

# PHYSICS 1B – Fall 2009

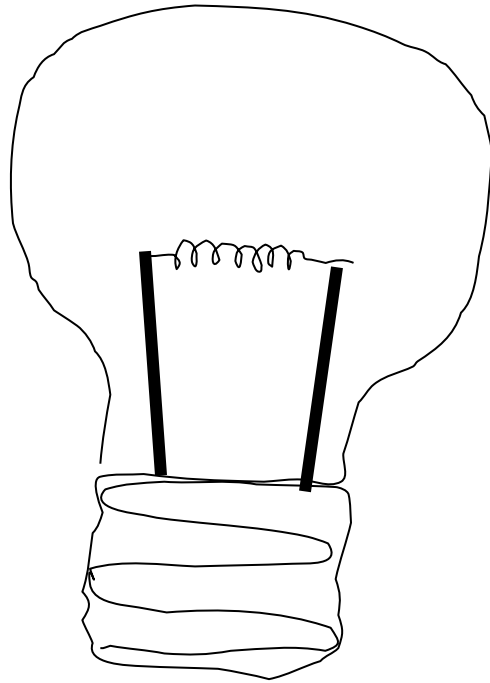


## Electricity & Magnetism



Professor Brian Keating  
SERF Building. Room 333

## Resistance of a light bulb filament.



Thin tungsten coil.

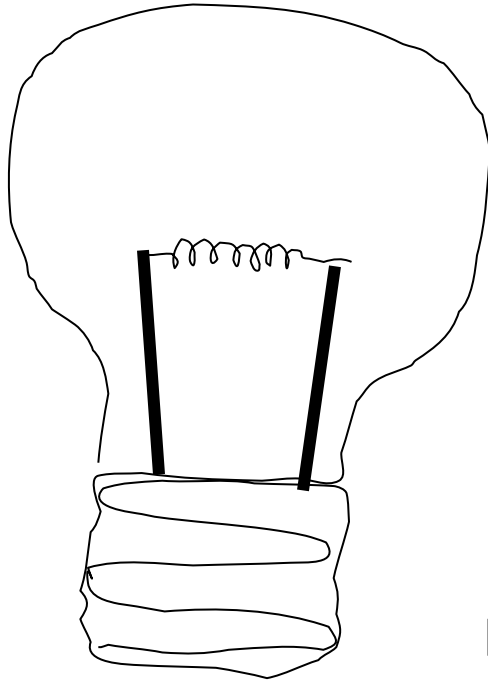
$$R = 150 \, \Omega$$

$$\rho = 73 \times 10^{-8} \, \Omega\text{-m (at 2000 C)}$$

$$L = 0.5 \, \text{m}$$

Find the diameter of the wire.

# Resistance of a light bulb filament.



Thin tungsten coil.

$$R = 150 \, \Omega$$

$$\rho = 73 \times 10^{-8} \, \Omega\text{-m (at 2000 C)}$$

$$L = 0.5 \, \text{m}$$

Find the diameter of the wire.

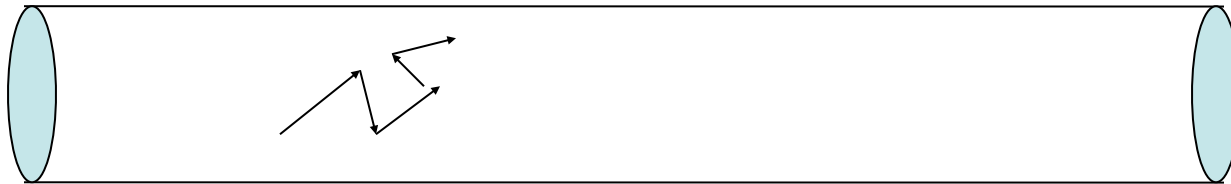
$$R = \frac{\rho L}{A} = \frac{4\rho L}{\pi d^2}$$

$$d = \sqrt{\frac{4\rho L}{\pi R}} = \sqrt{\frac{4(73 \times 10^{-8})(0.5)}{\pi(150)}} = 5.5 \times 10^{-5} \, \text{m}$$

$$55 \, \mu\text{m}$$

# Ch 17.6

## Temperature dependence of resistance metal conductors



At higher  $T$  the collisions with the lattice are more frequent.

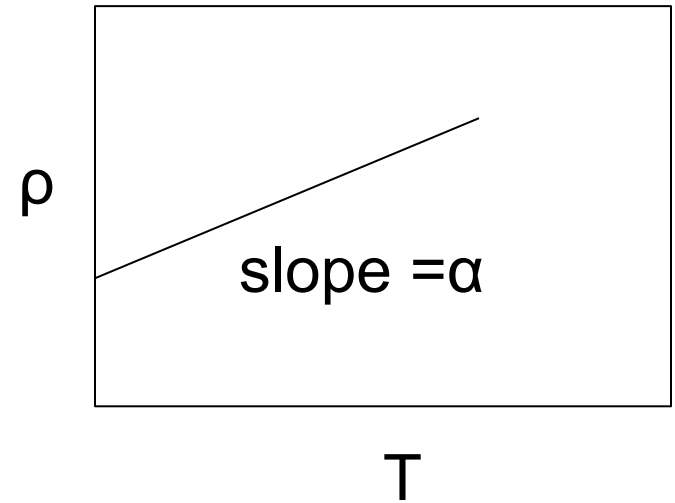
$v_D$  becomes lower

$R$  becomes larger

# Temperature coefficient of resistivity

For small changes in T

$$\rho = \rho_o [1 + \alpha(T - T_o)]$$



Material	$\alpha$ (C $^\circ$ ) $^{-1}$ near 20 $^\circ$ C
Copper	$3.9 \times 10^{-3}$
Tungsten	$4.5 \times 10^{-3}$
Silicon	$-7.5 \times 10^{-3}$

## Thermometry

A platinum resistance thermometer uses the change in resistance to measure temperature. If a student with the flu has a temperature rise of  $4.5^{\circ}\text{C}$  measured with a platinum resistance thermometer and the initial  $R = 50.00$  ohms. What is the final resistance?  $\alpha = 3.92 \times 10^{-3} \text{ }^{\circ}\text{C}^{-1}$

## Thermometry

A platinum resistance thermometer uses the change in resistance to measure temperature. If a student with the flu has a temperature rise of  $4.5^{\circ}\text{C}$  measured with a platinum resistance thermometer and the initial  $R = 50.00$  ohms. What is the final resistance?  $\alpha = 3.92 \times 10^{-3} \text{ }^{\circ}\text{C}^{-1}$

$$R \propto \rho$$

## Thermometry

A platinum resistance thermometer uses the change in resistance to measure temperature. If a student with the flu has a temperature rise of  $4.5^{\circ}\text{C}$  measured with a platinum resistance thermometer and the initial  $R = 50.00$  ohms. What is the final resistance?  $\alpha = 3.92 \times 10^{-3} \text{ }^{\circ}\text{C}^{-1}$

$$R \propto \rho$$

$$R = R_o [1 + \alpha(T - T_o)]$$

$$R = 50.00 [1 + 3.92 \times 10^{-3} (4.5)]$$

$$R = 50.00 [1.018] = 50.88 \Omega$$

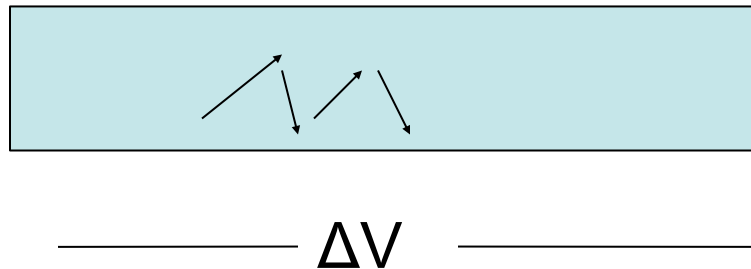


# 17.8

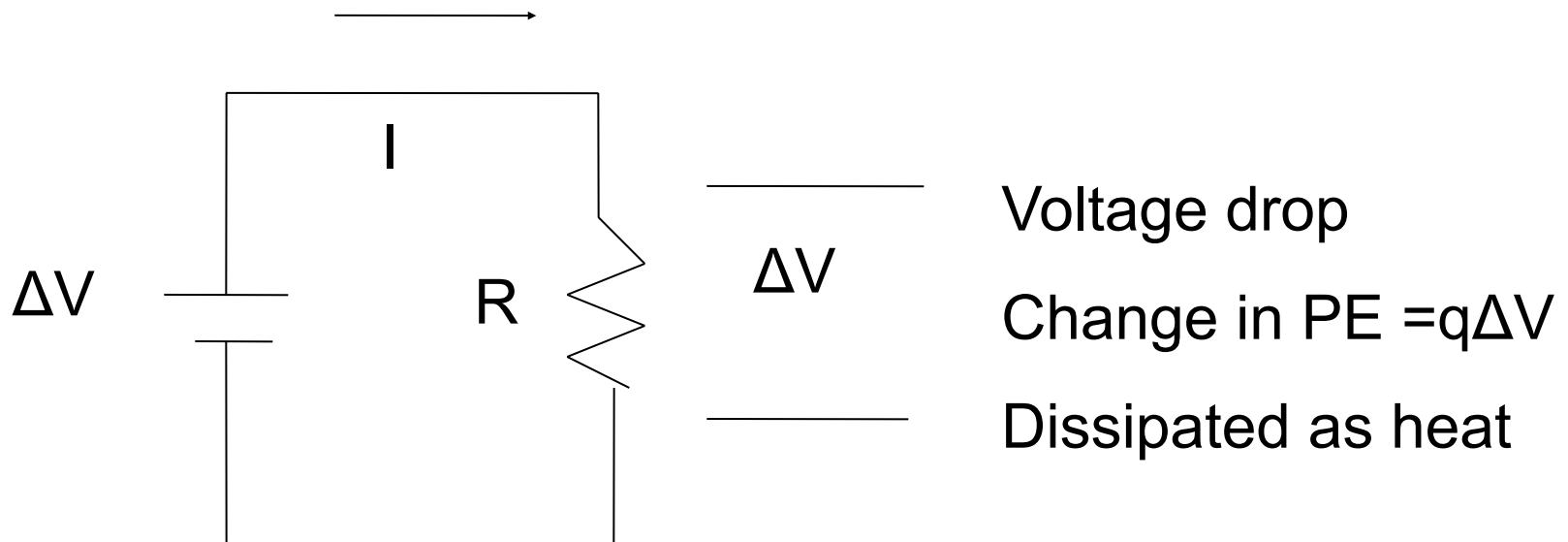
## Electrical energy, power

The power dissipated in a resistor is due to collisions of charge carriers with the lattice.

Electrical potential energy is converted to  
Kinetic energy is converted into heat.



# Energy dissipated in a resistor



# Power dissipated in a resistor

$$P = \frac{\textit{work}}{\textit{time}} = \frac{q\Delta V}{\Delta t}$$

Three equivalent relations for the power

# Power dissipated in a resistor

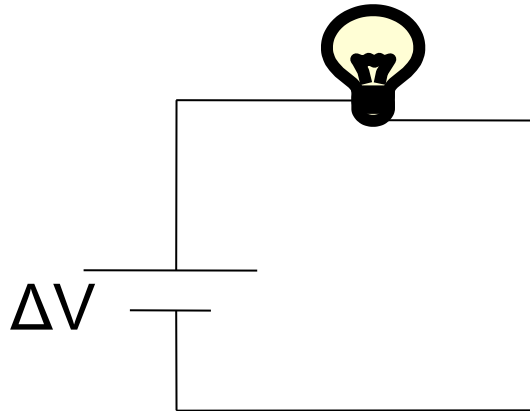
$$P = \frac{\textit{work}}{\textit{time}} = \frac{q\Delta V}{\Delta t}$$

$$P = I\Delta V$$

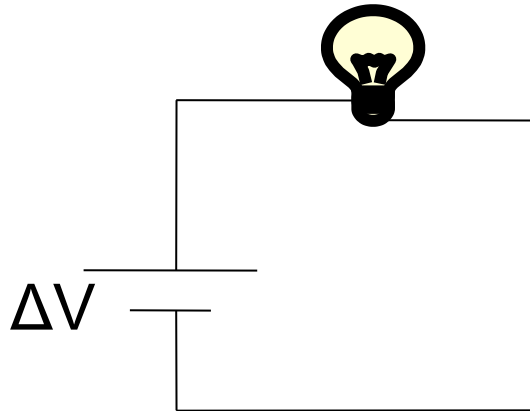
$$P = I(IR) = I^2R$$

$$P = \left(\frac{\Delta V}{R}\right)\Delta V = \frac{\Delta V^2}{R}$$

Three equivalent relations for the power



A lightbulb has an output of 100 W when connected to a 120V household outlet. What is the resistance of the filament?



A lightbulb has an output of 100 W when connected to a 120V household outlet. What is the resistance of the filament?

$$P = \frac{\Delta V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{120^2}{100} = 144\Omega$$

A heating element in an electric range is rated at 2000 W. Find the current required if the voltage is 240 V. Find the resistance of the heating element.



A heating element in an electric range is rated at 2000 W. Find the current required if the voltage is 240 V. Find the resistance of the heating element.



$$P = IV$$

$$I = \frac{P}{V} = \frac{2000}{240} = 8.3A$$



A heating element in an electric range is rated at 2000 W. Find the current required if the voltage is 240 V. Find the resistance of the heating element.



$$P = IV$$

$$I = \frac{P}{V} = \frac{2000}{240} = 8.3A$$

$$P = I^2R$$

$$R = \frac{P}{I^2} = \frac{2000}{8.3^2} = 29\Omega$$

## Cost of electrical power

$$\text{Kilowatt hour} = 1\text{kW} \times 1\text{hr} = 1000\text{J/s}(3600\text{s}) = 3.6 \times 10^6\text{J}$$

1kW hr costs ~ \$0.15

How much does it cost to keep a 100W light on for 24 hrs?

Cost of electrical power

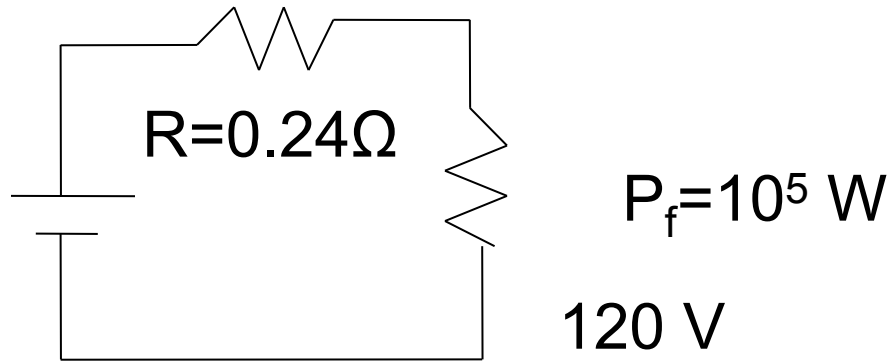
Kilowatt hour =  $1\text{kW} \times 1\text{hr} = 1000\text{J/s}(3600\text{s}) = 3.6 \times 10^6\text{J}$

1kW hr costs ~ \$0.15

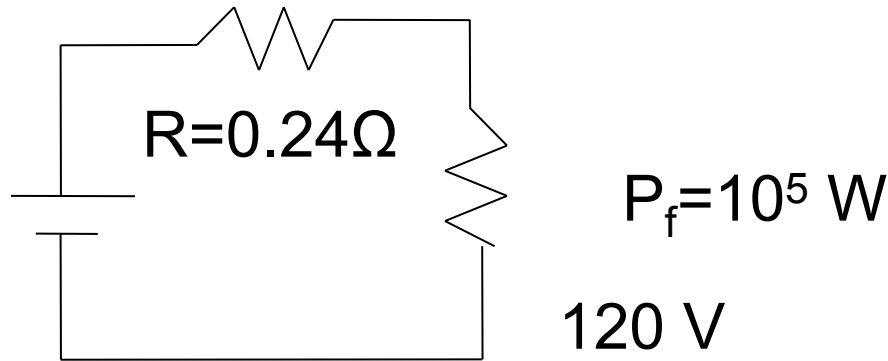
How much does it cost to keep a 100W light on for 24 hrs?

$$\text{Cost} = \frac{\$}{\text{kwhr}} \text{kwhr} = 0.15(0.10)(24) = \$0.36$$

A 10 km copper power cable with a resistance of  $0.24 \Omega$  leads from a power plant to a factory. If the factory uses 100 kW of power at a voltage of 120 V how much power would be dissipated in the cable.

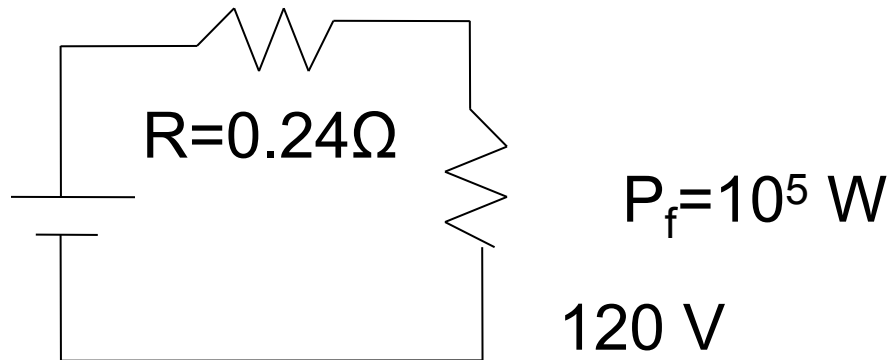


A 10 km copper power cable with a resistance of  $0.24 \Omega$  leads from a power plant to a factory. If the factory uses 100 kW of power at a voltage of 120 V how much power would be dissipated in the cable.



$$P_f = I \Delta V_f$$

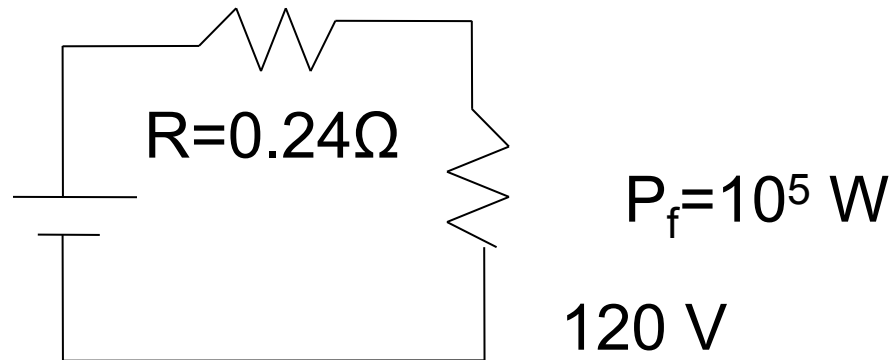
A 10 km copper power cable with a resistance of  $0.24 \Omega$  leads from a power plant to a factory. If the factory uses  $100 \text{ kW}$  of power at a voltage of  $120 \text{ V}$  how much power would be dissipated in the cable.



$$P_f = I \Delta V_f$$
$$I = \frac{P_f}{\Delta V_f} = \frac{10^5}{120} = 8.3 \times 10^2 \text{ A}$$

A large current is required to provide this power at low voltage

A 10 km copper power cable with a resistance of  $0.24 \Omega$  leads from a power plant to a factory. If the factory uses  $100 \text{ kW}$  of power at a voltage of  $120 \text{ V}$  how much power would be dissipated in the cable.



