Chapter 21
Alternating Current Circuits and Electromagnetic Waves

Problem Solutions

21.1 (a) $\Delta V_{max} = \sqrt{2}(\Delta V_{rms}) = \sqrt{2}(100\text{ V}) = \boxed{141\text{ V}}$

(b) $I_{rms} = \dfrac{\Delta V_{rms}}{R} = \dfrac{100\text{ V}}{5.00\ \Omega} = \boxed{20.0\text{ A}}$

(c) $I_{max} = \dfrac{\Delta V_{max}}{R} = \dfrac{141\text{ V}}{5.00\ \Omega} = \boxed{28.3\text{ A}}$ or $I_{max} = \sqrt{2}\,I_{rms} = \sqrt{2}(20.0\text{ A}) = \boxed{28.3\text{ A}}$

(d) $\mathcal{P}_{av} = I_{rms}^2 R = (20.0\text{ A})^2(5.00\ \Omega) = 2.00\times10^3\text{ W} = \boxed{2.00\text{ kW}}$

21.11 $I_{rms} = \dfrac{\Delta V_{rms}}{X_C} = 2\pi f C\left(\dfrac{\Delta V_{max}}{\sqrt{2}}\right) = \pi f C(\Delta V_{max})\sqrt{2}$

so $C = \dfrac{I}{\pi f(\Delta V_{max})\sqrt{2}} = \dfrac{0.75\text{ A}}{\pi(60\text{ Hz})(170\text{ V})\sqrt{2}} = 1.7\times10^{-5}\text{ F} = \boxed{17\ \mu\text{F}}$

21.15 The ratio of inductive reactance at $f_2 = 50.0\text{ Hz}$ to that at $f_1 = 60.0\text{ Hz}$ is

$$\frac{(X_L)_2}{(X_L)_1} = \frac{2\pi f_2 L}{2\pi f_1 L} = \frac{f_2}{f_1},\ \text{so}\ (X_L)_2 = \frac{f_2}{f_1}(X_L)_1 = \frac{50.0\text{ Hz}}{60.0\text{ Hz}}(54.0\,\Omega) = 45.0\,\Omega$$

The maximum current at $f_2 = 50.0\text{ Hz}$ is then

$$I_{max} = \frac{\Delta V_{max}}{X_L} = \frac{\sqrt{2}(\Delta V_{rms})}{X_L} = \frac{\sqrt{2}(100\text{ V})}{45.0\ \Omega} = \boxed{3.14\text{ A}}$$

21.19 $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0\text{ Hz})(40.0\times 10^{-6}\text{ F})} = 66.3\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (0 - 66.3\ \Omega)^2} = 83.1\ \Omega$$

(a) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{30.0\text{ V}}{83.1\ \Omega} = \boxed{0.361\text{ A}}$

(b) $\Delta V_{R,\text{rms}} = I_{\text{rms}} R = (0.361\text{ A})(50.0\ \Omega) = \boxed{18.1\text{ V}}$

(c) $\Delta V_{C,\text{rms}} = I_{\text{rms}} X_C = (0.361\text{ A})(66.3\ \Omega) = \boxed{23.9\text{ V}}$

(d) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{0 - 66.3\ \Omega}{50.0\ \Omega}\right) = -53.0°$

so, $\boxed{\text{the voltage lags behind the current by } 53.0°}$

21.33 The resonance frequency of the circuit should match the broadcast frequency of the station.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ gives } L = \frac{1}{4\pi^2 f_0^2 C},$$

or $$L = \frac{1}{4\pi^2(88.9\times 10^6\text{ Hz})^2(1.40\times 10^{-12}\text{ F})} = 2.29\times 10^{-6}\text{ H} = \boxed{2.29\ \mu\text{H}}$$

21.41 (a) At 90% efficiency, $(\mathcal{P}_{\text{av}})_{output} = 0.90(\mathcal{P}_{\text{av}})_{input}$

Thus, if $(\mathcal{P}_{\text{av}})_{output} = 1\,000\text{ kW}$

the input power to the primary is $(\mathcal{P}_{\text{av}})_{input} = \dfrac{(\mathcal{P}_{\text{av}})_{output}}{0.90} = \dfrac{1\,000\text{ kW}}{0.90} = \boxed{1.1\times 10^3\text{ kW}}$

