HOMEWORK ASSIGNMENT #4

October 29, 2009

1. Evaluate the Fourier transform of the function $f(x) = \frac{1}{(x^2+a^2)^2}$

2. Using the representation

$$K_\nu(x) = \frac{1}{2}\left(\frac{x}{2}\right)^\nu \int_0^\infty e^{-t-\frac{x^2}{4t}} \, t^{-\nu-1} \, dt,$$

determine the leading behavior of this function as $x \to 0$, with ν fixed.

3. Using a different representation of the same function, viz.

$$K_\nu(x) = \int_0^\infty e^{-x\cosh t} \cosh(\nu t) \, dt,$$

determine the leading behavior of $K_\nu(x)$ as $\nu \to \infty$, with x fixed. Check that the answer obtained here tallies with the one obtained in Problem 2.

4. The Faxén integral $Fi(\alpha, \beta; x)$ is given by

$$I(x) = \int_0^\infty e^{-t+xt^\alpha} \cdot t^{\beta-1} \, dt \qquad (\beta > 0,\ 0 < \alpha < 1).$$

Determine the asymptotic behavior of this integral for $x \gg 1$.

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In contrast, what would the corresponding behavior of the integral

$$J(x) = \int_0^\infty e^{-t-xt^\alpha} \, t^{\beta-1} \, dt \qquad \text{be?}$$

5. Using Schläfli's representation for the Bessel function $J_\nu(z)$, show that, for $\nu \gg 1$,

$$J_\nu(\nu) \approx \frac{2^{1/3}}{3^{2/3}\,\Gamma(2/3)} \, \frac{1}{\nu^{1/3}} .$$

6. Show that, for $s \gg 1$,

$$\int_{-\infty}^{\infty} \frac{\cos(st)}{(1+t^2)^s} \, dt \approx \sqrt{\frac{\pi\sqrt{2}\,c}{s}} \, \frac{e^{-cs}}{(2c)^s} ,$$

where $c = \sqrt{2} - 1$.

[Hint: convert the given integral into a contour integral of the form

$$-i \int_{-i\infty}^{i\infty} e^{s f(z)} \, dz ,$$

and apply the method of steepest descent.]