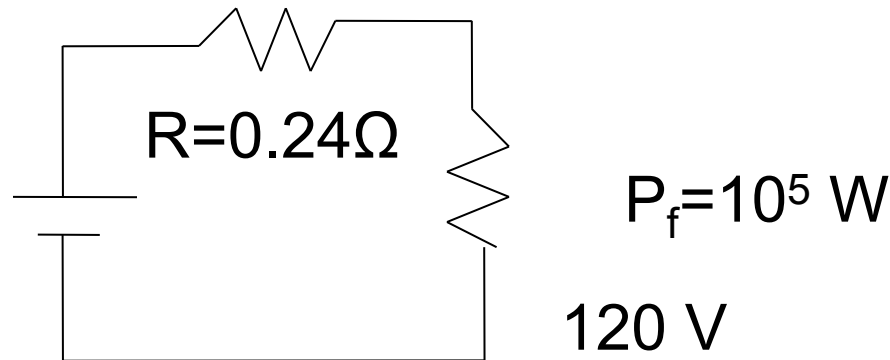
$$P_f = I \Delta V_f$$

$$I = \frac{P_f}{\Delta V_f} = \frac{10^5}{120} = 8.3 \times 10^2 \text{ A}$$

A large current is required to provide this power at low voltage

$$P_c = I^2 R_c = (8.3 \times 10^2)^2 (0.24) = 1.6 \times 10^5 \text{ W}$$

A 10 km copper power cable with a resistance of  $0.24 \Omega$  leads from a power plant to a factory. If the factory uses  $100 \text{ kW}$  of power at a voltage of  $120 \text{ V}$  how much power would be dissipated in the cable.



$$P_f = I \Delta V_f$$

$$I = \frac{P_f}{\Delta V_f} = \frac{10^5}{120} = 8.3 \times 10^2 \text{ A}$$

A large current is required to provide this power at low voltage

$$P_c = I^2 R_c = (8.3 \times 10^2)^2 (0.24) = 1.6 \times 10^5 \text{ W}$$

Very lossy cable



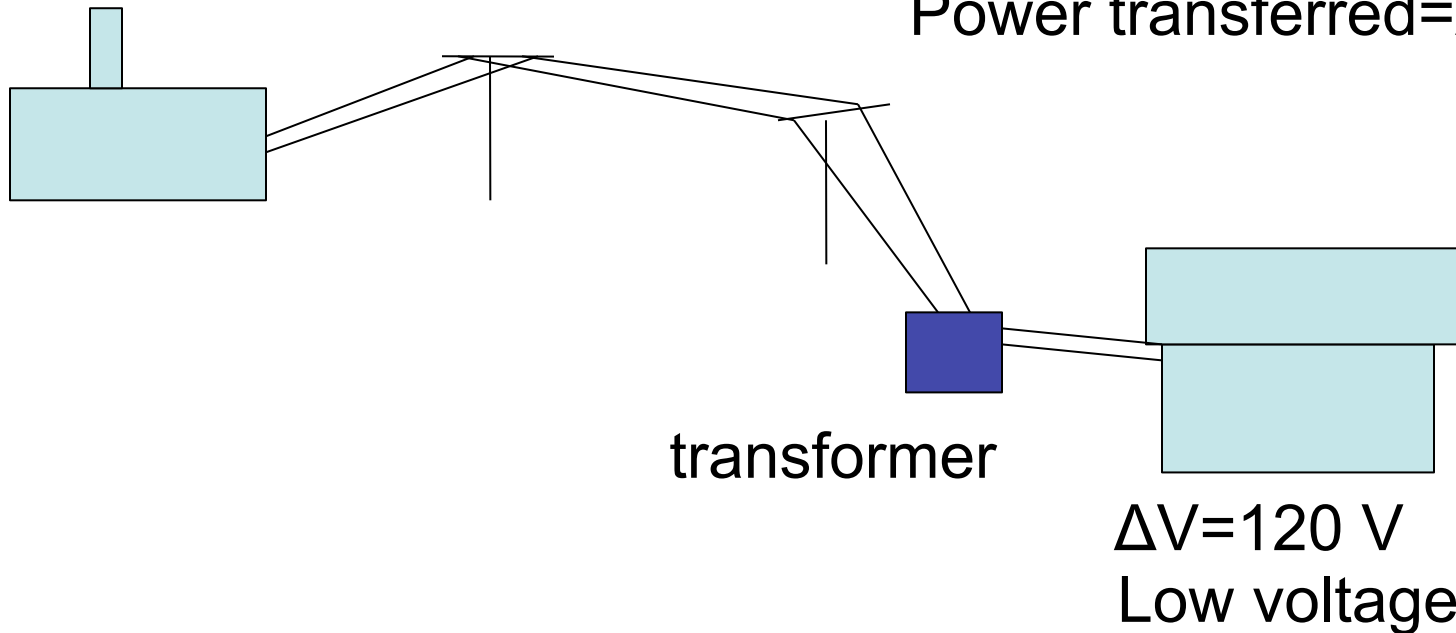
# Power Transmission

High voltage

$$\Delta V_{\text{trans}} = 10^5 \text{ V}$$

$$\text{Power loss} = I^2 R_{\text{wire}}$$

$$\text{Power transferred} = \Delta V_{\text{trans}} I$$



High voltage transmission- power transmitted with lower current. Therefore lower  $I^2R$  loss in the line.

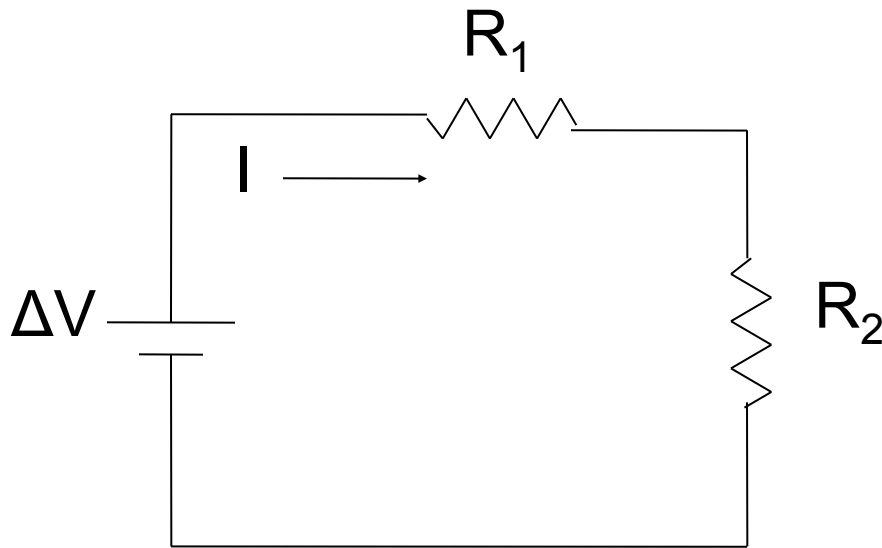
# Chapter 18

- Resistors in Series
- Resistors in Parallel
- Combinations of Parallel and Series
- Combinations of Capacitors and Resistors

## Ch 18. 2

### Resistors in Series

What is the equivalent resistance  $R_{eq}$  ?

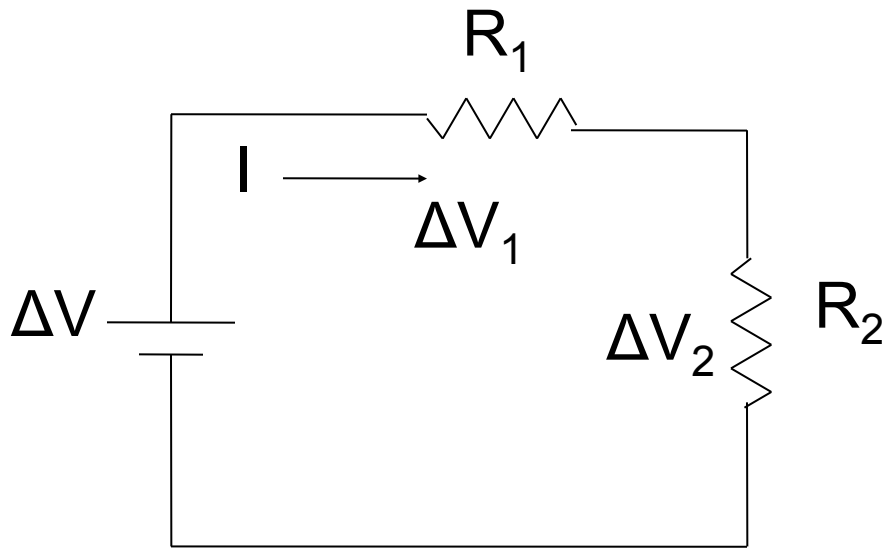


$I$  same,  $\Delta V$  different

## Ch 18. 2

### Resistors in Series

What is the equivalent resistance  $R_{eq}$  ?



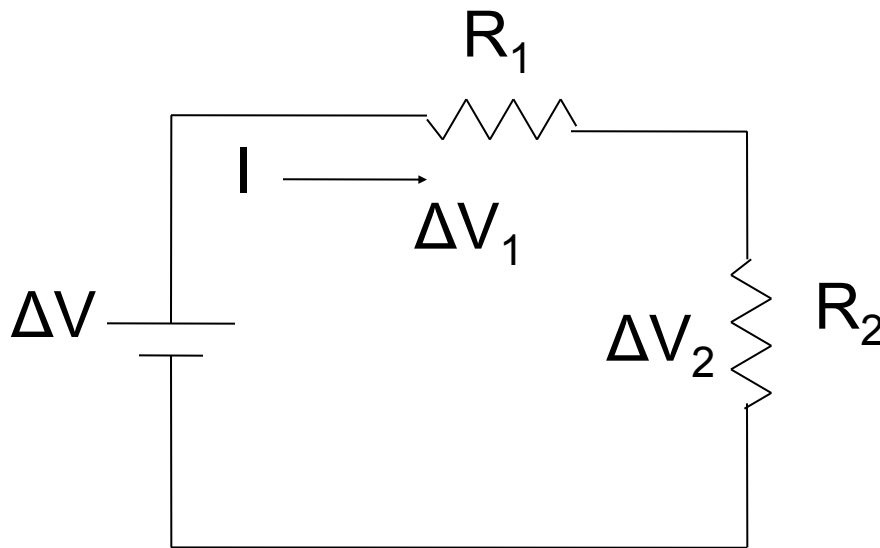
I same,  $\Delta V$  different

$$\Delta V = \Delta V_1 + \Delta V_2$$

## Ch 18. 2

### Resistors in Series

What is the equivalent resistance  $R_{eq}$  ?



I same,  $\Delta V$  different

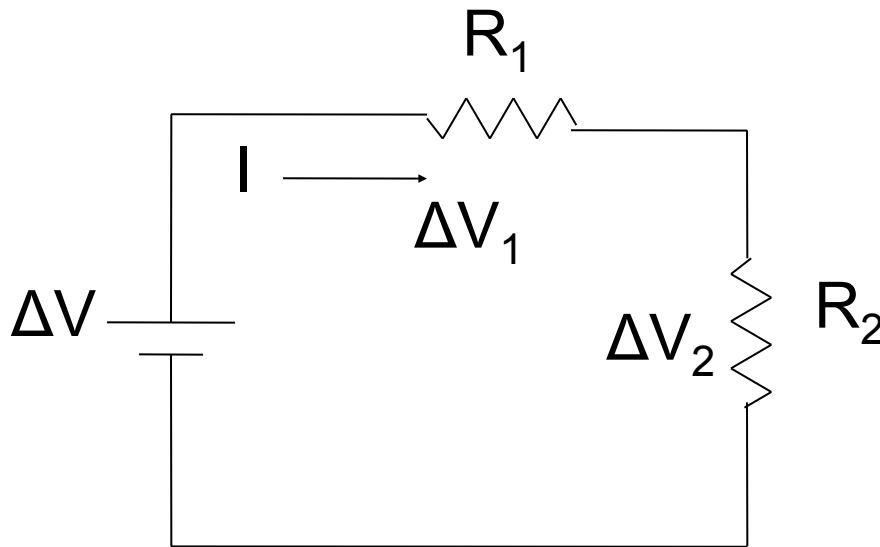
$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = IR_{eq} = IR_1 + IR_2$$

## Ch 18. 2

### Resistors in Series

What is the equivalent resistance  $R_{\text{eq}}$  ?



$I$  same,  $\Delta V$  different

$$\Delta V = \Delta V_1 + \Delta V_2$$

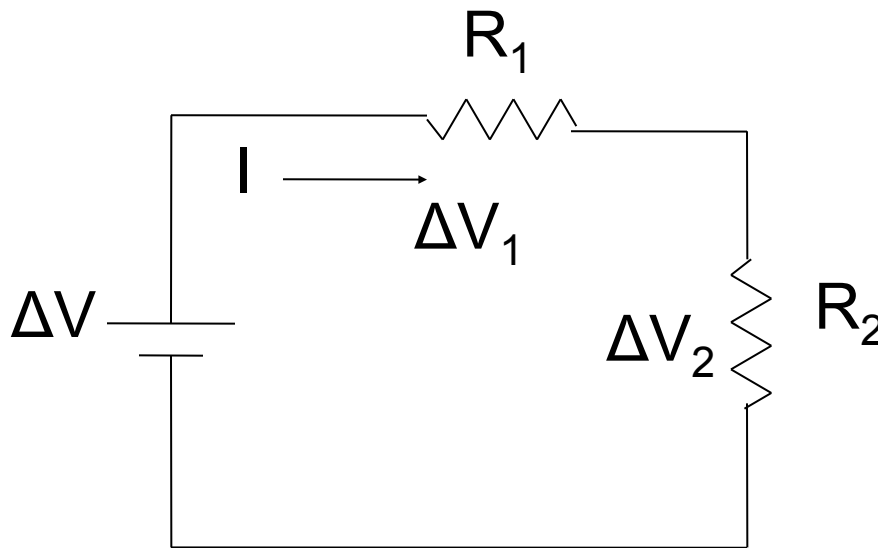
$$\Delta V = IR_{\text{eq}} = IR_1 + IR_2$$

$$R_{\text{eq}} = R_1 + R_2$$

## Ch 18. 2

### Resistors in Series

What is the equivalent resistance  $R_{eq}$  ?