(b) $I_{1,\text{rms}} = \dfrac{(\mathcal{P}_{\text{av}})_{input}}{\Delta V_{1,\text{rms}}} = \dfrac{1.1\times 10^3\text{ kW}}{\Delta V_{1,\text{rms}}} = \dfrac{1.1\times 10^6\text{ W}}{3\,600\text{ V}} = \boxed{3.1\times 10^2\text{ A}}$

(c) $I_{2,\text{rms}} = \dfrac{(\mathcal{P}_{\text{av}})_{output}}{\Delta V_{2,\text{rms}}} = \dfrac{1\,000\text{ kW}}{\Delta V_{1,\text{rms}}} = \dfrac{1.0\times 10^6\text{ W}}{120\text{ V}} = \boxed{8.3\times 10^3\text{ A}}$

21.45 (a) The frequency of an electromagnetic wave is $f = c/\lambda$, where c is the speed of light, and λ is the wavelength of the wave. The frequencies of the two light sources are then

Red: $$f_{red} = \frac{c}{\lambda_{red}} = \frac{3.00\times 10^{8}\ \text{m/s}}{660\times 10^{-9}\ \text{m}} = \boxed{4.55\times 10^{14}\ \text{Hz}}$$

and

Infrared: $$f_{IR} = \frac{c}{\lambda_{IR}} = \frac{3.00\times 10^{8}\ \text{m/s}}{940\times 10^{-9}\ \text{m}} = \boxed{3.19\times 10^{14}\ \text{Hz}}$$

(b) The intensity of an electromagnetic wave is proportional to the square of its amplitude. If 67% of the incident intensity of the red light is absorbed, then the intensity of the emerging wave is $(100\% - 67\%) = 33\%$ of the incident intensity, or $I_f = 0.33 I_i$. Hence, we must have

$$\frac{E_{\max, f}}{E_{\max, i}} = \sqrt{\frac{I_f}{I_i}} = \sqrt{0.33} = \boxed{0.57}$$

21.46 If I_0 is the incident intensity of a light beam, and I is the intensity of the beam after passing through length L of a fluid having concentration C of absorbing molecules, the Beer-Lambert law states that $\log_{10}(I/I_0) = -\varepsilon CL$ where ε is a constant.

For 660-nm light, the absorbing molecules are oxygenated hemoglobin. Thus, if 33% of this wavelength light is transmitted through blood, the concentration of oxygenated hemoglobin in the blood is

$$C_{HBO2} = \frac{-\log_{10}(0.33)}{\varepsilon L} \qquad [1]$$

The absorbing molecules for 940-nm light are deoxygenated hemoglobin, so if 76% of this light is transmitted through the blood, the concentration of these molecules in the blood is

$$C_{HB} = \frac{-\log_{10}(0.76)}{\varepsilon L} \qquad [2]$$

Dividing equation [2] by equation [1] gives the ratio of deoxygenated hemoglobin to oxygenated hemoglobin in the blood as

$$\frac{C_{HB}}{C_{HBO2}} = \frac{\log_{10}(0.76)}{\log_{10}(0.33)} = 0.25 \quad \text{or} \quad C_{HB} = 0.25\,C_{HBO2}$$

Since all the hemoglobin in the blood is either oxygenated or deoxygenated, it is necessary that $C_{HB} + C_{HBO2} = 1.00$, and we now have $0.25C_{HBO2} + C_{HBO2} = 1.0$. The fraction of hemoglobin that is oxygenated in this blood is then

$$C_{HBO2} = \frac{1.0}{1.0 + 0.25} = 0.80 \quad \text{or} \quad \boxed{80\%}$$

Someone with only 80% oxygenated hemoglobin in the blood is probably in serious trouble needing supplemental oxygen immediately.

21.48 At Earth's location, the wave fronts of the solar radiation are spheres whose radius is the Sun-Earth distance. Thus, from $Intensity = \frac{\tilde{A}_{av}}{A} = \frac{\tilde{A}_{av}}{4\pi r^2}$, the total power is

$$P_{av} = (Intensity)(4\pi r^2) = \left(1\,340\ \frac{\text{W}}{\text{m}^2}\right)\left[4\pi\left(1.49 \times 10^{11}\ \text{m}\right)^2\right] = \boxed{3.74 \times 10^{26}\ \text{W}}$$