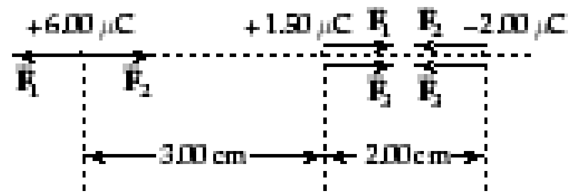
Problem Solutions

15.1 Since the charges have opposite signs, the force is one of $\boxed{\text{attraction}}$.
Its magnitude is

$$F = \frac{k_e|q_1 q_2|}{r^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(4.5\times10^{-9}\ \mathrm{C}\right)\left(2.8\times10^{-9}\ \mathrm{C}\right)}{\left(3.2\ \mathrm{m}\right)^2} = \boxed{1.1\times10^{-8}\ \mathrm{N}}$$

15.10 The forces are as shown in the sketch at the right.



$$F_1 = \frac{k_e q_1 q_2}{r_{12}^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(6.00\times10^{-6}\ \mathrm{C}\right)\left(1.50\times10^{-6}\ \mathrm{C}\right)}{\left(3.00\times10^{-2}\ \mathrm{m}\right)^2} = 89.9\ \mathrm{N}$$

$$F_2 = \frac{k_e q_1 |q_3|}{r_{13}^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(6.00\times10^{-6}\ \mathrm{C}\right)\left(2.00\times10^{-6}\ \mathrm{C}\right)}{\left(5.00\times10^{-2}\ \mathrm{m}\right)^2} = 43.2\ \mathrm{N}$$

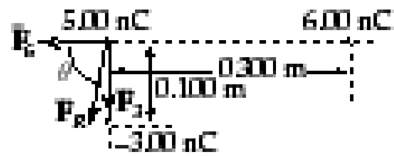
$$F_3 = \frac{k_e q_2 |q_3|}{r_{23}^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(1.50\times10^{-6}\ \mathrm{C}\right)\left(2.00\times10^{-6}\ \mathrm{C}\right)}{\left(2.00\times10^{-2}\ \mathrm{m}\right)^2} = 67.4\ \mathrm{N}$$

The net force on the $6\,\mu\mathrm{C}$ charge is $F_6 = F_1 - F_2 = \boxed{46.7\ \text{N (to the left)}}$

The net force on the $1.5\,\mu\mathrm{C}$ charge is $F_{1.5} = F_1 + F_3 = \boxed{157\ \text{N (to the right)}}$

The net force on the $-2\,\mu\mathrm{C}$ charge is $F_{-2} = F_2 + F_3 = \boxed{111\ \text{N (to the left)}}$

15.11 In the sketch at the right, F_R is the resultant of the forces F_6 and F_3 that are exerted on the charge at the origin by the 6.00 nC and the –3.00 nC charges respectively.



$$F_6 = \left(8.99\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(6.00\times 10^{-9}\ \mathrm{C}\right)\left(5.00\times 10^{9}\ \mathrm{C}\right)}{\left(0.300\ \mathrm{m}\right)^2}$$

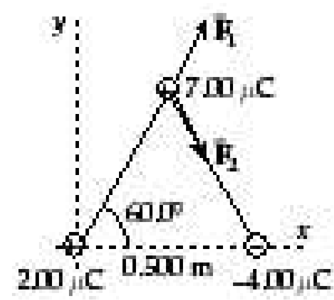
$$= 3.00\times 10^{-6}\ \mathrm{N}$$

$$F_3 = \left(8.99\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(3.00\times 10^{-9}\ \mathrm{C}\right)\left(5.00\times 10^{9}\ \mathrm{C}\right)}{\left(0.100\ \mathrm{m}\right)^2} = 1.35\times 10^{-5}\ \mathrm{N}$$

The resultant is $F_R = \sqrt{\left(F_6\right)^2 + \left(F_3\right)^2} = 1.38\times 10^{-5}\ \mathrm{N}$ at $\theta = \tan^{-1}\left(\frac{F_3}{F_6}\right) = 77.5°$

or $\quad \mathbf{F}_R = \boxed{1.38\times 10^{-5}\ \mathrm{N}\ \text{at}\ 77.5°\ \text{below}\ -x\ \text{axis}}$