$I$  same,  $\Delta V$  different

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = IR_{eq} = IR_1 + IR_2$$

$$R_{eq} = R_1 + R_2$$

For  $N$  resistors in series

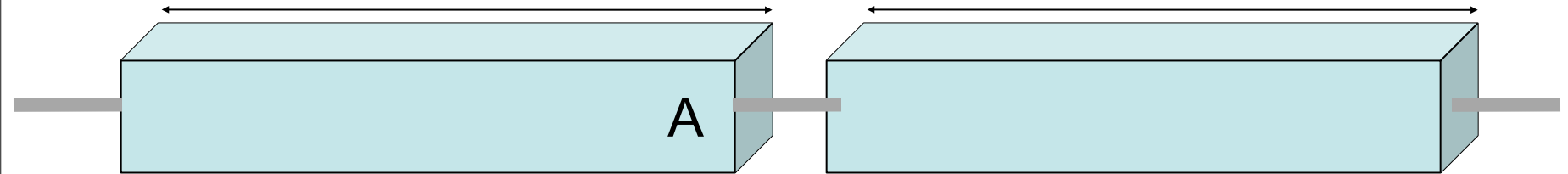
$$R_{eq} = R_1 + R_2 + \dots + R_N$$

$R_{eq}$  is larger  
than any  $R$

# Why is the **series law** easy to understand?

- Recall that the resistance of a resistor is

$$R = \rho \frac{L}{A}$$



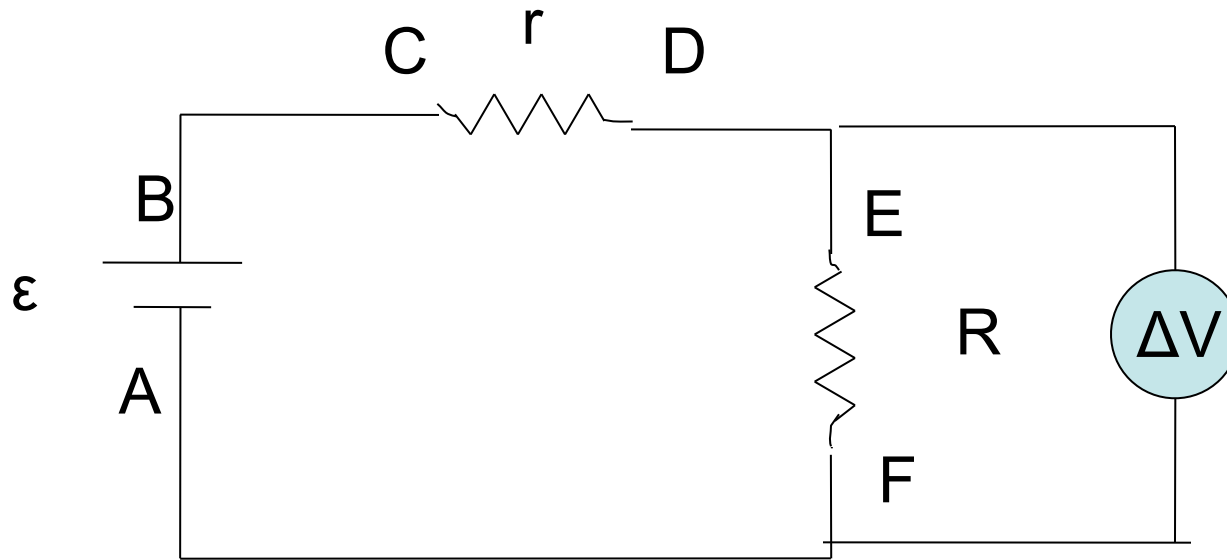
$$R \sim L$$

$$R_{\text{tot}} \sim L_1 + L_2$$



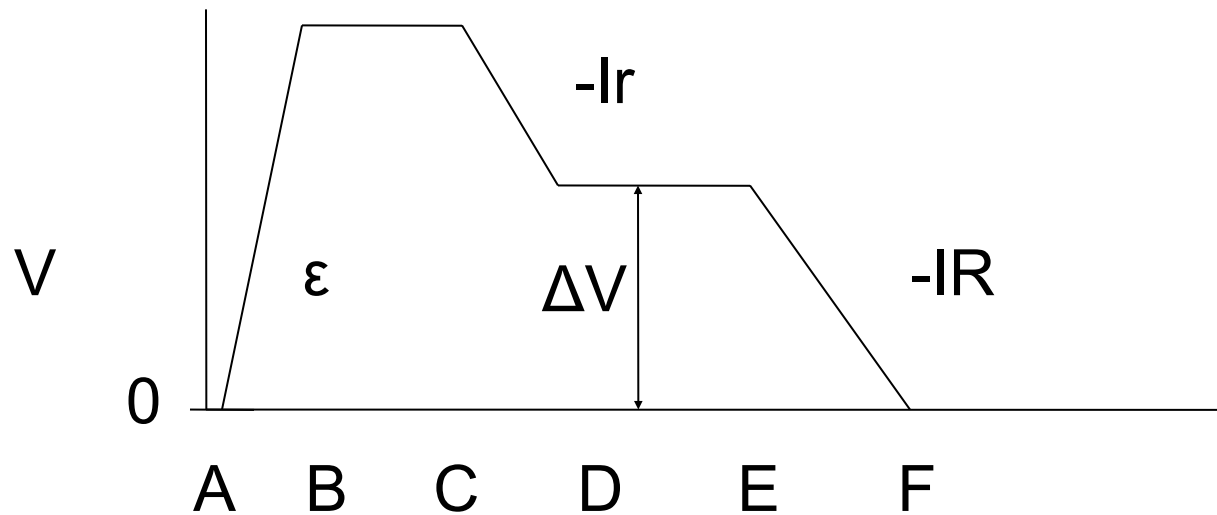
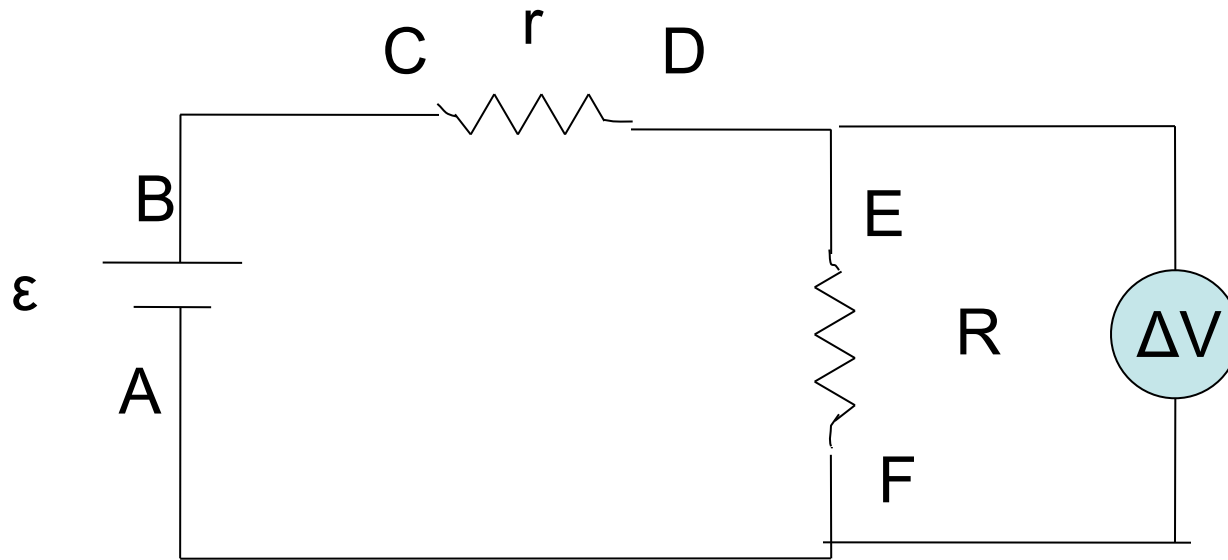
## Why do we care?

Consider Simple Circuit: Two resistors in Series



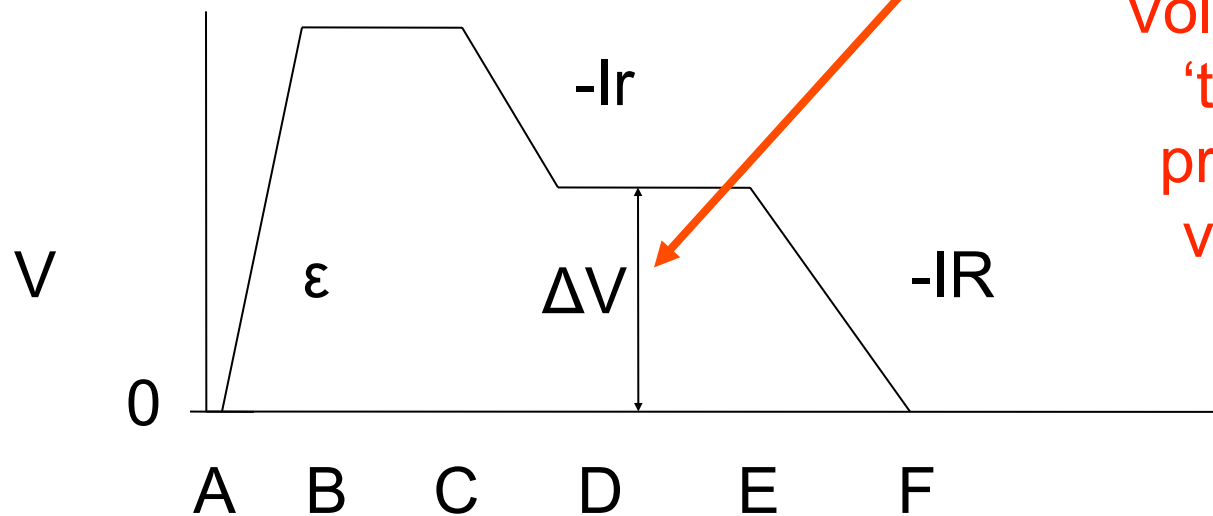
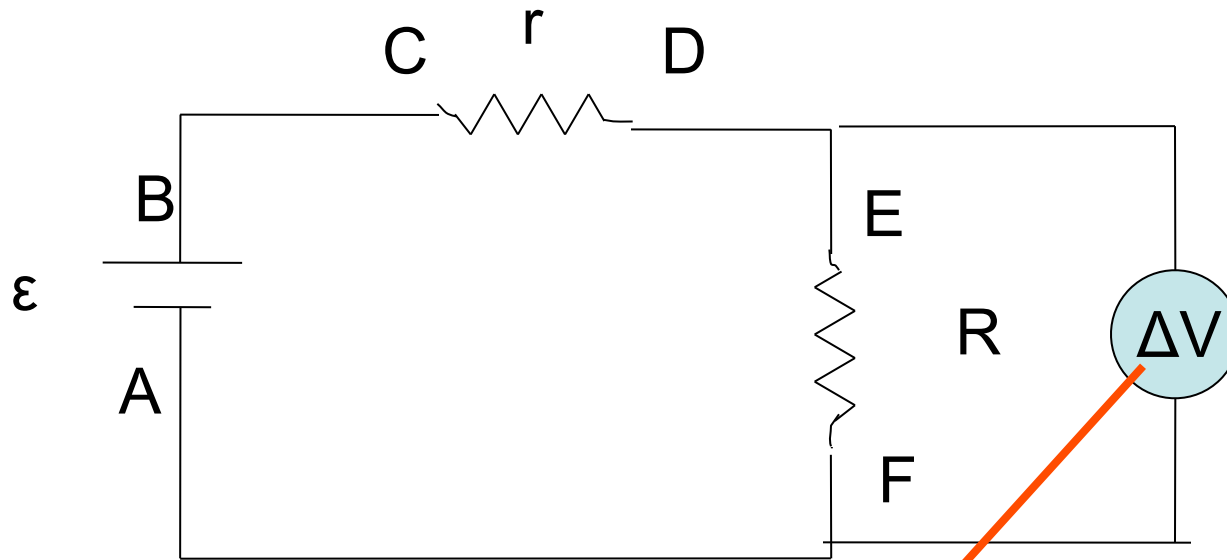
# Why do we care?

Consider Simple Circuit: Two resistors in Series



# Why do we care?

Consider Simple Circuit: Two resistors in Series



Voltage can be 'tailored' to produce any voltage we desire!

# Why is the **parallel law** easy to understand?

- Recall that the resistance of a resistor is

$$R = \rho \frac{L}{A}$$

$$R \sim 1/\text{Area}$$



# Why is the **parallel law** easy to understand?

- Recall that the resistance of a resistor is

$$R = \rho \frac{L}{A}$$

$$R \sim 1/\text{Area}$$



$$A_{\text{tot}} = A_1 + A_2$$

# Why is the **parallel law** easy to understand?

- Recall that the resistance of a resistor is

$$R = \rho \frac{L}{A}$$

$$R \sim 1/\text{Area}$$



$$A_{\text{tot}} = A_1 + A_2$$

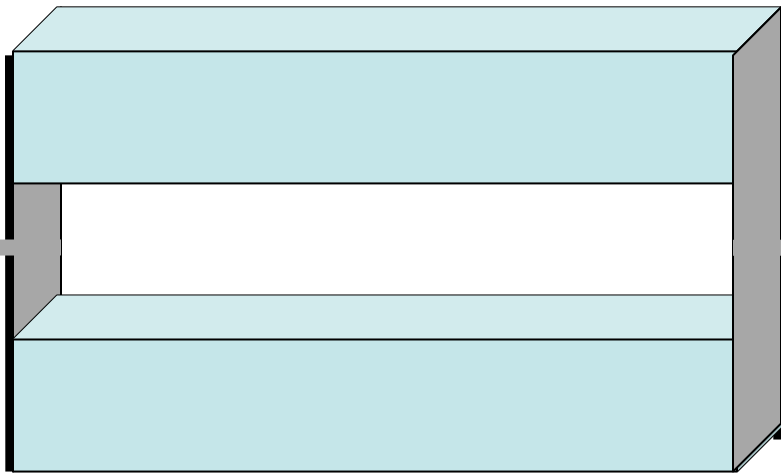
$$A_{\text{tot}} = 1/R_1 + 1/R_2$$

# Why is the **parallel law** easy to understand?

- Recall that the resistance of a resistor is

$$R = \rho \frac{L}{A}$$

$$R \sim 1/\text{Area}$$

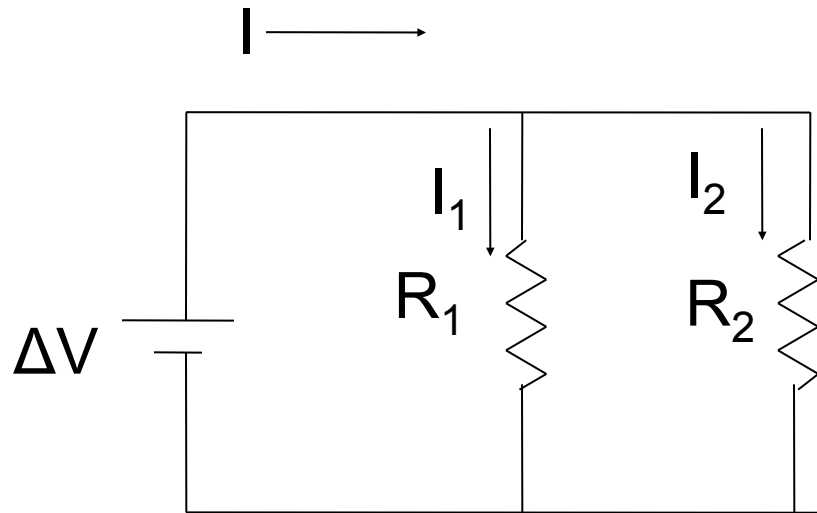


$$A_{\text{tot}} = A_1 + A_2$$

$$A_{\text{tot}} = 1/R_1 + 1/R_2$$

$$R_{\text{tot}} \sim 1/A_{\text{tot}} \sim 1/(1/R_1 + 1/R_2)$$

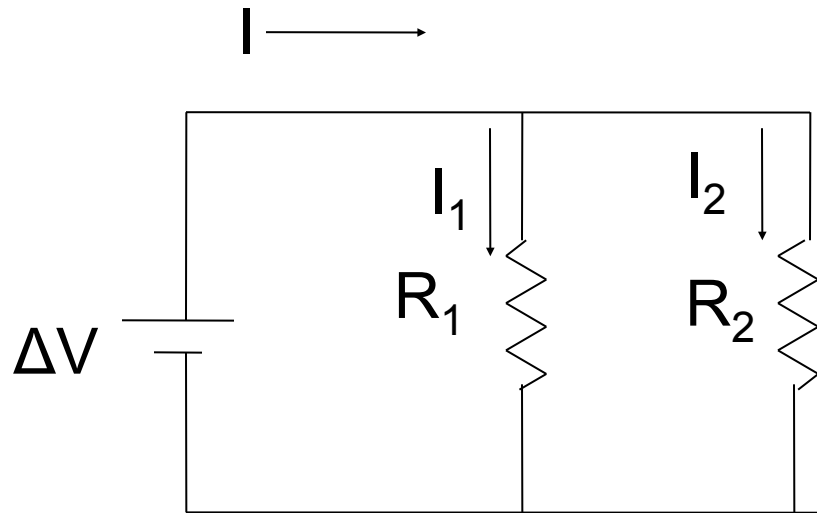
Resistors in parallel,  $\Delta V$  same,  $I$  different



$$I = I_1 + I_2$$



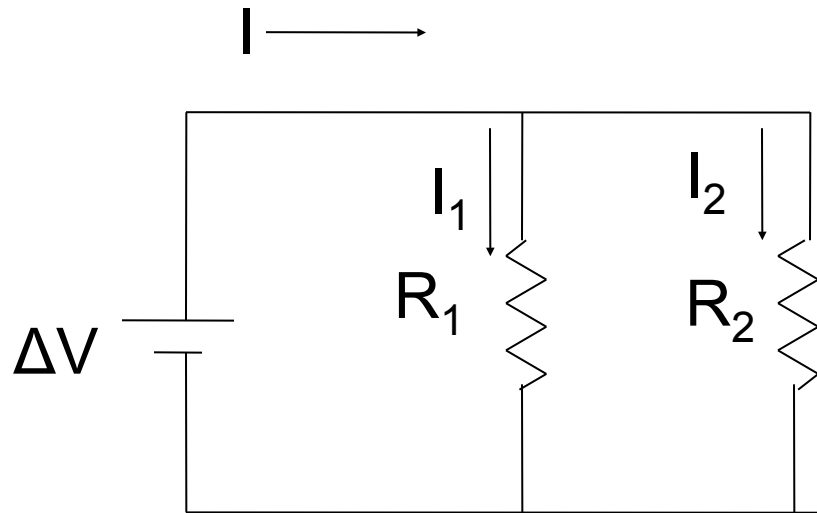
Resistors in parallel,  $\Delta V$  same,  $I$  different



$$I = I_1 + I_2$$

$$\frac{\Delta V}{R_{eq}} = I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

# Resistors in parallel, $\Delta V$ same, $I$ different

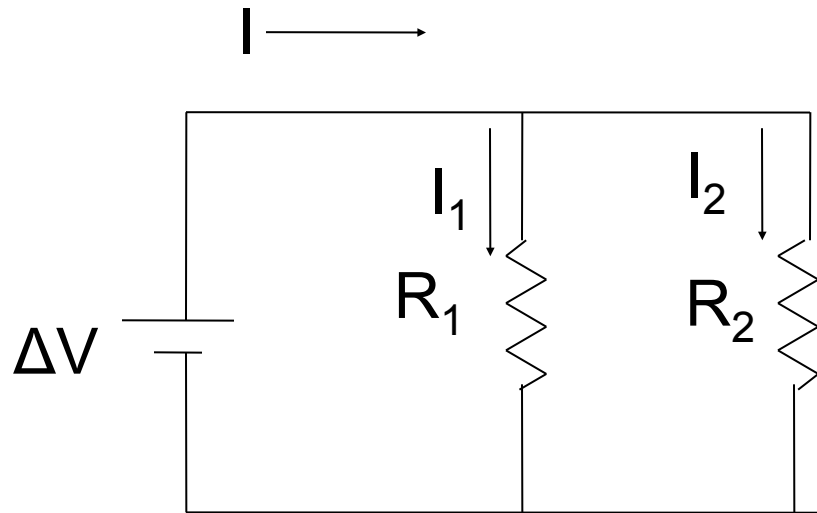


$$I = I_1 + I_2$$

$$\frac{\Delta V}{R_{eq}} = I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

# Resistors in parallel, $\Delta V$ same, $I$ different



$$I = I_1 + I_2$$

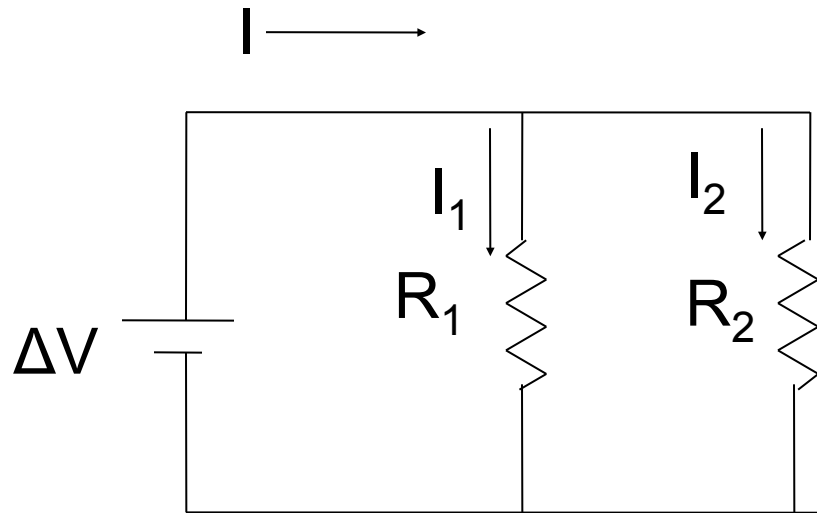
$$\frac{\Delta V}{R_{eq}} = I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For  $N$  resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

# Resistors in parallel, $\Delta V$ same, $I$ different



$$I = I_1 + I_2$$

$$\frac{\Delta V}{R_{eq}} = I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For  $N$  resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

