Chapter 16
Electrical Energy and Capacitance

Problem Solutions

16.1 (a) The work done is $W = F \cdot s\cos\theta = (qE)\cdot s\cos\theta$, or

$$W = (1.60\times10^{-19}\ \text{C})(200\ \text{N/C})(2.00\times10^{-2}\ \text{m})\cos0° = \boxed{6.40\times10^{-19}\ \text{J}}$$

(b) The change in the electrical potential energy is

$$\Delta PE_e = -W = \boxed{-6.40\times10^{-19}\ \text{J}}$$

(c) The change in the electrical potential is

$$\Delta V = \frac{\Delta PE_e}{q} = \frac{-6.40\times10^{-19}\ \text{J}}{1.60\times10^{-19}\ \text{C}} = \boxed{-4.00\ \text{V}}$$

16.3 The work done by the agent moving the charge out of the cell is

$$W_{input} = -W_{field} = -(-\Delta PE_e) = +q(\Delta V)$$

$$= (1.60\times10^{-19}\ \text{C})\left(+90\times10^{-3}\ \frac{\text{J}}{\text{C}}\right) = \boxed{1.4\times10^{-20}\ \text{J}}$$

16.5 $E = \dfrac{|\Delta V|}{d} = \dfrac{25\,000\ \text{J/C}}{1.5\times10^{-2}\ \text{m}} = \boxed{1.7\times10^{6}\ \text{N/C}}$

16.8 From conservation of energy, $\dfrac{1}{2}mv_f^2 - 0 = |q(\Delta V)|$ or $v_f = \sqrt{\dfrac{2|q(\Delta V)|}{m}}$

(a) For the proton, $v_f = \sqrt{\dfrac{2|(1.60\times10^{-19}\ \text{C})(-120\ \text{V})|}{1.67\times10^{-27}\ \text{kg}}} = \boxed{1.52\times10^{5}\ \text{m/s}}$

(b) For the electron, $v_f = \sqrt{\dfrac{2\left|\left(-1.60\times 10^{-19}\ \text{C}\right)\left(+120\ \text{V}\right)\right|}{9.11\times 10^{-31}\ \text{kg}}} = \boxed{6.49\times 10^{6}\ \text{m/s}}$

16.12 $V = V_1 + V_2 = k_e\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right)$ where $r_1 = 0.60\ \text{m} - 0 = 0.60\ \text{m}$, and

$r_2 = 0.60\ \text{m} - 0.30\ \text{m} = 0.30\ \text{m}$. Thus,

$$V = \left(8.99\times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{3.0\times 10^{-9}\ \text{C}}{0.60\ \text{m}} + \frac{6.0\times 10^{-9}\ \text{C}}{0.30\ \text{m}}\right) = \boxed{2.2\times 10^{2}\ \text{V}}$$

16.15 (a)

$$V = \sum_i \frac{k_e q_i}{r_i} = \left(8.99\times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{5.00\times 10^{-9}\ \text{C}}{0.175\ \text{m}} - \frac{3.00\times 10^{-9}\ \text{C}}{0.175\ \text{m}}\right) = \boxed{103\ \text{V}}$$

(b)

$$PE = \frac{k_e q_1 q_2}{r_{12}} = \left(8.99\times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(5.00\times 10^{-9}\ \text{C}\right)\left(-3.00\times 10^{-9}\ \text{C}\right)}{0.350\ \text{m}} = \boxed{-3.85\times 10^{-7}\ \text{J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges (that is, bring them up to a state of zero potential energy).

16.19 From conservation of energy, $\left(KE + PE_e\right)_f = \left(KE + PE_e\right)_i$, which gives

$$0 + \frac{k_e Q q}{r_f} = \frac{1}{2} m_\alpha v_i^2 + 0 \quad \text{or} \quad r_f = \frac{2 k_e Q q}{m_\alpha v_i^2} = \frac{2 k_e (79e)(2e)}{m_\alpha v_i^2}$$

$$r_f = \frac{2\left(8.99\times 10^{9}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(158)\left(1.60\times 10^{-19}\ \text{C}\right)^2}{\left(6.64\times 10^{-27}\ \text{kg}\right)\left(2.00\times 10^{7}\ \text{m/s}\right)^2} = \boxed{2.74\times 10^{-14}\ \text{m}}$$

