HOMEWORK ASSIGNMENT # 5

November 05, 2009

1. Solve the differential equation

$$x y'' - (x+1) y' + y = -1$$

so that $y(1) = y'(1) = 0$.

2. We have seen by now that it is no big a deal to solve a differential equation with <u>constant</u> coefficients, with $f(x) \sim e^{rx}$ [so long as $r \neq r_1, r_2, \ldots$ of the complementary function].

How about if $f(x) \sim r^m e^{rx}$ $(m = 1, 2, 3, \ldots)$ [instead of the mere e^{rx}?

What would the solution then be?

In the light of this, solve the differential equation

$$y'' + 3y' + 2y = x e^{x}.$$

3. Next, solve the differential equation

$$x^2 y'' - 2y = x.$$

[In this one, you may have to use the Wronskian $W\{y_1, y_2\}$.]

4. Develop the series solutions of the Hermite equation

$$y'' - 2xy' + 2\alpha y = 0 \qquad (0 \leq x \leq \infty),$$

expanded around the point $x = 0$.

Examine the convergence of these solutions as $x \longrightarrow \infty$, and determine the values of α for which one or the other of the two series becomes a polynomial. Write down the first three terms of the resulting polynomials.

5. The spherically symmetric solutions of the hydrogen atom are determined by a function $u(\rho)$ which satisfies the (associated) Laguerre equation

$$\rho u'' + (2-\rho)u' + (\lambda - 1)u = 0 \qquad (0 \leq \rho \leq \infty);$$

here, ρ is a measure of the radial distance r and λ a measure of the total energy E of the atom.

Develop a series solution of this equation that is regular at $\rho = 0$, writing terms up to ρ^3 explicitly.

Examine the convergence of this series and choose λ so that the series terminates and becomes a polynomial.

6. Check that the function $\sin x / x$ is a solution of the differential equation

$$x^2 y'' + 2xy' + x^2 y = 0.$$

With $\sin x / x$ known to be a solution, what is the other solution of this equation?