$R_{eq}$  is smaller than any  $R$

# PHYSICS 1B – Fall 2009



## Electricity & Magnetism



Professor Brian Keating

Monday 11/2

SERF Building. Room 333

# Comparisons: Resistors & Capacitors

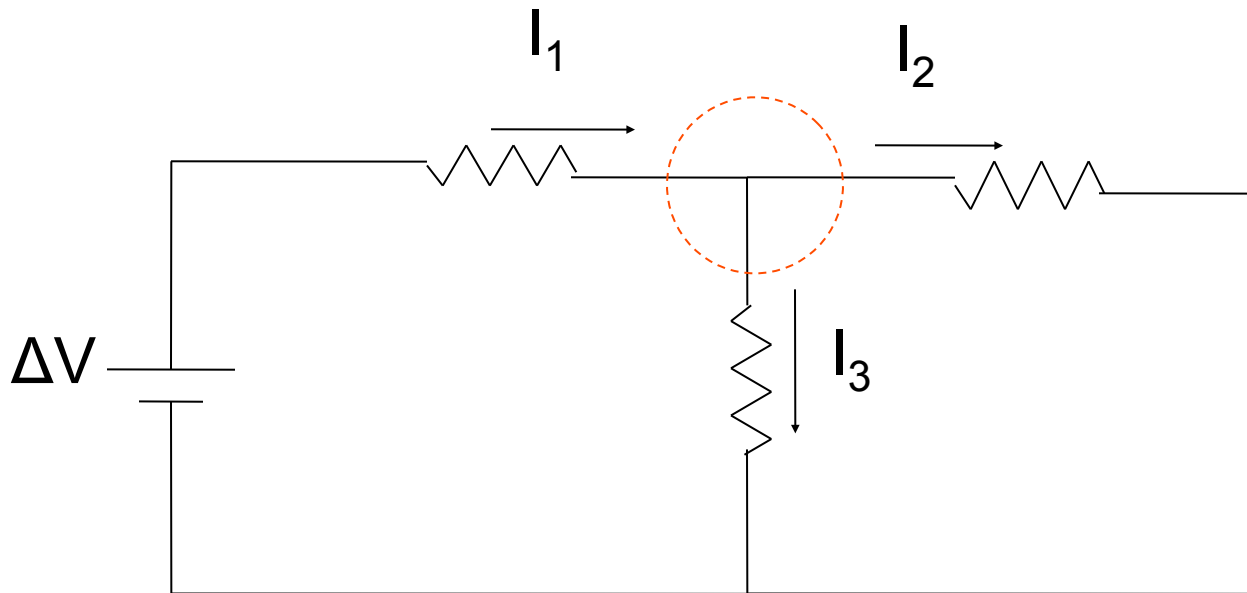
- Resistors in series are like capacitors in parallel.
- Resistors in parallel are like capacitors in series.
- This is because  $R \sim L$  and  $C \sim 1/L$
- And because  $R \sim 1/A$  and  $C \sim A$

# Ch 18 Kirchoff's 2 Rules

1. Junction rule
2. Loop rule

## Rule #1. “*Junction rule*”

The current flowing into a junction is equal to the current flowing out.

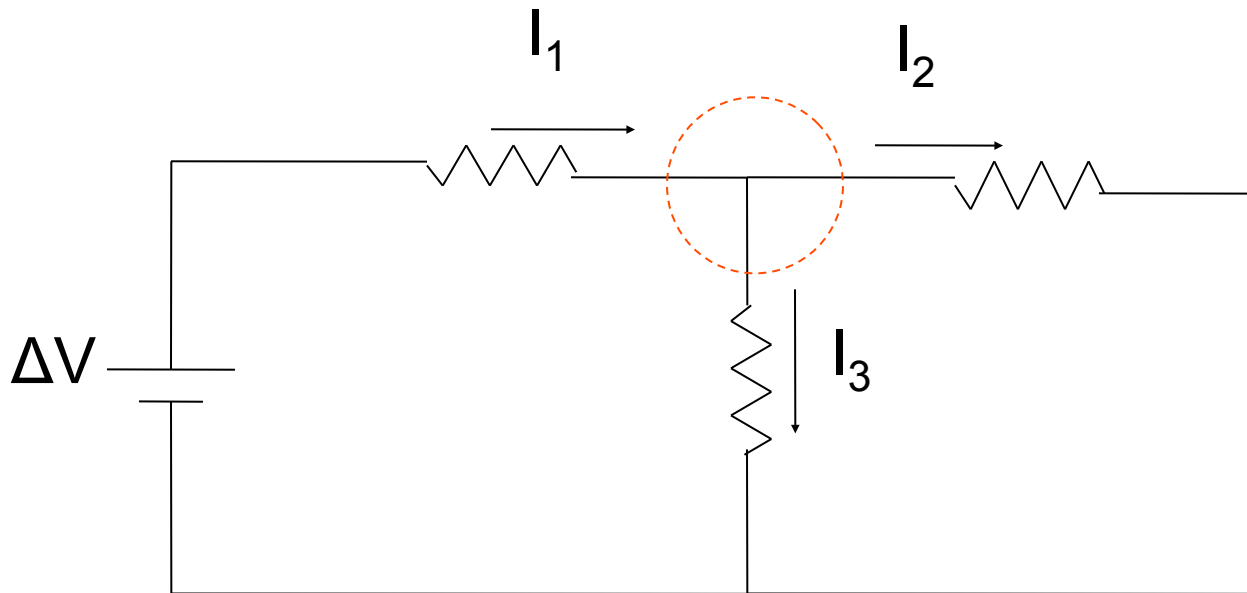


$$I_1 = I_2 + I_3$$



## Rule #1. “Junction rule”

The current flowing into a junction is equal to the current flowing out.

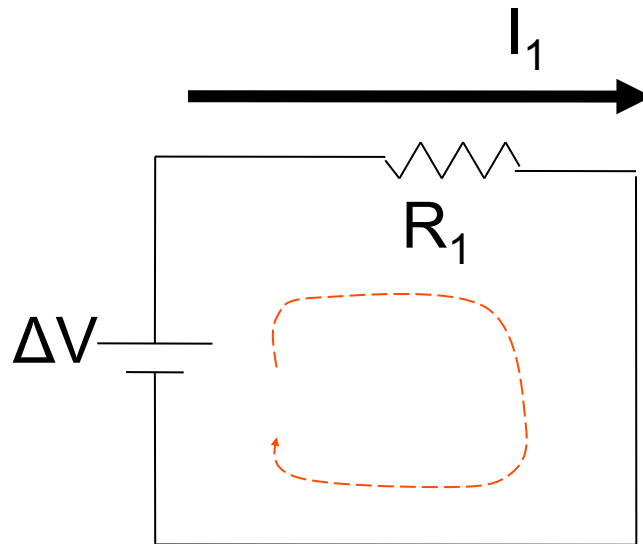


$$I_1 = I_2 + I_3$$

This comes from ‘conservation of charge’

## #2. Loop rule

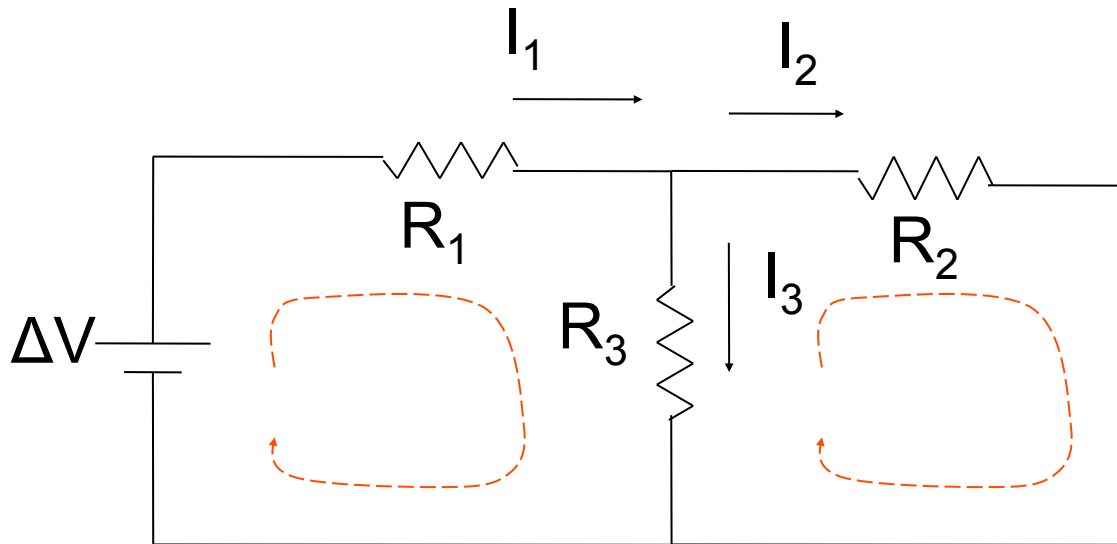
*“The sum of voltage differences in going around a closed current loop is equal to zero”*



$$\sum_{loop} \Delta V_i = 0$$

## #2. Loop rule

The sum of voltage differences in going around a closed current loop is equal to zero



$$\sum_{loop} \Delta V_i = 0$$

# Voltage changes in traversing the loop

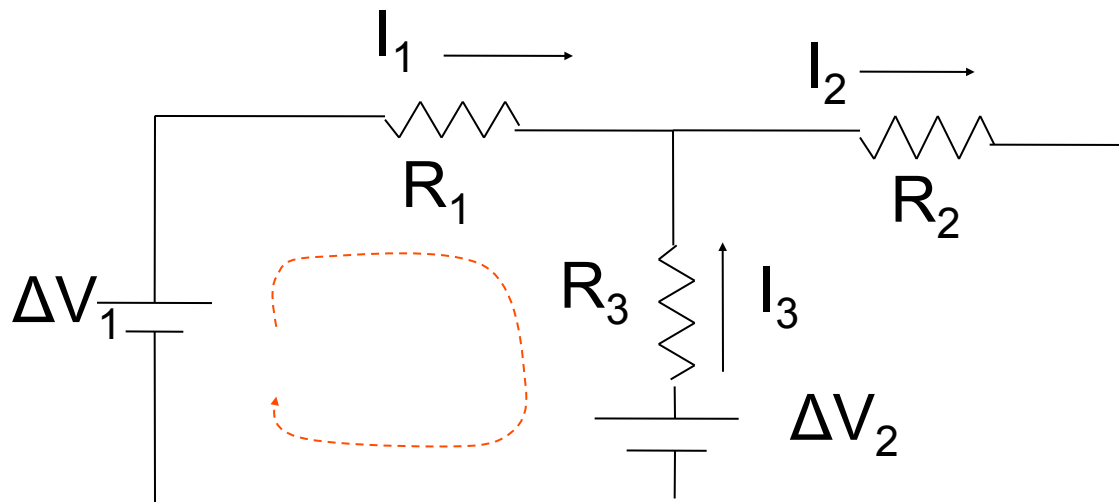
Choose a current direction

- $IR$ , current in traversal direction

+ $IR$  current in opposite direction

+ $\Delta V$  voltage increases along traversal direction

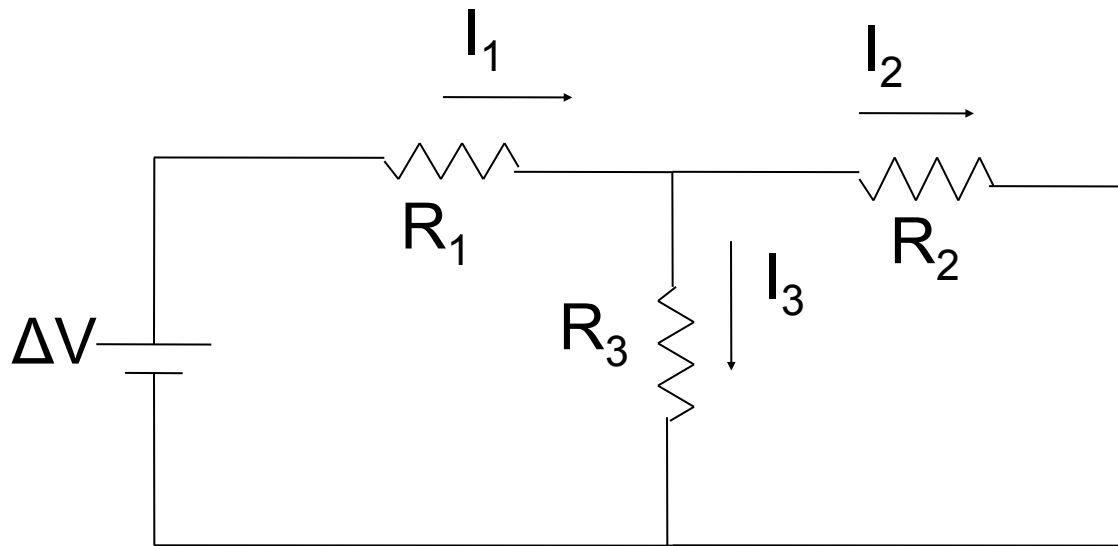
- $\Delta V$  voltage decreases along traversal direction



$$\Delta V_1 - I_1 R_1 + I_3 R_3 - \Delta V_2 = 0$$

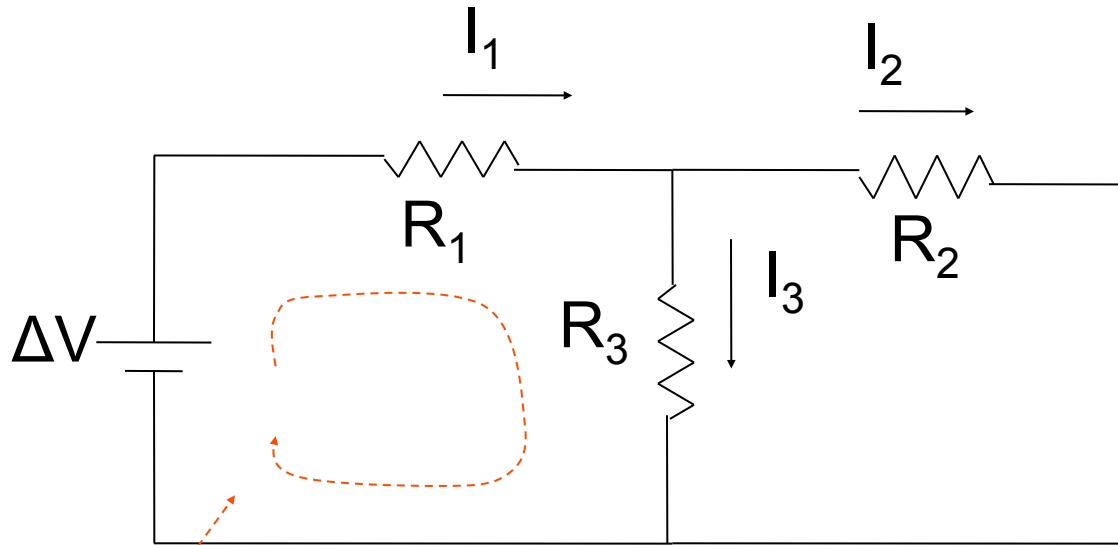
If  $I$  is negative when you solve the equations, the current flows in the opposite direction than you chose.