16.22 (a) $Q = C(\Delta V) = \left(4.00\times 10^{-6}\ \text{F}\right)(12.0\ \text{V}) = 48.0\times 10^{-6}\ \text{C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C(\Delta V) = (4.00\times10^{-6}\text{ F})(1.50\text{ V}) = 6.00\times10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

16.23 (a) $C = \epsilon_0 \dfrac{A}{d} = \left(8.85\times10^{-12}\ \dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\dfrac{(1.0\times10^6\text{ m}^2)}{(800\text{ m})} = \boxed{1.1\times10^{-8}\text{ F}}$

(b)

$$Q_{max} = C(\Delta V)_{max} = C(E_{max}d)$$

$$= (1.11\times10^{-8}\text{ F})(3.0\times10^6\text{ N/C})(800\text{ m}) = \boxed{27\text{ C}}$$

16.25 (a) $E = \dfrac{\Delta V}{d} = \dfrac{20.0\text{ V}}{1.80\times10^{-3}\text{ m}} = 1.11\times10^4\text{ V/m} = \boxed{11.1\text{ kV/m}}$ directed toward the negative plate

(b) $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(7.60\times10^{-4}\text{ m}^2)}{1.80\times10^{-3}\text{ m}}$

$$= 3.74\times10^{-12}\text{ F} = \boxed{3.74\text{ pF}}$$

(c) $Q = C(\Delta V) = (3.74\times10^{-12}\text{ F})(20.0\text{ V}) = 7.47\times10^{-11}\text{ C} = \boxed{74.7\text{ pC}}$ on one plate and $\boxed{-74.7\text{ pC}}$ on the other plate.

16.29 (a) For series connection, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$

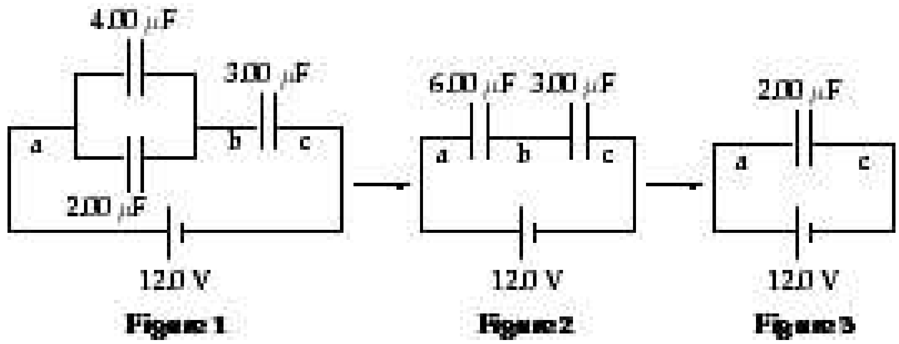
$$Q = C_{eq}(\Delta V) = \left(\frac{C_1C_2}{C_1+C_2}\right)\Delta V$$

$$= \left[\frac{(0.050\ \mu\text{F})(0.100\ \mu\text{F})}{0.050\ \mu\text{F} + 0.100\ \mu\text{F}}\right](400\text{ V}) = \boxed{13.3\ \mu\text{C on each}}$$

(b) $Q_1 = C_1(\Delta V) = (0.050\ \mu\text{F})(400\text{ V}) = \boxed{20.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V) = (0.100\ \mu\text{F})(400\text{ V}) = \boxed{40.0\ \mu\text{C}}$

16.31 (a) Using the rules for combining capacitors in series and in parallel, the circuit is reduced in steps as shown below. The equivalent capacitor is shown to be a $\boxed{2.00\ \mu\text{F}}$ capacitor.