15.13 The forces on the 7.00 μC charge are shown at the right.



$$F_1 = \left(8.99\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(7.00\times 10^{-6}\ \mathrm{C}\right)\left(2.00\times 10^{-6}\ \mathrm{C}\right)}{\left(0.500\ \mathrm{m}\right)^2}$$

$$= 0.503\ \mathrm{N}$$

$$F_2 = \left(8.99\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(7.00\times 10^{-6}\ \mathrm{C}\right)\left(4.00\times 10^{-6}\ \mathrm{C}\right)}{\left(0.500\ \mathrm{m}\right)^2}$$

$$= 1.01\ \mathrm{N}$$

Thus, $\Sigma F_x = \left(F_1 + F_2\right)\cos 60.0° = 0.755\ \mathrm{N}$

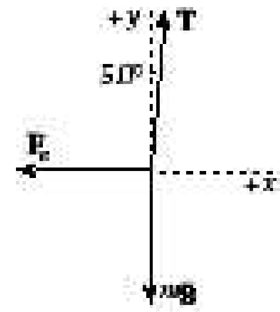
and $\Sigma F_y = \left(F_1 - F_2\right)\sin 60.0° = -0.436\ \mathrm{N}$

The resultant force on the 7.00 μC charge is

$$F_R = \sqrt{\left(\Sigma F_x\right)^2 + \left(\Sigma F_y\right)^2} = 0.872\ \mathrm{N}\ \text{at}\ \theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = -30.0°$$

or $\quad \mathbf{F}_R = \boxed{0.872\ \mathrm{N}\ \ \text{at}\ 30.0°\ \text{below the}\ +x\ \text{axis}}$

15.15 Consider the free-body diagram of one of the spheres given at the right. Here, T is the tension in the string and F_e is the repulsive electrical force exerted by the other sphere.



$$\Sigma F_y = 0 \Rightarrow T\cos 5.0° = mg\text{, or } T = \frac{mg}{\cos 5.0°}$$

$$\Sigma F_x = 0 \Rightarrow F_e = T\sin 5.0° = mg\tan 5.0°$$

At equilibrium, the distance separating the two spheres is $r = 2L\sin 5.0°$.

Thus, $F_e = mg\tan 5.0°$ becomes $\dfrac{k_e q^2}{(2L\sin 5.0°)^2} = mg\tan 5.0°$ and yields

$$q = (2L\sin 5.0°)\sqrt{\frac{mg\tan 5.0°}{k_e}}$$

$$= \left[2(0.300\text{ m})\sin 5.0°\right]\sqrt{\frac{(0.20\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 5.0°}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{7.2\text{ nC}}$$

15.17 For the object to "float" it is necessary that the electrical force support the weight, or

$$qE = mg \quad\text{ or }\quad m = \frac{qE}{g} = \frac{(24\times 10^{-6}\text{ C})(610\text{ N/C})}{9.8\text{ m/s}^2} = \boxed{1.5\times 10^{-3}\text{ kg}}$$

15.20 (a) The magnitude of the force on the electron is $F = |q|E = eE$, and the acceleration is

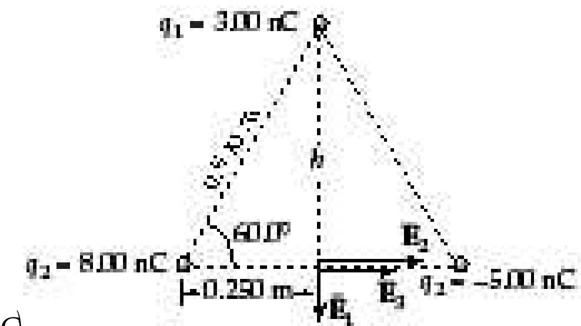
$$a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{(1.60\times 10^{-19}\text{ C})(300\text{ N/C})}{9.11\times 10^{-31}\text{ kg}} = \boxed{5.27\times 10^{13}\text{ m/s}^2}$$

(b) $v = v_0 + at = 0 + (5.27\times 10^{13}\text{ m/s}^2)(1.00\times 10^{-8}\text{ s}) = \boxed{5.27\times 10^5\text{ m/s}}$

15.24 The altitude of the triangle is

$$h = (0.500\ \text{m})\sin 60.0° = 0.433\ \text{m}$$

and the magnitudes of the fields due to each of the charges are



$$E_1 = \frac{k_e q_1}{h^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-9}\ \text{C}\right)}{(0.433\ \text{m})^2}$$

$$= 144\ \text{N/C}$$

$$E_2 = \frac{k_e q_2}{r_2^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times10^{-9}\ \text{C}\right)}{(0.250\ \text{m})^2} = 1.15\times10^3\ \text{N/C}$$

and $$E_3 = \frac{k_e |q_3|}{r_3^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-9}\ \text{C}\right)}{(0.250\ \text{m})^2} = 719\ \text{N/C}$$