Not all loop equations are independent



only 2 of these equations are independent

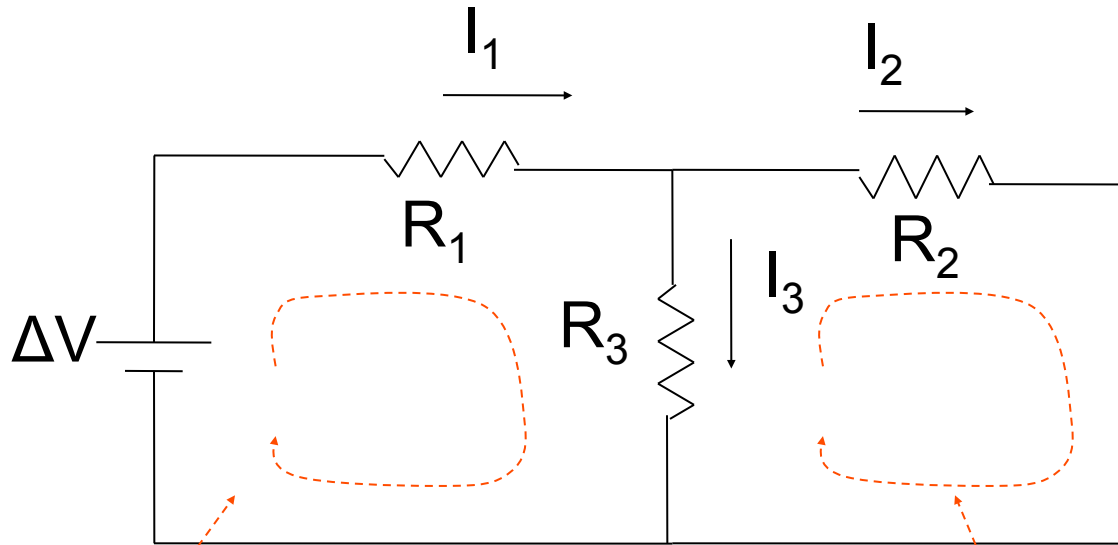
Not all loop equations are independent



$$\Delta V - I_1 R_1 - I_3 R_3 = 0$$

only 2 of these equations are independent

Not all loop equations are independent

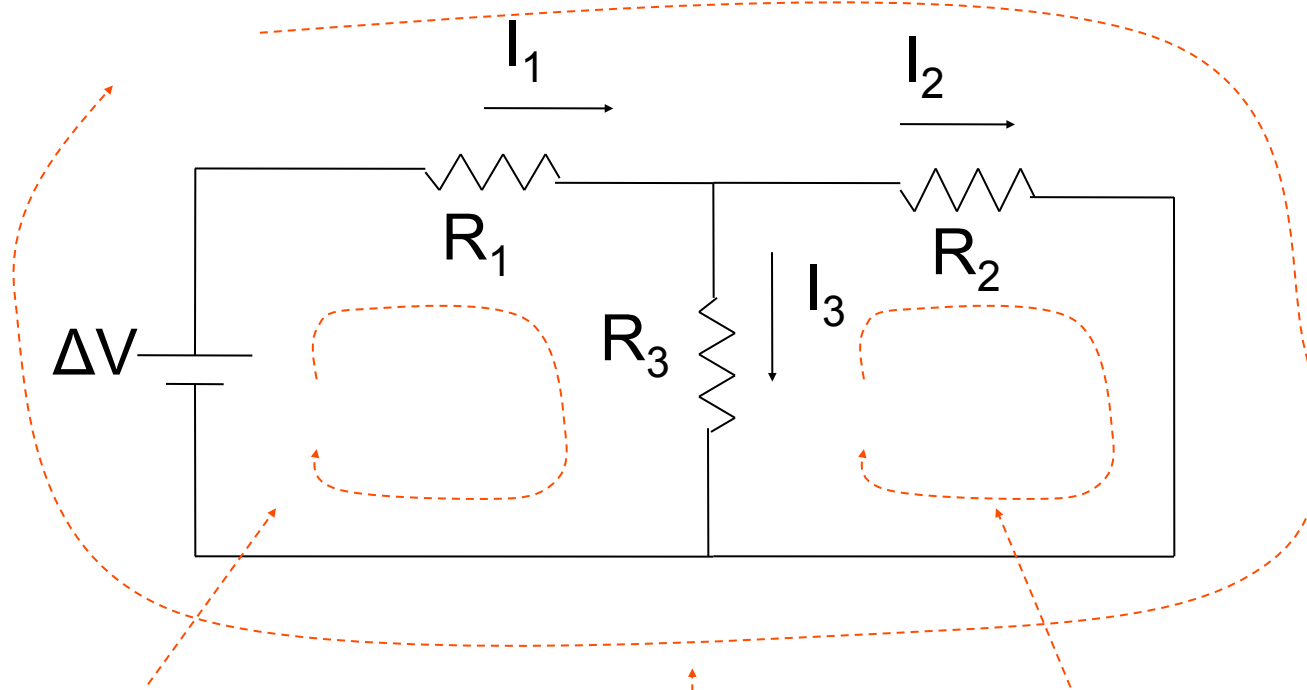


$$\Delta V - I_1 R_1 - I_3 R_3 = 0$$

$$I_3 R_3 - I_2 R_2 = 0$$

only 2 of these equations are independent

Not all loop equations are independent



$$\Delta V - I_1 R_1 - I_3 R_3 = 0$$

$$I_3 R_3 - I_2 R_2 = 0$$

$$\Delta V - I_1 R_1 - I_2 R_2 = 0$$

only 2 of these equations are independent





## Using Kirchoff's rules

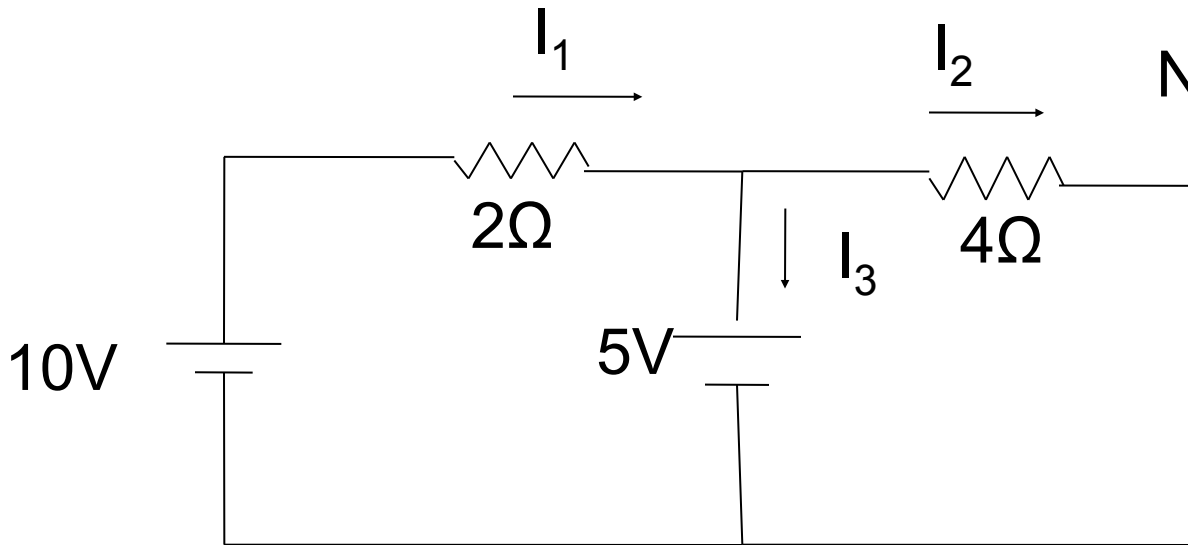
- (1) Write the equations for the junction rule.
- (2) Write the equations for the loop rule. Choose a direction for currents. If the current is negative then it flows in the opposite direction. Use as many equations as necessary to solve for all unknown quantities. (for  $n$  unknowns need  $n$  equations).
- (3) Solve the set of equations for  $n$  unknown quantities.