(b) From Figure 3: $Q_{ac} = C_{ac}(\Delta V)_{ac} = (2.00\ \mu\text{F})(12.0\ \text{V}) = 24.0\ \mu\text{C}$

From Figure 2: $Q_{ab} = Q_{bc} = Q_{ac} = 24.0\ \mu\text{C}$

Thus, the charge on the $3.00\ \mu\text{F}$ capacitor is $Q_3 = \boxed{24.0\ \mu\text{C}}$

Continuing to use Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ab}}{C_{ab}} = \dfrac{24.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 4.00\ \text{V}$

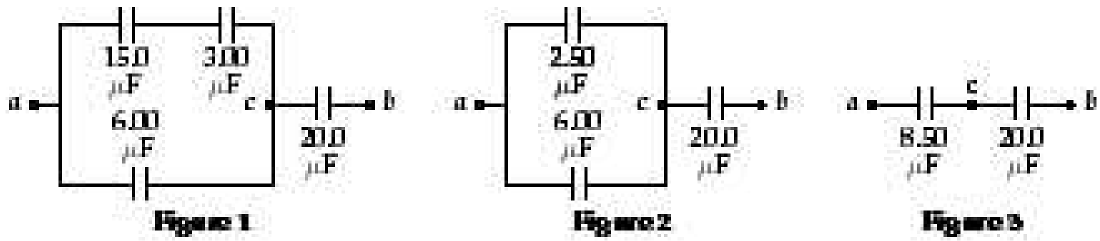
and $(\Delta V)_3 = (\Delta V)_{bc} = \dfrac{Q_{bc}}{C_{bc}} = \dfrac{24.0\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{8.00\ \text{V}}$

From Figure 1, $(\Delta V)_4 = (\Delta V)_2 = (\Delta V)_{ab} = \boxed{4.00\ \text{V}}$

and $Q_4 = C_4(\Delta V)_4 = (4.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V)_2 = (2.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{8.00\ \mu\text{C}}$

16.33



(a) The equivalent capacitance of the upper branch between points *a* and *c* in Figure 1 is

$$C_s = \frac{(15.0\ \mu\text{F})(3.00\ \mu\text{F})}{15.0\ \mu\text{F} + 3.00\ \mu\text{F}} = 2.50\ \mu\text{F}$$

Then, using Figure 2, the total capacitance between points *a* and *c* is

$$C_{ac} = 2.50\ \mu\text{F} + 6.00\ \mu\text{F} = 8.50\ \mu\text{F}$$

From Figure 3, the total capacitance is

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}} \right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b)

$$Q_{ab} = Q_{ac} = Q_{cb} = (\Delta V)_{ab}\, C_{eq}$$

$$= (15.0\ \text{V})(5.96\ \mu\text{F}) = 89.5\ \mu\text{C}$$

Thus, the charge on the $20.0\ \mu\text{C}$ is $Q_{20} = Q_{cb} = \boxed{89.5\ \mu\text{C}}$

$$(\Delta V)_{ac} = (\Delta V)_{ab} - (\Delta V)_{bc} = 15.0\ \text{V} - \left(\frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} \right) = 10.53\ \text{V}$$

Then, $Q_6 = (\Delta V)_{ac}(6.00\ \mu F) = \boxed{63.2\ \mu\text{C}}$ and

$$Q_{15} = Q_3 = (\Delta V)_{ac}(2.50\ \mu F) = \boxed{26.3\ \mu\text{C}}$$

16.35

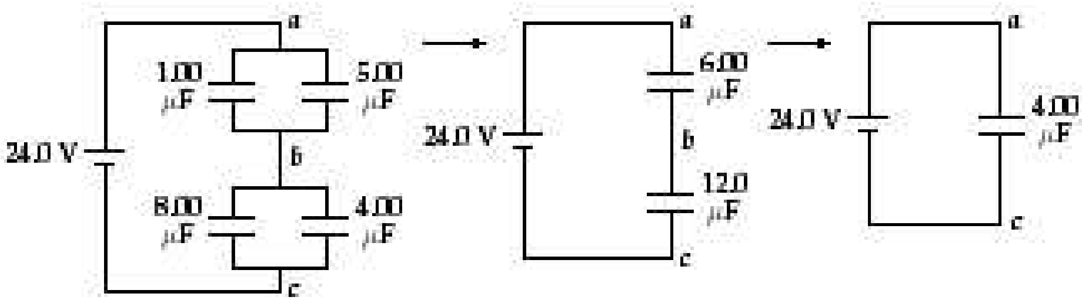


Figure 1 Figure 2 Figure 3

The circuit may be reduced in steps as shown above.