Thus, $\Sigma E_x = E_2 + E_3 = 1.87\times10^3\ \text{N/C}$ and $\Sigma E_y = -E_1 = -144\ \text{N/C}$ giving

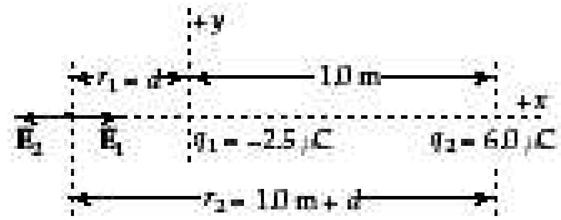
$$E_R = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2} = 1.88\times10^3\ \text{N/C}$$

and

$$\theta = \tan^{-1}\left(\Sigma E_y/\Sigma E_x\right) = \tan^{-1}(-0.0769) = -4.40°$$

Hence $\boxed{\mathbf{E}_R = 1.88\times10^3\ \text{N/C at } 4.40° \text{ below the } +x \text{ axis}}$

15.27 If the resultant field is zero, the contributions from the two charges must be in opposite directions and also have equal magnitudes. Choose the line connecting the charges as the x-axis, with the origin at the –2.5 μC charge. Then, the two contributions will have opposite directions only in the regions $x<0$ and $x>1.0$ m. For the magnitudes to be equal, the point must be nearer the smaller charge. Thus, the point of zero resultant field is on the x-axis at $x<0$.



Requiring equal magnitudes gives $\frac{k_e|q_1|}{r_1^2}=\frac{k_e|q_2|}{r_2^2}$ or $\frac{2.5\,\mu\text{C}}{d^2}=\frac{6.0\,\mu\text{C}}{(1.0\text{ m}+d)^2}$

Thus, $(1.0\text{ m}+d)\sqrt{\frac{2.5}{6.0}}=d$

Solving for d yields

$d=1.8$ m, or $\boxed{1.8\text{ m to the left of the }-2.5\,\mu\text{C charge}}$

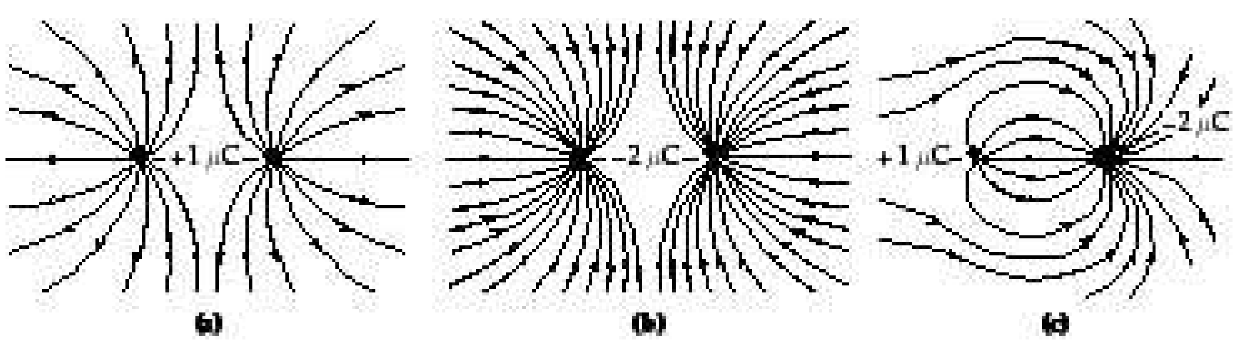
15.28 The magnitude of q_2 is three times the magnitude of q_1 because 3 times as many lines emerge from q_2 as enter q_1. $|q_2|=3|q_1|$

(a) Then, $\boxed{q_1/q_2=-1/3}$

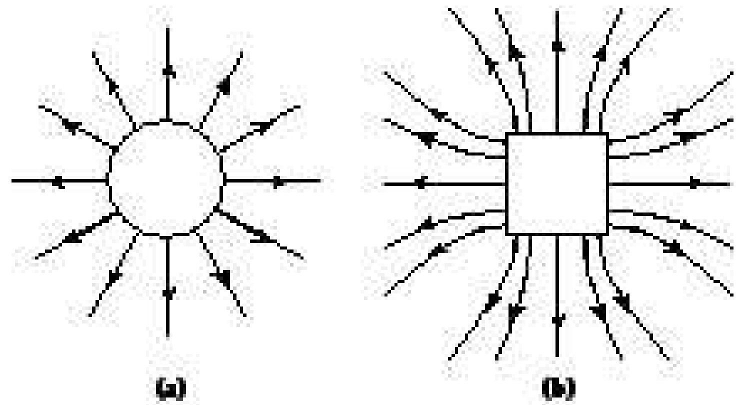
(b) $\boxed{q_2>0}$ because lines emerge from it,

and $\boxed{q_1<0}$ because lines terminate on it.

