2^4 + m_1^4}_{} - 2M^2 m_2^2 + 2M^2 m_1^2 - 2m_1^2 m_2^2 - 4M^2 m_1^2$$

$$4M^2 p^2 = \quad " \quad - 2M^2(m_1^2 + m_2^2) - 2m_1^2 m_2^2$$

$$= M^4 + (m_1^2 - m_2^2)^2 - M^2(\mu^2 + \delta m^2)$$

$$= M^4 + \mu^2 \delta m^2 - M^2(\mu^2 + \delta m^2)$$

$$= (M^2 - \mu^2)\cdot(M^2 - \delta m^2) \Rightarrow \boxed{P = \sqrt{\frac{(M^2-\mu^2)(M^2-\delta m^2)}{4M^2}}}$$

② (b) continued

(b) $D^{*+} \to D^0 \pi^+$

$M = 2010$ MeV $\quad m_1 = 1864$ MeV $\quad m_2 = 140$ MeV

$\mu = 2004$ MeV $\quad \delta m = 1724$ MeV

$E_{D^0} = 1864$ MeV $\quad T_{D^0} = 0.42$ MeV

$E_{\pi^+} = 146$ MeV $\quad T_{\pi^+} = 6$ MeV

$P = 40$ MeV

(C) Transverse component remains unchanged.
Longitudinal component ($P_z = P\cos\Theta$)
gets boosted to:

$$P_z' = \gamma(P_z + \beta E) = \gamma(P\cos\theta + \beta E)$$

Total momentum in the labframe:

$$P' = \sqrt{P_T^2 + P_z'^2} = \sqrt{P^2\sin^2\Theta + \gamma^2(P\cos\theta + \beta E)^2}$$

q = momentum of D^* in lab frame

$$q = \gamma m_{D^*}\beta \implies \gamma\beta = q/m_{D^*}$$

$$\gamma = \frac{E_{D^*}}{m_{D^*}} = \sqrt{q^2 + m_{D^*}^2}/m_{D^*}$$

(2) b continued:

Combining it:

$$P' = \sqrt{P^2(1-\cos^2\theta) + \left(\cos\theta \frac{\sqrt{q^2+m_{D^*}^2}\,P}{m_{D^*}} + \frac{qE}{m_{D^*}}\right)^2}$$

Note: $P \simeq 40\,\text{MeV} \ll E = 146\,\text{MeV}$

as long as $\frac{q}{m_{D^*}} \gg \frac{P}{E} \simeq \frac{1}{4}$ we get

P' to be roughly independent of $\cos\theta$!!!

i.e. for $q \gg 500\,\text{MeV}$ $\cos\theta$ doesn't matter much.

In that case $P' \simeq \frac{qE}{m_{D^*}}$

3) $y = \frac{1}{2} \ln \frac{E+p_L}{E-p_L}$

HW1 p. 8

(a) I had a typo in the question (3(a)). My apologies for that you were meant to show that:

$$\frac{d^3p}{E} = \frac{1}{2}\, dp_T^2\, dy\, d\phi$$

Here's how:

$$\frac{d^3p}{E} = \frac{dp_T^2\, dp_L\, d\phi}{2E}$$

Nothing more than volume element in cylindrical coord. syst.

Let x be defined as:

$$x = \frac{p_L}{E} \Rightarrow y = \frac{1}{2} \ln \frac{1+x}{1-x}$$

$$\frac{dy}{dx} = \frac{1}{2}\, \frac{1-x}{1+x}\, \frac{1-\cancel{x}+1+\cancel{x}}{(1-\cancel{x})^2} = \cancel{(1-x)^2}$$

$\cancel{dy = (1-x)^{-2}\, dx}$

$$\frac{dy}{dx} = \frac{1}{2}\, \frac{\cancel{1-x}}{1+x}\, \frac{2}{(1-x)^{\cancel{2}}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{1-x^2}}$$

3 a) continued

Next we want to see $\frac{dx}{dP_L}$

$$x = \frac{P_L}{\sqrt{P_L^2 + m^2 + P_T^2}} = \frac{P_L}{E}$$

$$\frac{dx}{dP_L} = \frac{E - \frac{1}{2E} 2P_L^2}{E^2} = \boxed{\frac{1-x^2}{E} = \frac{dx}{dP_L}}$$

Now combine:

$$\frac{dy}{dP_L} = \frac{dy}{dx}\frac{dx}{dP_L} = \frac{1-x^2}{E} \cdot \frac{1}{1-x^2} \cdot$$

$$\frac{dy}{dP_L} = \frac{1}{E}$$

$$\boxed{dy = \frac{1}{E} dP_L}$$

$$\frac{dP_T^2 \, dP_L \, d\phi}{2E} = \frac{dP_T^2 \, dy \, d\phi}{2} \quad \text{qed}$$

③ (b) continued:

Longitudinal boost means:

$$P_L \to P_L' = \gamma\,(P_L + \beta E)$$

$$E \to E' = \gamma\,(E + \beta P_L)$$

$$P_T \to P_T' = P_T$$

You ~~are asked to~~ first calculate $y \to y'$

$$y' = \frac{1}{2} \ln\left[\frac{E' + P_L'}{E' - P_L'}\right] = \frac{1}{2} \ln \frac{(E' + P_L')^2}{\underbrace{E'^2 - P_L'^2}_{= P_T^2 + m^2}}$$

$$E'^2 = P_L'^2 + P_T'^2 + m^2 \Rightarrow$$

denominator is thus independend of the boost

To show that the Δy between two particles is boost invariant: $\Delta y = y_1 - y_2$

$$\Delta y \to \Delta y' = \frac{1}{2} \ln(E_1' + P_{L_1}')^2 - \frac{1}{2} \ln(E_2' + P_{L2}')^2 + \text{terms that are boost invariant}$$