Using the Figure 3, $Q_{ac} = (4.00\ \mu\text{F})(24.0\ \text{V}) = 96.0\ \mu\text{C}$

Then, in Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ac}}{C_{ab}} = \dfrac{96.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 16.0\ \text{V}$

and $(\Delta V)_{bc} = (\Delta V)_{ac} - (\Delta V)_{ab} = 24.0\ \text{V} - 16.0\ \text{V} = 8.00\ \text{V}$

Finally, using Figure 1, $Q_1 = C_1(\Delta V)_{ab} = (1.00\ \mu\text{F})(16.0\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$Q_5 = (5.00\ \mu\text{F})(\Delta V)_{ab} = \boxed{80.0\ \mu\text{C}}$, $Q_8 = (8.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{64.0\ \mu\text{C}}$

and $Q_4 = (4.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{32.0\ \mu\text{C}}$

16.43 The capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00\times10^{4}\ \text{m}^2)}{5.00\times10^{3}\ \text{m}} = 3.54\times10^{-13}\ \text{F}$$

and the stored energy is

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.54\times10^{-13}\ \text{F})(12.0\ \text{V})^2 = \boxed{2.55\times10^{-11}\ \text{J}}$$

16.45 The capacitance of this parallel plate capacitor is

$$C = \epsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{\left(1.0\times10^{6}\ \text{m}^2\right)}{(800\ \text{m})} = 1.1\times10^{-8}\ \text{F}$$

With an electric field strength of $E = 3.0\times10^{6}\ \text{N/C}$ and a plate separation of $d = 800\ \text{m}$, the potential difference between plates is

$$\Delta V = Ed = \left(3.0\times10^{6}\ \text{V/m}\right)(800\ \text{m}) = 2.4\times10^{9}\ \text{V}$$

Thus, the energy available for release in a lightning strike is

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(1.1\times10^{-8}\ \text{F}\right)\left(2.4\times10^{9}\ \text{V}\right)^2 = \boxed{3.2\times10^{10}\ \text{J}}$$

16.47 The initial capacitance (with air between the plates) is $C_i = Q/(\Delta V)_i$, and the final capacitance (with the dielectric inserted) is $C_f = Q/(\Delta V)_f$ where Q is the constant quantity of charge stored on the plates.

Thus, the dielectric constant is $\kappa = \dfrac{C_f}{C_i} = \dfrac{(\Delta V)_i}{(\Delta V)_f} = \dfrac{100\ \text{V}}{25\ \text{V}} = \boxed{4.0}$

16.49 (a) The dielectric constant for Teflon® is $\kappa = 2.1$, so the capacitance is

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{(2.1)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(175\times10^{-4}\ \text{m}^2\right)}{0.040\,0\times10^{-3}\ \text{m}}$$

$$C = 8.13\times10^{-9}\ \text{F} = \boxed{8.13\ \text{nF}}$$

(b) For Teflon®, the dielectric strength is $E_{max} = 60.0\times10^{6}\ \text{V/m}$, so the maximum voltage is

$$V_{max} = E_{max}d = \left(60.0\times10^{6}\ \text{V/m}\right)\left(0.040\,0\times10^{-3}\ \text{m}\right)$$

$$V_{max} = 2.40\times10^{3}\ \text{V} = \boxed{2.40\ \text{kV}}$$

16.60 From $Q = C(\Delta V)$, the capacitance of the capacitor with air between the plates is

$$C_0 = \frac{Q_0}{\Delta V} = \frac{150\ \mu\text{C}}{\Delta V}$$

After the dielectric is inserted, the potential difference is held to the original value, but the charge changes to $Q = Q_0 + 200\ \mu\text{C} = 350\ \mu\text{C}$. Thus, the capacitance with the dielectric slab in place is

$$C = \frac{Q}{\Delta V} = \frac{350\ \mu\text{C}}{\Delta V}$$

The dielectric constant of the dielectric slab is therefore

$$\kappa = \frac{C}{C_0} = \left(\frac{350\ \mu\text{C}}{\Delta V}\right)\left(\frac{\Delta V}{150\ \mu\text{C}}\right) = \frac{350}{150} = \boxed{2.33}$$