Find  $I_1$ ,  $I_2$ ,  $I_3$

No. equations needed =

no. Junction =

No. loop =



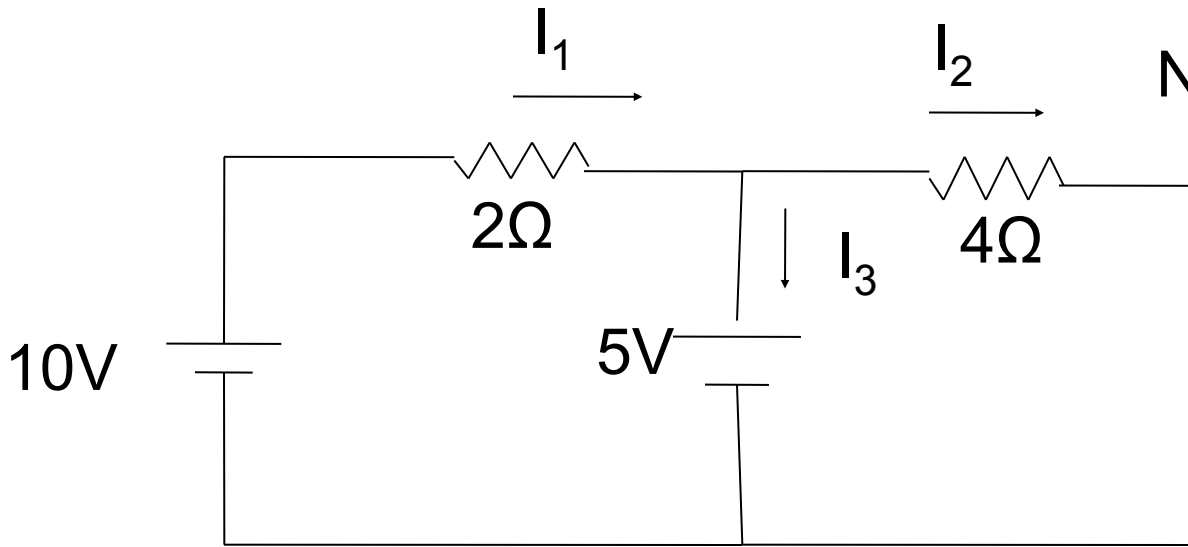
$$V=IR$$

Find  $I_1$ ,  $I_2$ ,  $I_3$

No. equations needed = 3

no. Junction =

No. loop =



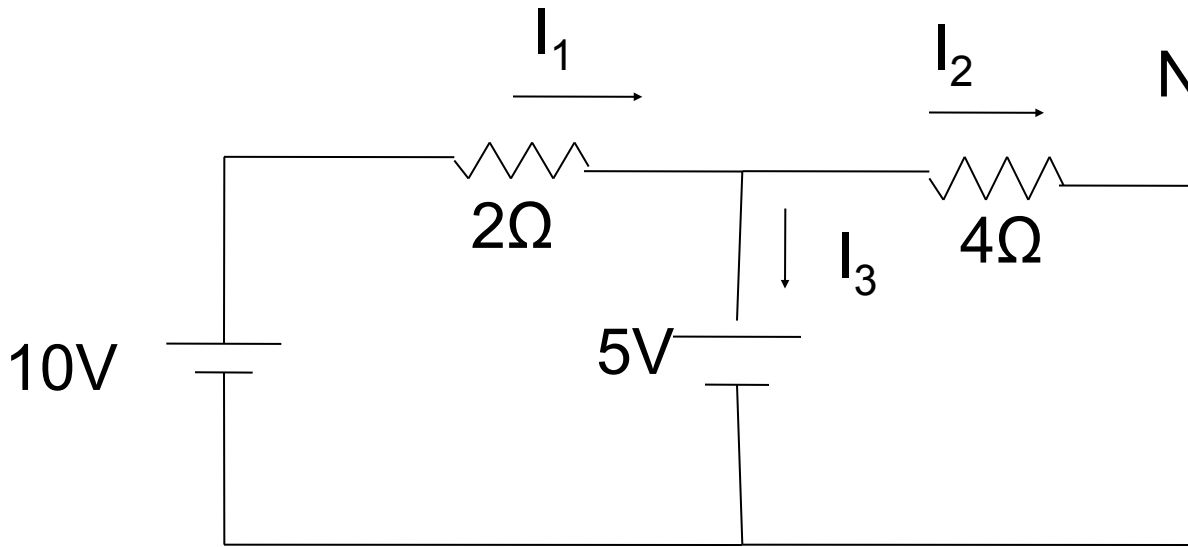
$$V=IR$$

Find  $I_1$ ,  $I_2$ ,  $I_3$

No. equations needed = 3

no. Junction = 1

No. loop =



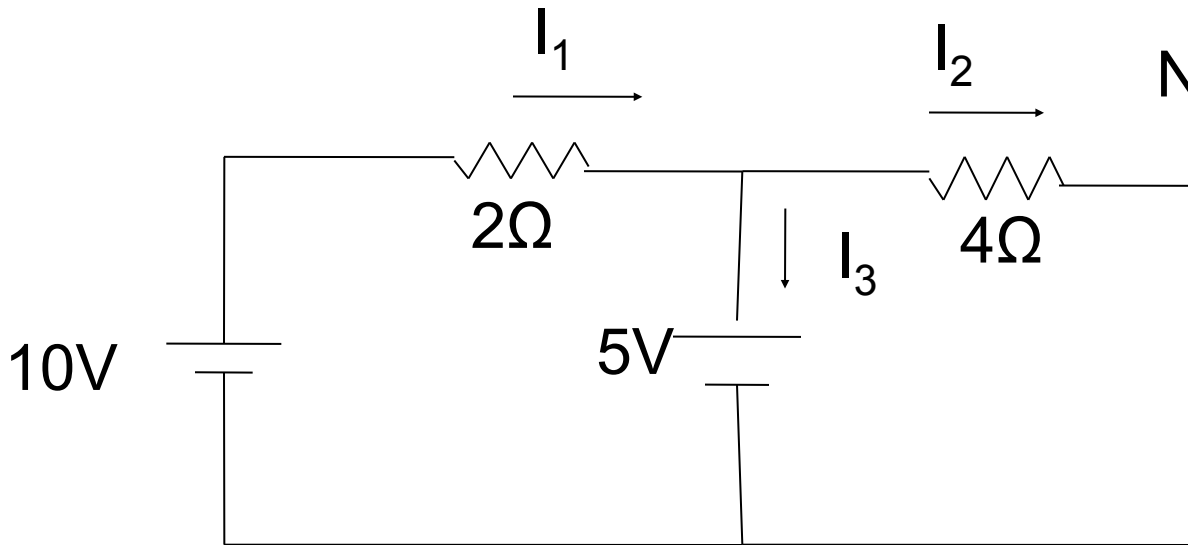
$$V=IR$$

Find  $I_1$ ,  $I_2$ ,  $I_3$

No. equations needed = 3

no. Junction = 1

No. loop = 2



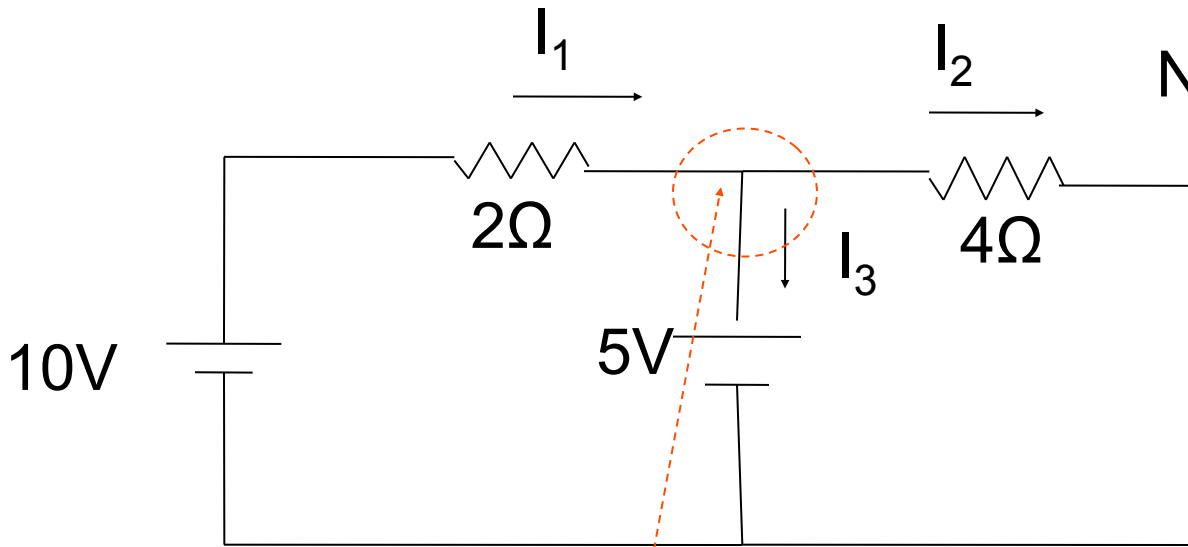
$$V=IR$$

Find  $I_1$ ,  $I_2$ ,  $I_3$

No. equations needed = 3

no. Junction = 1

No. loop = 2



$$V=IR$$

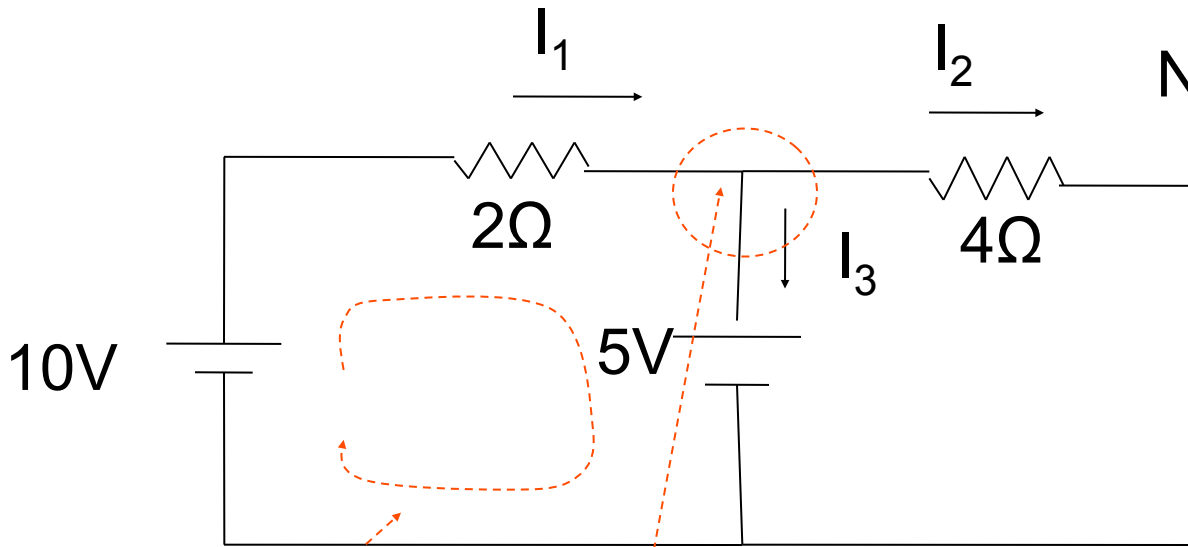
$$I_1 = I_2 + I_3$$

Find  $I_1, I_2, I_3$

No. equations needed = 3

no. Junction = 1

No. loop = 2



$V=IR$

$$10 - 2I_1 - 5 = 0$$

$$I_1 = I_2 + I_3$$

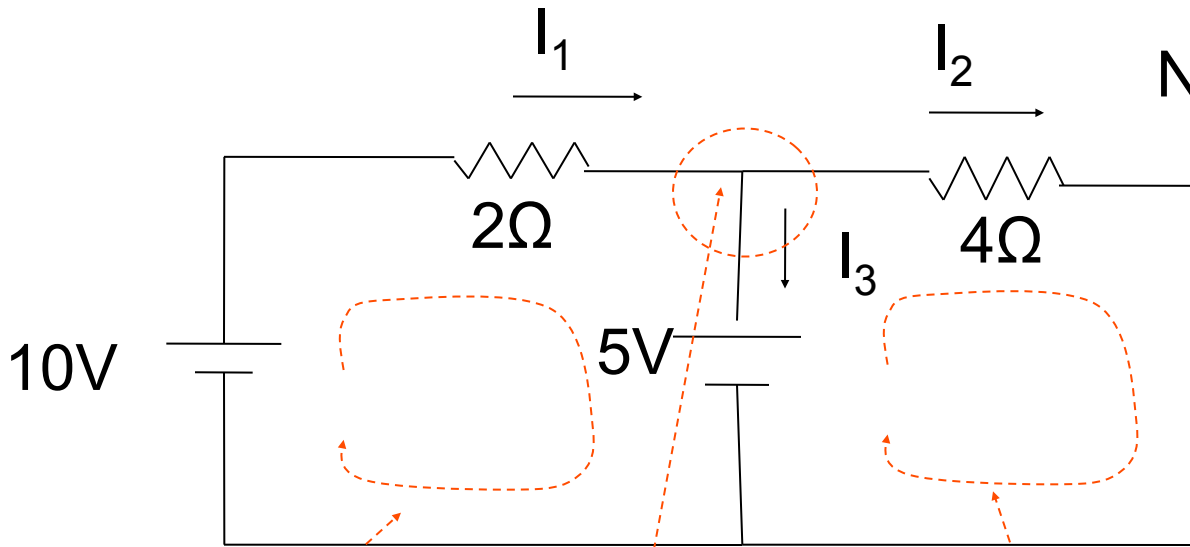


Find  $I_1, I_2, I_3$

No. equations needed = 3

no. Junction = 1

No. loop = 2



$V=IR$

$$10 - 2I_1 - 5 = 0$$

$$I_1 = I_2 + I_3$$

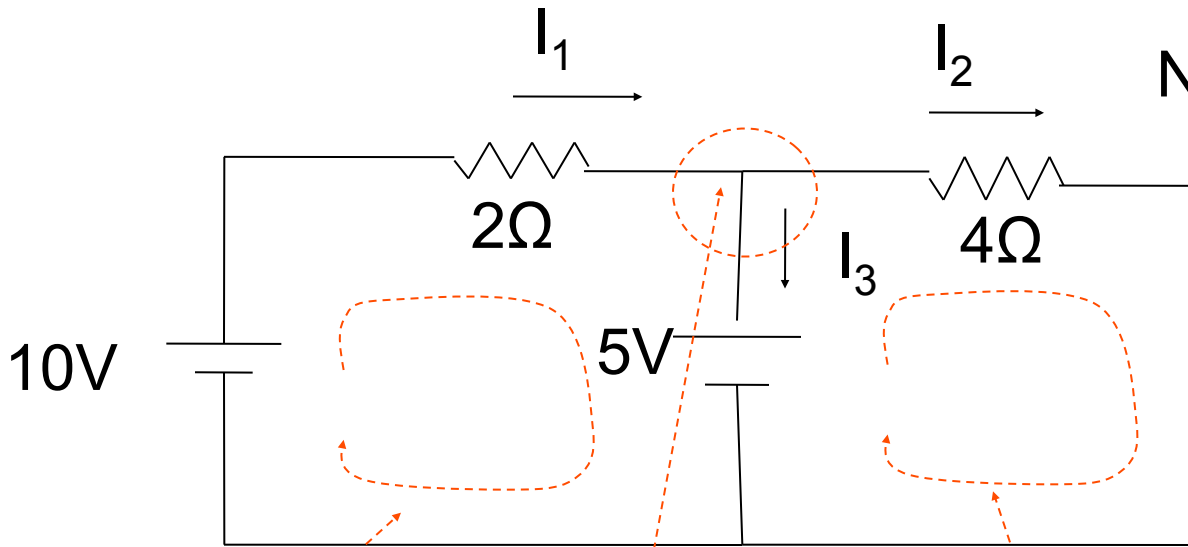
$$5 - 4I_2 = 0$$

Find  $I_1, I_2, I_3$

No. equations needed = 3

no. Junction = 1

No. loop = 2



$$V=IR$$

$$10 - 2I_1 - 5 = 0$$

$$I_1 = I_2 + I_3$$

$$5 - 4I_2 = 0$$

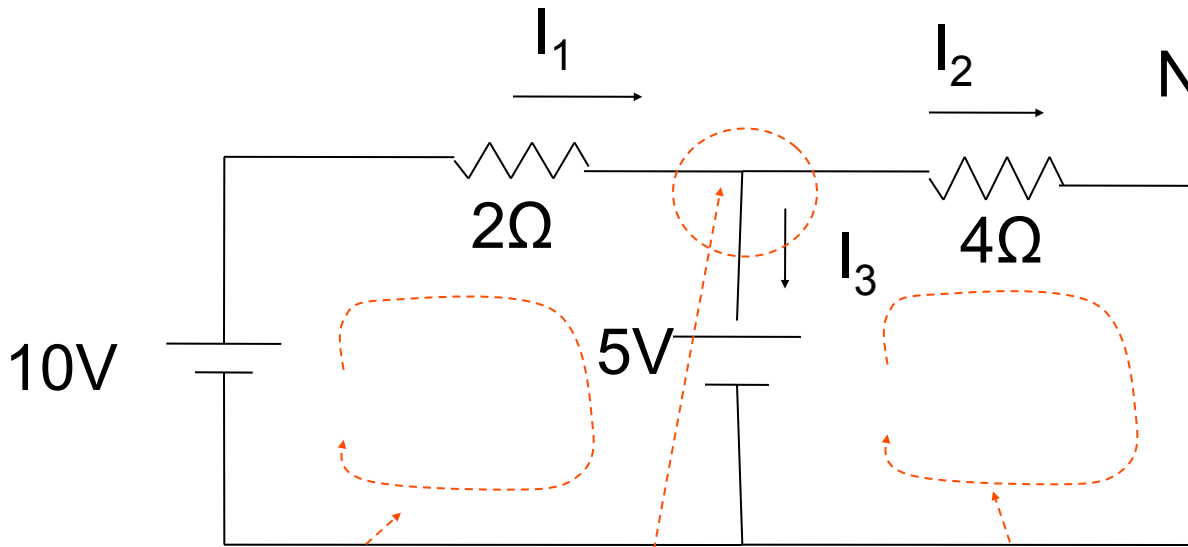
$$I_1 = \frac{10 - 5}{2} = 2.5A$$

Find  $I_1$ ,  $I_2$ ,  $I_3$

No. equations needed = 3

no. Junction = 1

No. loop = 2



$$V=IR$$

$$10 - 2I_1 - 5 = 0$$

$$I_1 = I_2 + I_3$$

$$5 - 4I_2 = 0$$

$$I_1 = \frac{10 - 5}{2} = 2.5A$$

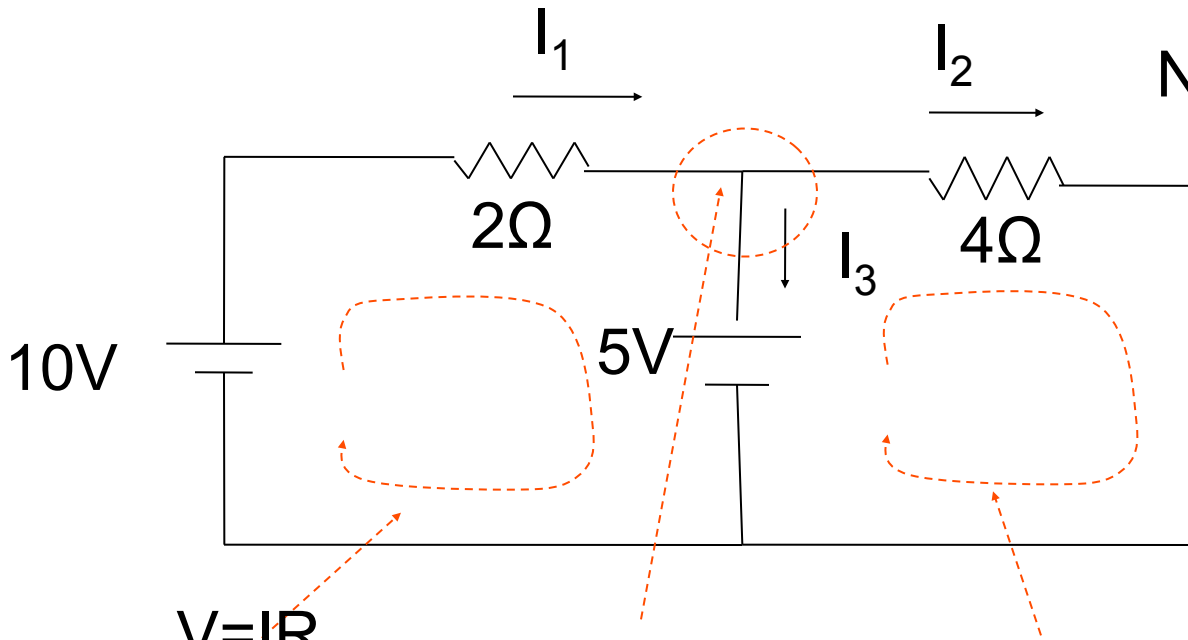
$$I_2 = \frac{5}{4} = 1.25A$$

Find  $I_1$ ,  $I_2$ ,  $I_3$

No. equations needed = 3

no. Junction = 1

No. loop = 2



$$V=IR$$

$$10 - 2I_1 - 5 = 0$$

$$I_1 = \frac{10 - 5}{2} = 2.5A$$

$$I_1 = I_2 + I_3$$

$$I_3 = I_1 - I_2$$

$$I_3 = 2.5 - 1.25 = 1.25A$$

$$5 - 4I_2 = 0$$

$$I_2 = \frac{5}{4} = 1.25A$$

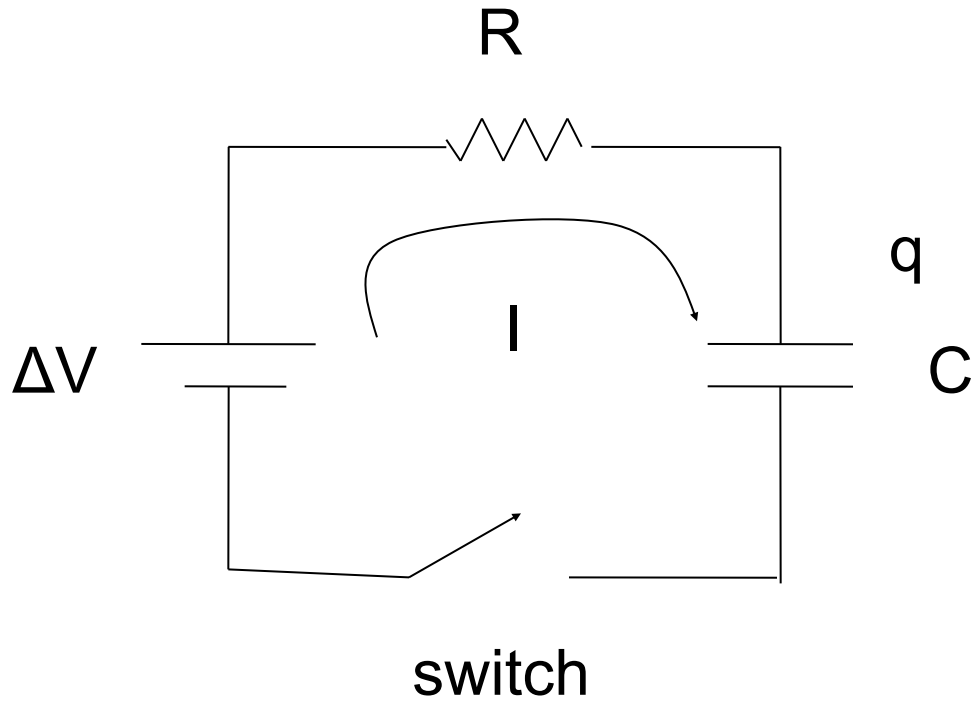
# Chapter 18.5 RC circuit

Time dependent currents and voltages.

Applications. clocks, timing circuits,  
computers.

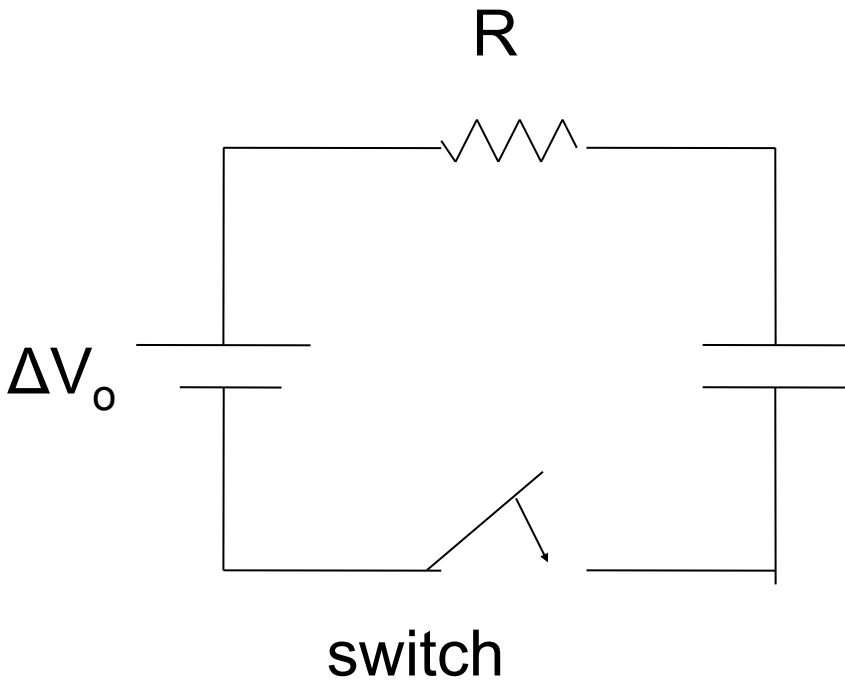
Time to charge and discharge of a capacitor

# RC circuit



When the switch is closed how does the current and voltage change with time?

# RC circuit

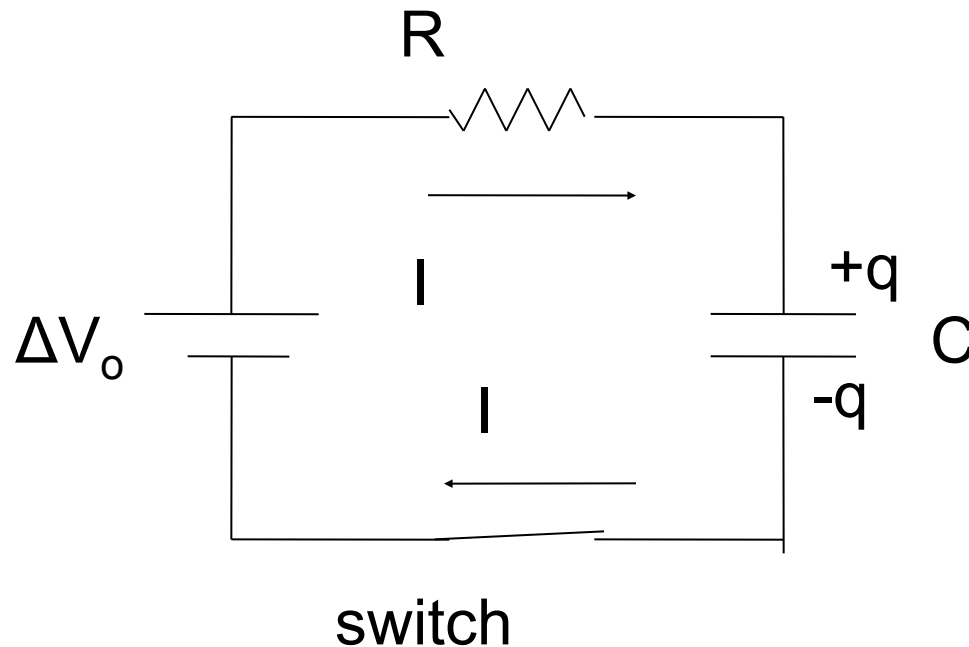


Switch off

Capacitor uncharged

$\Delta V_c = 0$

Charging



Switch on

$$\Delta V_c = \frac{q}{C}$$

$$\Delta V_0 - IR - \Delta V_c = 0$$

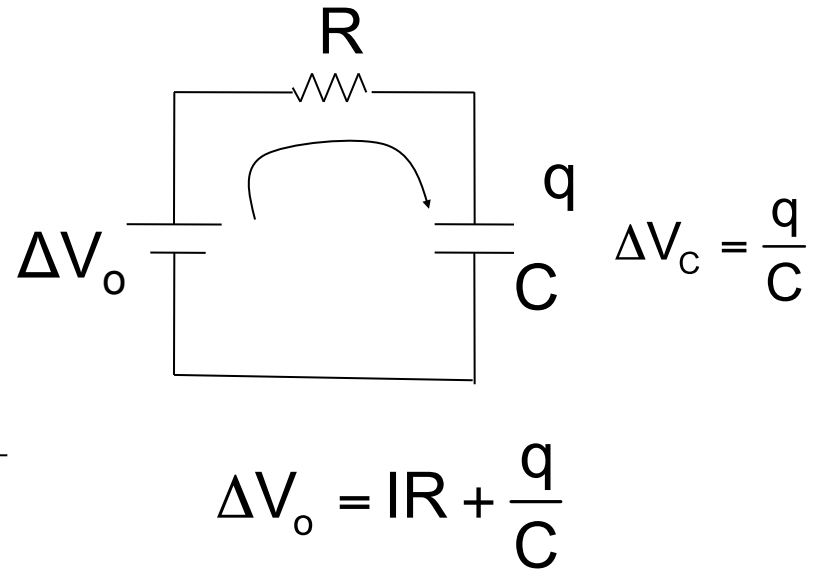
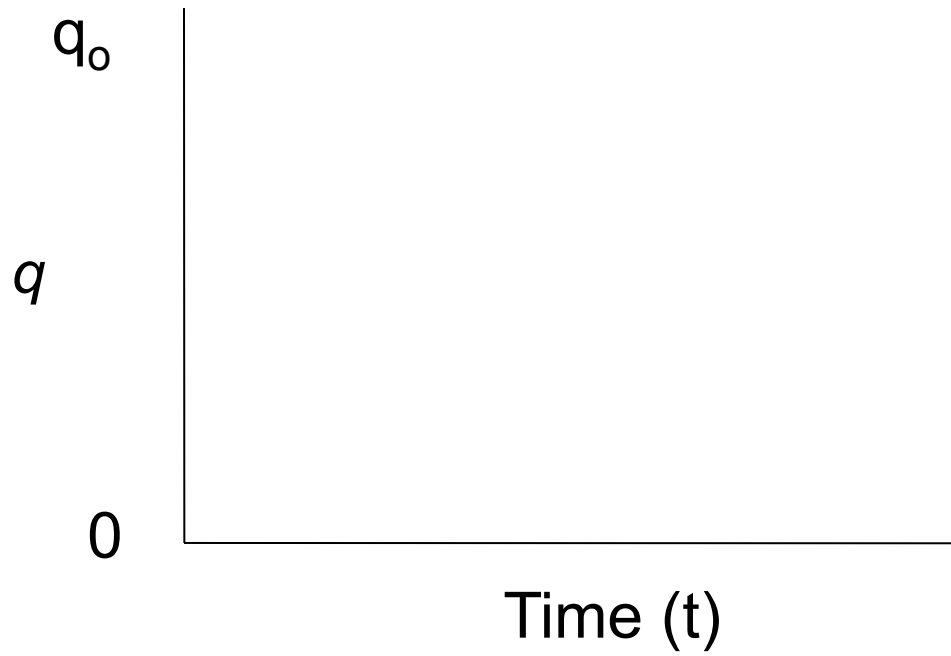
When the switch is initially closed the voltage on the capacitor is zero.

Charge is transferred to the capacitor at a rate  $I = dq/dt$ .