(3b) continued

Now look at $E' + P_L'^2$

$$E' + P_L' = \gamma E + \gamma\beta E + \gamma P_L + \gamma\beta P_L$$
$$= E(\gamma + \gamma\beta) + P_L(\gamma + \gamma\beta)$$
$$= (E + P_L)\,\gamma + \gamma\beta$$

Plug it into the log of y:

$$\Delta y' = \ln\left[(E_1 + P_{L1})(\gamma + \gamma\beta)\right] - \ln\left[(E_2 + P_{L2})(\gamma + \gamma\beta)\right]$$

+ terms that are boost invariant

$$= \ln(E_1 + P_{L1}) - \ln(E_2 + P_{L2})$$ + terms that are boost invariant

We have thus shown that the rapidity difference between two particles does not change under a boost along the z-axis.

3(c) continued:

(c) $y = \frac{1}{2} \ln \frac{E + P_L}{E - P_L} = \ln \frac{E + P_L}{\sqrt{P_T^2 + m^2}}$

For a given $|P_T|, |P_L|$ we obviously get

$$\left.\begin{aligned} y_{max} &= \ln \frac{E + |P_L|}{\sqrt{P_T^2 + m^2}} \\ y_{min} &= \ln \frac{E - |P_L|}{\sqrt{P_T^2 + m^2}} \end{aligned}\right\} \begin{aligned} y_{min} &= y(-|P_L|) \\ y_{max} &= y(+|P_L|) \end{aligned}$$

$$y_{min} = \ln \frac{E - |P_L|}{E - (-|P_L|)} = \ln \left[\left(\frac{E + |P_L|}{E - |P_L|} \right)^{-1} \right]$$

$y_{min} = - y_{max}$

In addition, we get maximum $|y|$ at a given E if $P_T = 0$

$$\Rightarrow \quad y_{max} = \ln \left[\frac{(E + P_L)_{Max}}{m} \right]$$

All we need to do now is relate $(E + P_L)_{Max}$ to the $\sqrt{s}$ of the $p\bar{p}$ collission.

(3c) continued:

Lets define "junk" = all of the $p\bar{p}$ collission other than one particle

$$p \longrightarrow \quad \longleftarrow \bar{p} \qquad \text{in cm}$$

$$\longleftarrow \quad \Longrightarrow \text{junk} \qquad \text{outgoing}$$

$$E + E_{junk} = \sqrt{s} \qquad E_{junk} = \sqrt{p_L^2 + m_X^2}$$

~~$E = E_{junk}$~~

$$\Rightarrow \quad E = \sqrt{s} - \sqrt{p_L^2 + m_X^2}$$

$$(E + p_L)_{Max} = \left(\sqrt{s} + p_L - \sqrt{p_L^2 + m_X^2}\right)_{Max}$$

This is max for $m_X = 0$

$$\Rightarrow \quad (E + p_L)_{Max} = \sqrt{s}$$

$$\Rightarrow \quad y_{Max} = \ln \frac{\sqrt{s}}{m} = \frac{1}{2} \ln \frac{s}{m^2}$$

$$y_{min} = -y_{max} = -\frac{1}{2} \ln \frac{s}{m^2} \qquad \text{qed}$$

(3 d) continued:

We have already shown that Δy is boost invariant.
And it is obvious that both ϕ, p_T are invariant under longitudinal boosts.

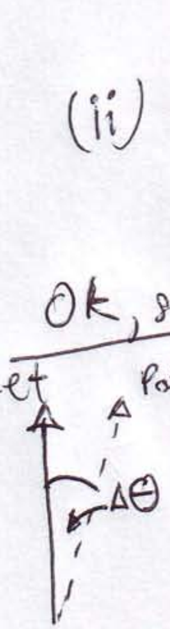
We then approximate $\eta \sim y$
and only need to show that jets are circular in η-ϕ when emitted straight off the z-axis, i.e. at $\Theta = 90^\circ$, i.e. in the x-y plane at $z = \emptyset$.

If we can show this, then

(i) jets will be circular for all y
(because Δy is boost invariant)

(ii) the size of a jet with given p_T is independent of Θ

Ok, so how do we show it at 90°?



$$\Delta\Theta \sim \frac{q_T}{q_L} \qquad \Delta\phi \sim \frac{q_T}{q_L \cdot \sin\Theta} = \frac{q_T}{q_L} \text{ for } \Theta = 90^\circ$$

$$\Rightarrow \Delta\Theta = \Delta\phi \text{ at } \Theta = 90^\circ$$

Next we need to understand $\Delta\theta$ in terms of $\Delta\eta$

$$\eta = \log \cot\frac{\theta}{2}$$

$$\frac{d\eta}{d\theta} = \tan\theta/2 \cdot \frac{d}{d\theta}\frac{\cos\theta/2}{\sin\theta/2} = \tan\theta/2 \;\frac{-\sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$= \frac{1}{2}\tan\theta/2 \cdot (-1) \cdot \frac{1}{\sin^2\theta/2}$$

$$\sin^2\theta/2 = (1-\cos\theta)/2$$

$$\tan\theta/2 = \frac{1-\cos\theta}{\sin\theta} \simeq 1-\cos\theta \quad \text{for } \theta = 90^\circ$$

$$\frac{d\eta}{d\theta} \simeq \frac{1}{2}(1-\cos\theta)(-1)\frac{2}{1-\cos\theta} = -1$$

$\therefore$ $\Delta\eta \approx \Delta\theta$ up to an inconsequential $-$ sign.

as $\Delta\theta \approx \Delta\phi$ $\therefore$ $\Delta\eta \approx \Delta\phi$ at $90^\circ = \theta$

qed