15.30 Rough sketches for these charge configurations are shown below.



15.32 (a) In the sketch for (a) at the right, note that there are no lines inside the sphere. On the outside of the sphere, the field lines are uniformly spaced and radially outward.



(b) In the sketch for (b) above, note that the lines are perpendicular to the surface at the points where they emerge. They should also be symmetrical about the symmetry axes of the cube. The field is zero inside the cube.

15.36 If the weight of the drop is balanced by the electric force, then $mg = |q|E = eE$ or the mass of the drop must be

$$m = \frac{eE}{g} = \frac{\left(1.6\times10^{-19}\ \text{C}\right)\left(3\times10^{4}\ \text{N/C}\right)}{9.8\ \text{m/s}^2} \approx 5\times10^{-16}\ \text{kg}$$

But, $m = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)$ and the radius of the drop is $r = \left[\frac{3m}{4\pi\rho}\right]^{1/3}$

$$r = \left[\frac{3\left(5\times10^{-16}\ \text{kg}\right)}{4\pi\left(858\ \text{kg/m}^3\right)}\right]^{1/3} = 5.2\times10^{-7}\ \text{m} \qquad \text{or} \qquad r \sim \boxed{1\ \mu\text{m}}$$

15.38 The flux through an area is $\Phi_E = EA\cos\theta$, where θ is the angle between the direction of the field E and the line perpendicular to the area A.

(a) $\Phi_E = EA\cos\theta = \left(6.2\times10^5\ \text{N/C}\right)\left(3.2\ \text{m}^2\right)\cos 0° = \boxed{2.0\times10^6\ \text{N}\cdot\text{m}^2/\text{C}}$

(b) In this case, $\theta = 90°$ and $\Phi_E = \boxed{0}$

15.43 We choose a spherical gaussian surface, concentric with the charged spherical shell and of radius r. Then, $\Sigma EA\cos\theta = E\left(4\pi r^2\right)\cos 0° = 4\pi r^2 E$.

(a) For $r > a$ (that is, outside the shell), the total charge enclosed by the gaussian surface is $Q = +q - q = 0$. Thus, Gauss's law gives $4\pi r^2 E = 0$, or $E = 0$.

(b) Inside the shell, $r < a$, and the enclosed charge is $Q = +q$.

Therefore, from Gauss's law, $4\pi r^2 E = \dfrac{q}{\epsilon_0}$, or $E = \dfrac{q}{4\pi \epsilon_0 r^2} = \dfrac{k_e q}{r^2}$

The field for $r < a$ is $\boxed{\vec{\mathbf{E}} = \dfrac{k_e q}{r^2} \text{ directed radially outward}}$.

15.46 Choose a very small cylindrical gaussian surface with one end inside the conductor. Position the other end parallel to and just outside the surface of the conductor.

Since, in static conditions, $E = 0$ at all points inside a conductor, there is no flux through the inside end cap of the gaussian surface. At all points outside, but very close to, a conductor the electric field is perpendicular to the conducting surface. Thus, it is parallel to the cylindrical side of the gaussian surface and no flux passes through this cylindrical side. The total flux through the gaussian surface is then $\Phi = EA$, where A is the cross-sectional area of the cylinder as well as the area of the end cap.

The total charge enclosed by the cylindrical gaussian surface is $Q = \sigma A$, where σ is the charge density on the conducting surface. Hence, Gauss's law gives

$$EA = \frac{\sigma A}{\epsilon_0} \quad \text{or} \quad E = \boxed{\frac{\sigma}{\epsilon_o}}$$

15.48 (a)

$$F = \frac{k_e |q_1||q_2|}{r^2} = \frac{k_e e^2}{r^2}$$

$$= \frac{\left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19} \text{ C}\right)^2}{\left(0.53\times 10^{-10} \text{ m}\right)^2} = \boxed{8.2\times 10^{-8} \text{ N}}$$

(b) $F = m_e a_c = m_e\left(v^2/r\right)$, so

$$v = \sqrt{\frac{r\cdot F}{m_e}} = \sqrt{\frac{\left(0.53\times 10^{-10} \text{ m}\right)\left(8.2\times 10^{-8} \text{ N}\right)}{9.11\times 10^{-31} \text{ kg}}} = \boxed{2.2\times 10^{6} \text{ m/s}}$$