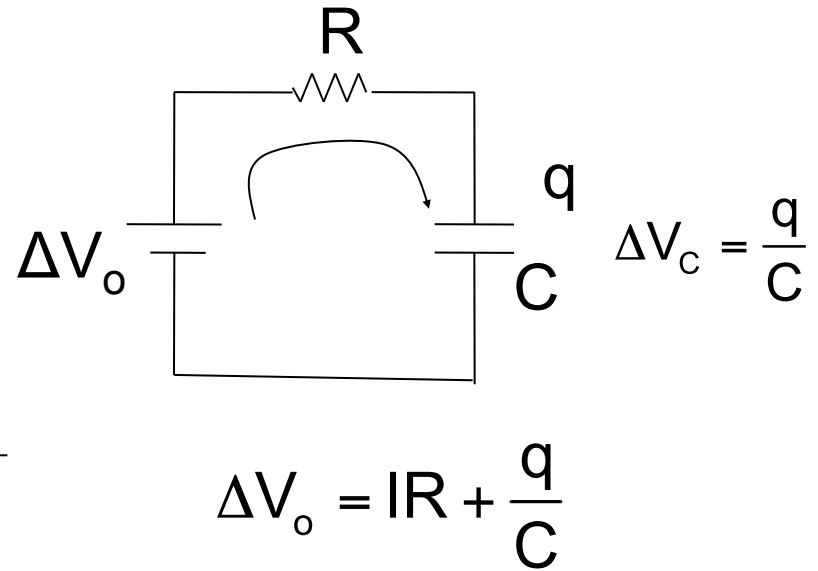
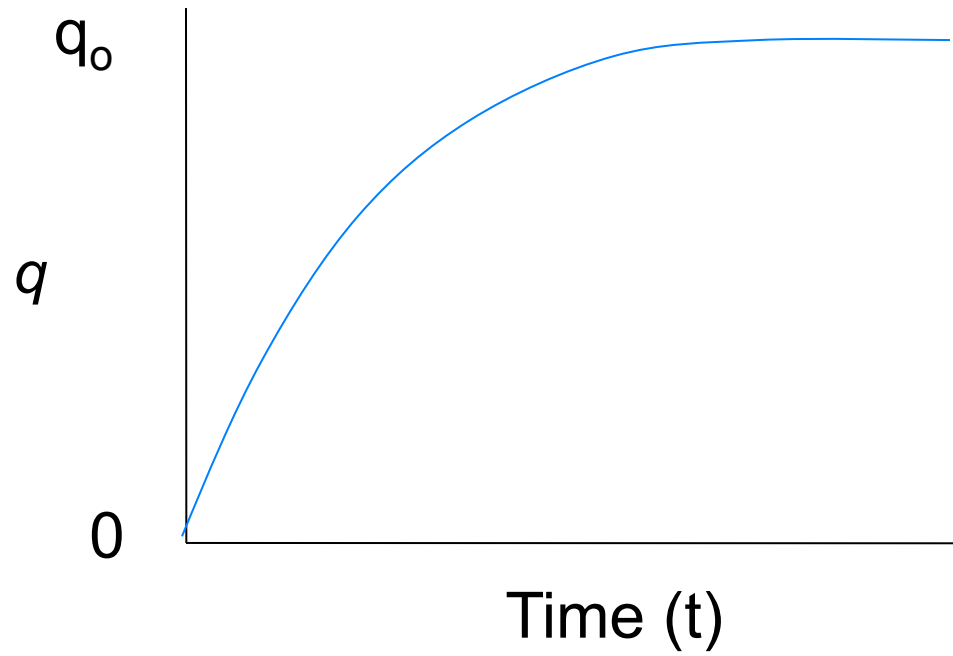
As the capacitor is charging the charge and voltage on the capacitor increases with time and the current decreases.



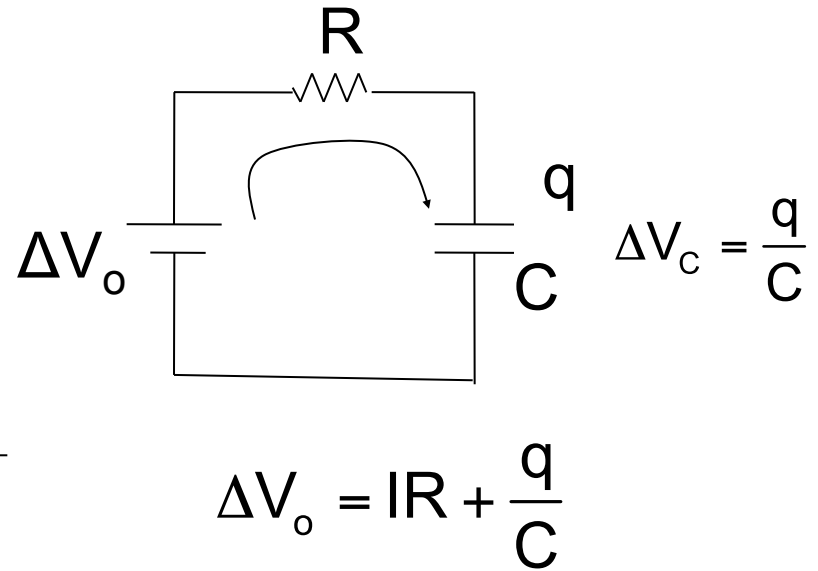
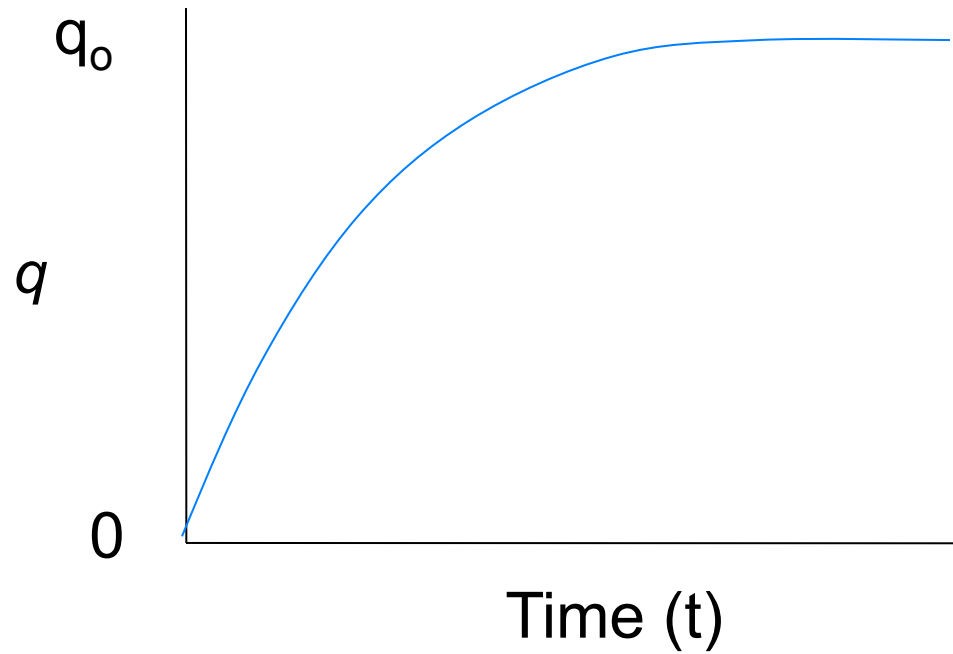
# Charging Capacitor



# Charging Capacitor



# Charging Capacitor

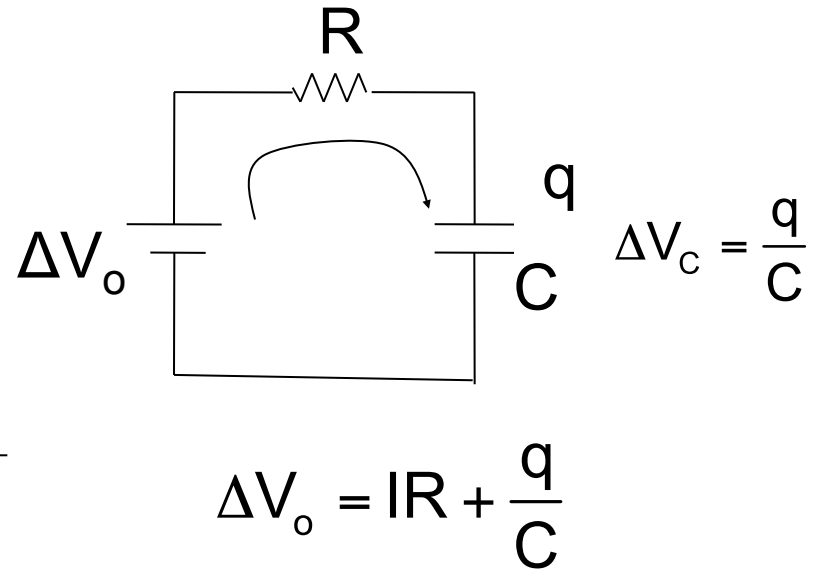
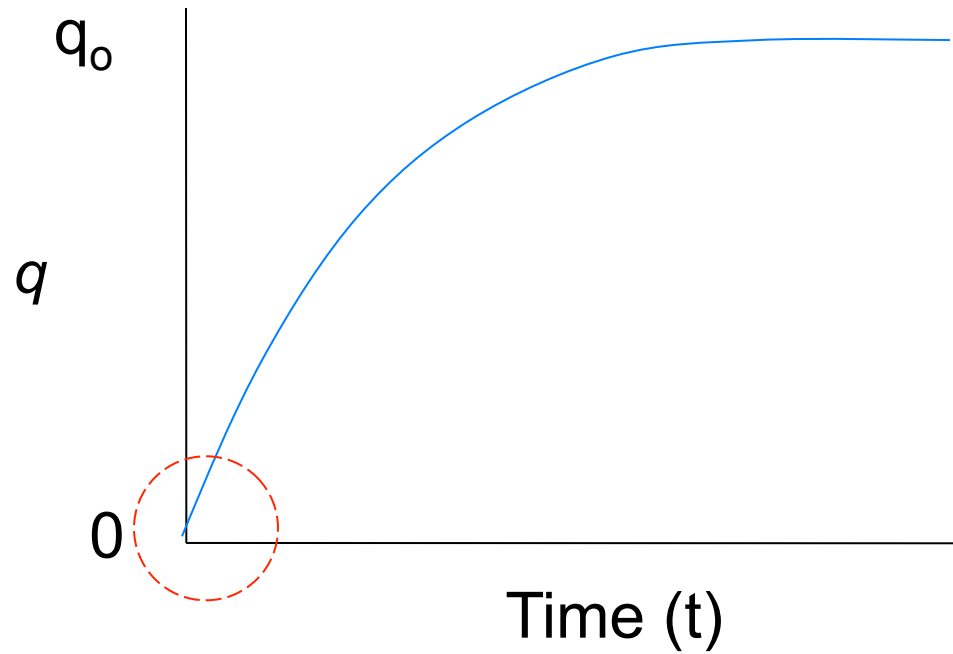


$q =$

$\Delta V_C =$

$I =$

# Charging Capacitor



$$\Delta V_0 = IR + \frac{q}{C}$$

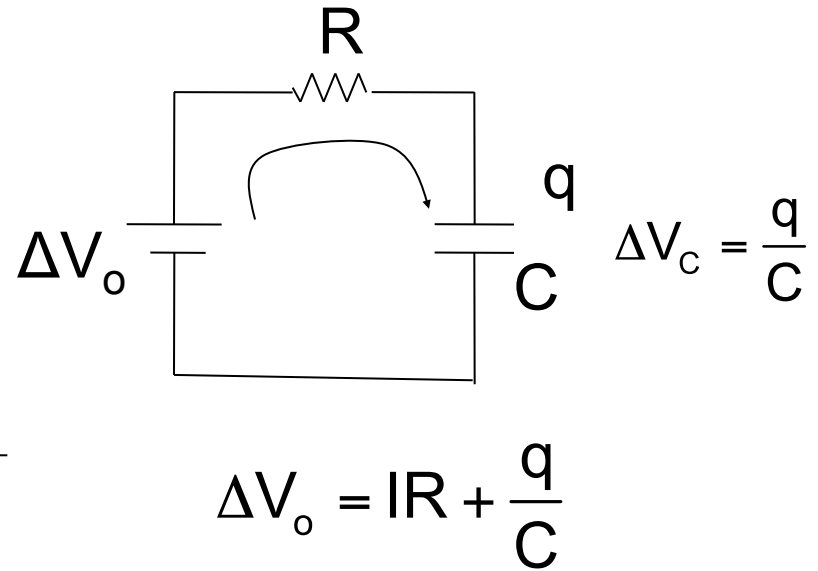
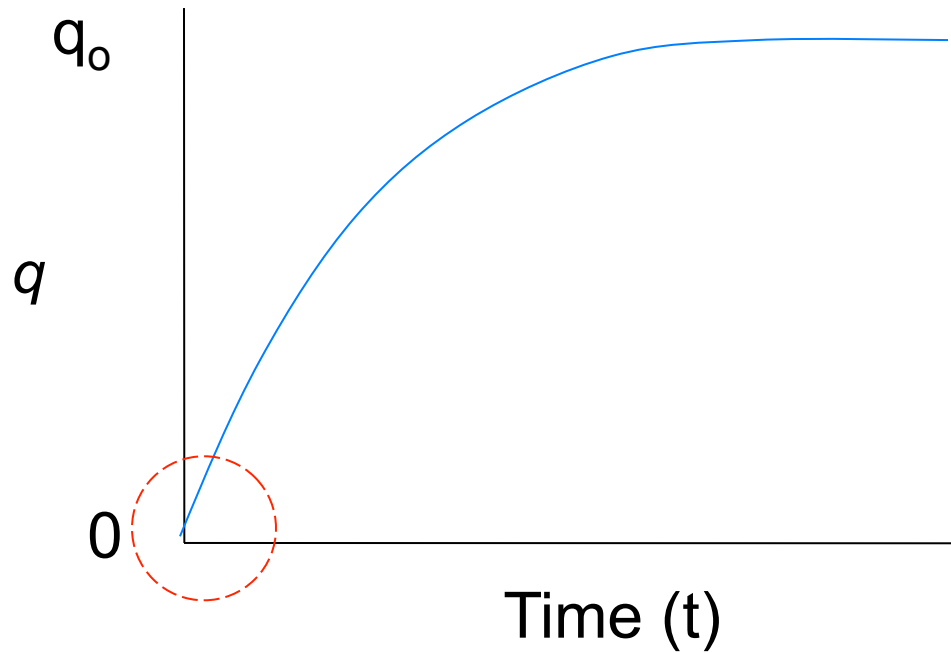
short times

$q =$

$\Delta V_C =$

$I =$

# Charging Capacitor



$$\Delta V_0 = IR + \frac{q}{C}$$

short times

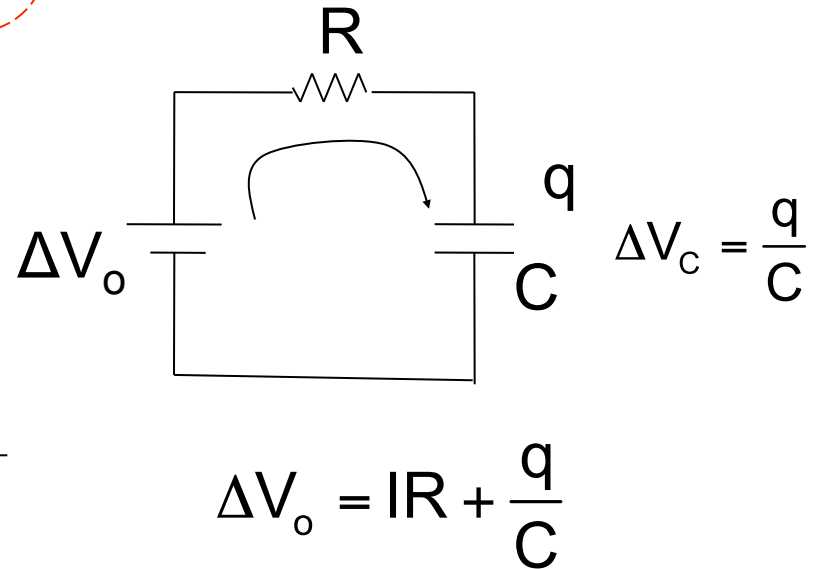
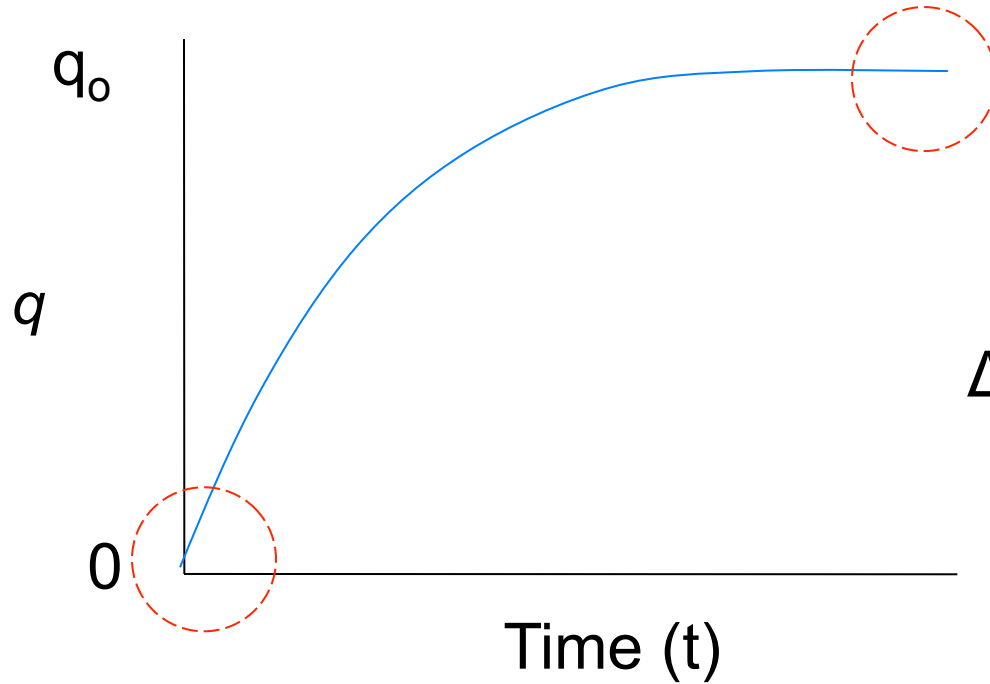
intermediate  
times

$q =$

$\Delta V_C =$

$I =$

# Charging Capacitor



$$\Delta V_C = \frac{q}{C}$$

$$\Delta V_0 = IR + \frac{q}{C}$$

short times

intermediate times

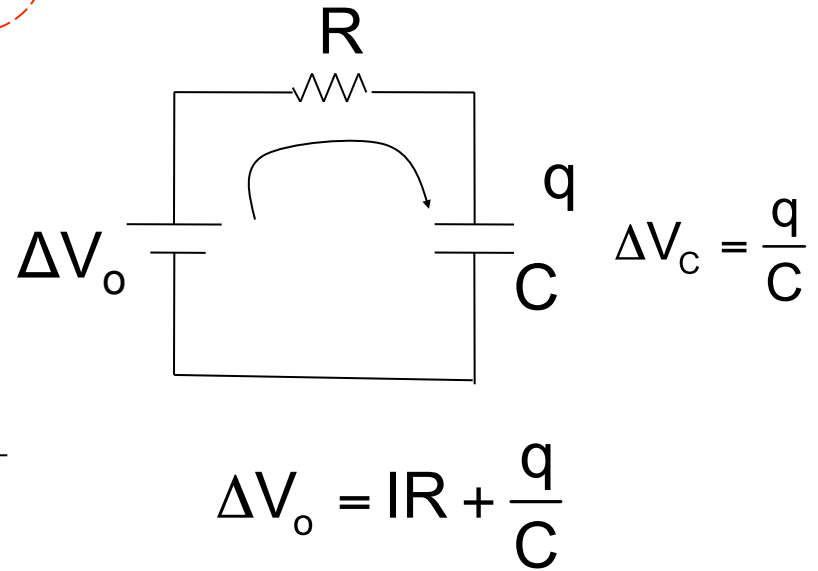
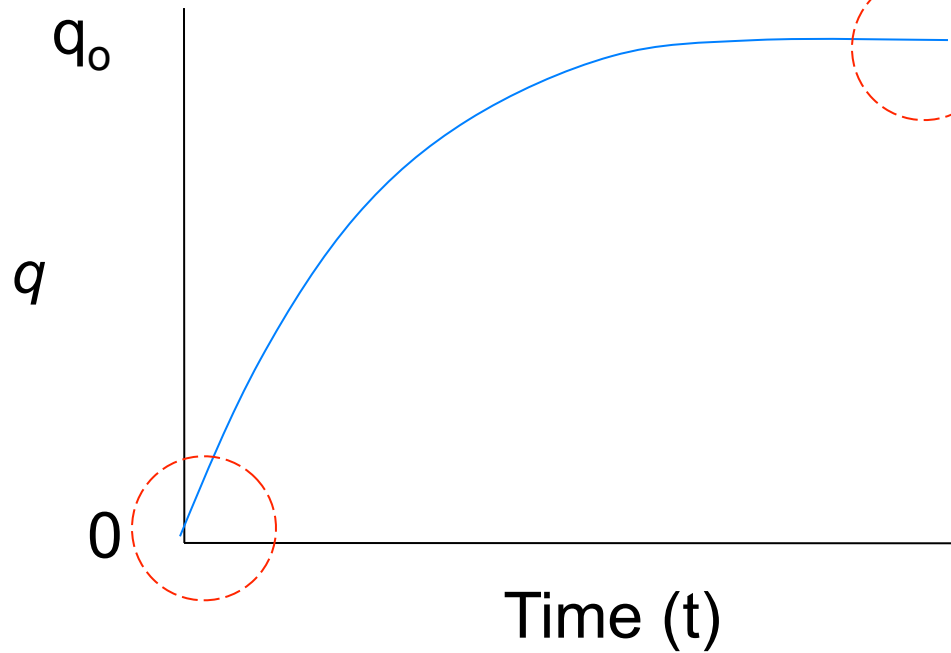
long times

$q =$

$\Delta V_C =$

$I =$

# Charging Capacitor



$$\Delta V_C = \frac{q}{C}$$

$$\Delta V_0 = IR + \frac{q}{C}$$

short times

intermediate  
times

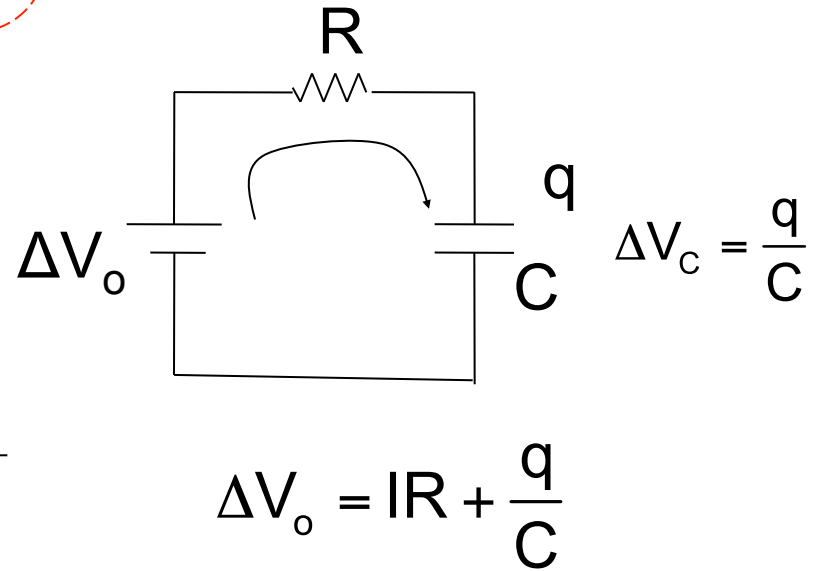
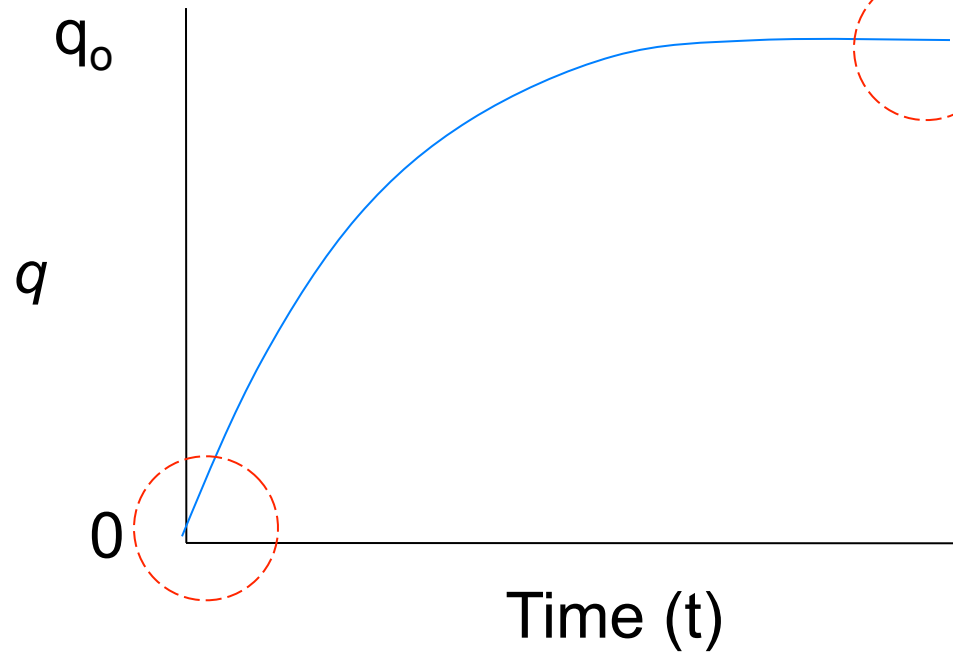
long times

$$q = \sim 0$$

$$\Delta V_C =$$

$$I =$$

# Charging Capacitor



$$\Delta V_0 = IR + \frac{q}{C}$$

short times

intermediate  
times

long times

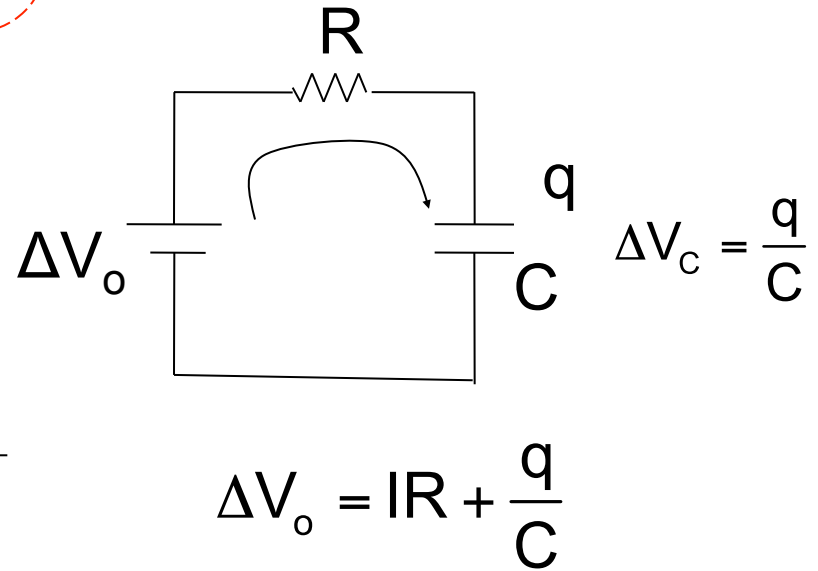
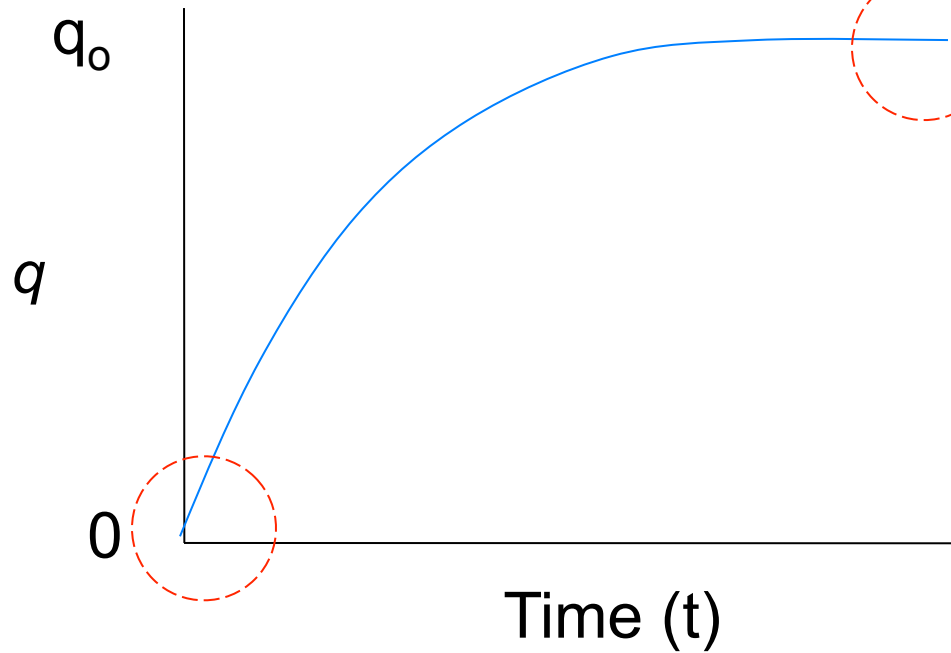
$$q = \sim 0$$

$$\Delta V_C = \approx 0$$

$$I =$$



# Charging Capacitor



$$\Delta V_0 = IR + \frac{q}{C}$$

short times

intermediate  
times

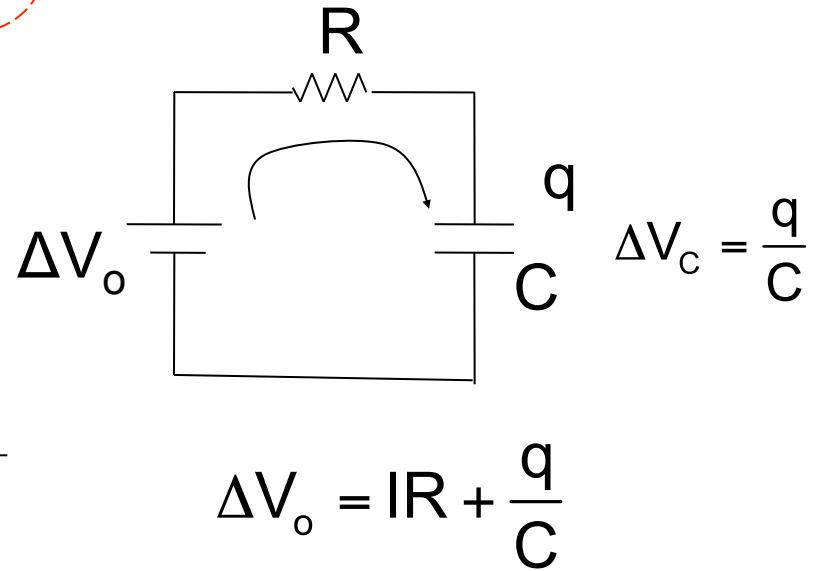
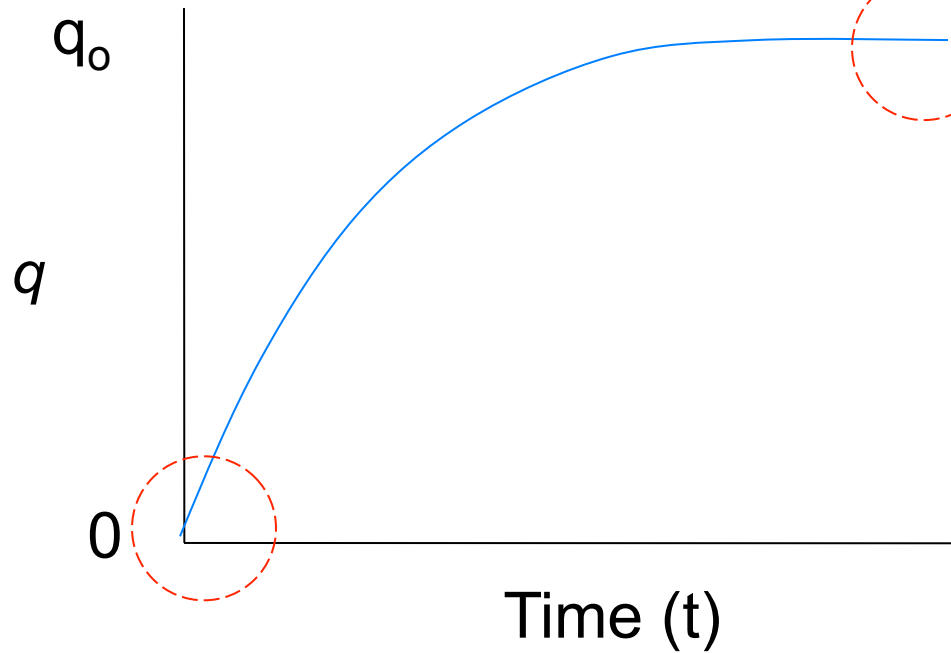
long times

$$q = \sim 0$$

$$\Delta V_C = \approx 0$$

$$I = \approx \frac{\Delta V_0}{R}$$

# Charging Capacitor



$$\Delta V_0 = IR + \frac{q}{C}$$

short times

intermediate  
times

long times

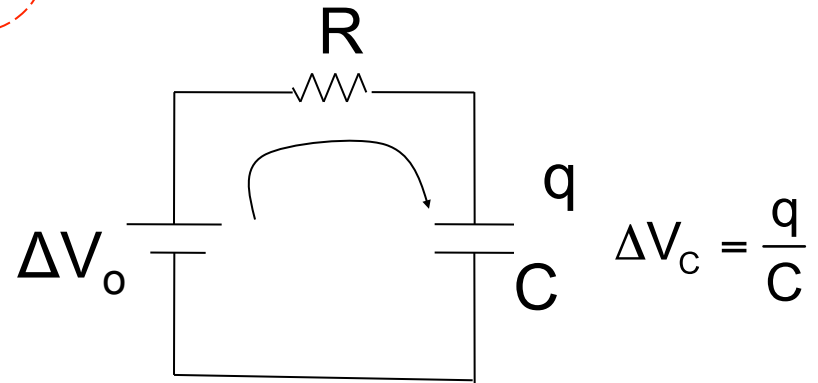
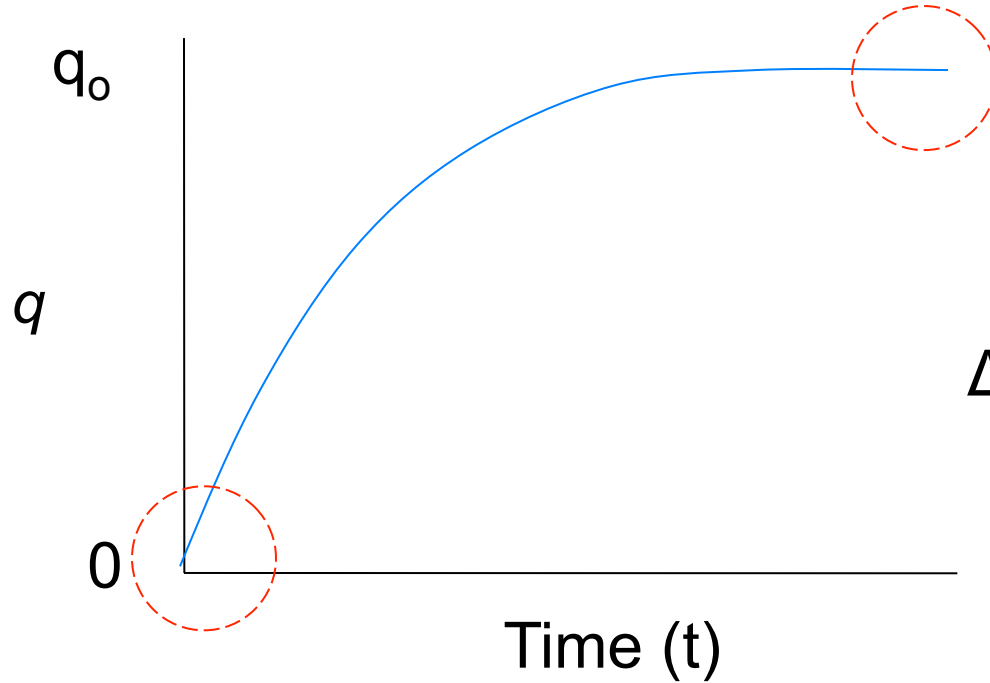
$$q = \sim 0$$

$$q = q_0$$

$$\Delta V_c = \approx 0$$

$$I = \approx \frac{\Delta V_0}{R}$$

# Charging Capacitor



$$\Delta V_C = \frac{q}{C}$$

$$\Delta V_0 = IR + \frac{q}{C}$$

short times

intermediate  
times

long times

$$q = \sim 0$$

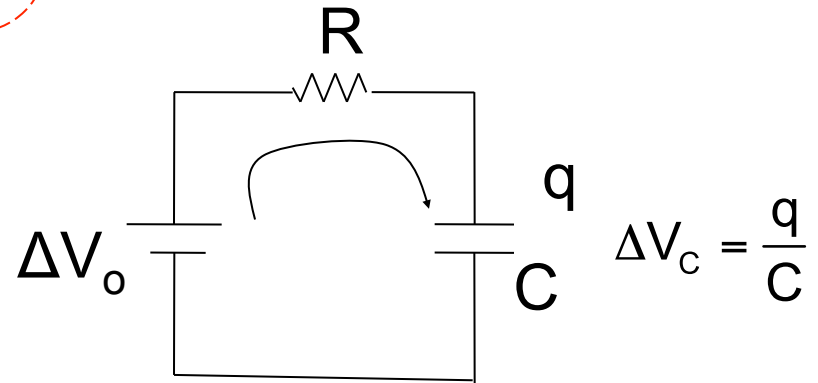
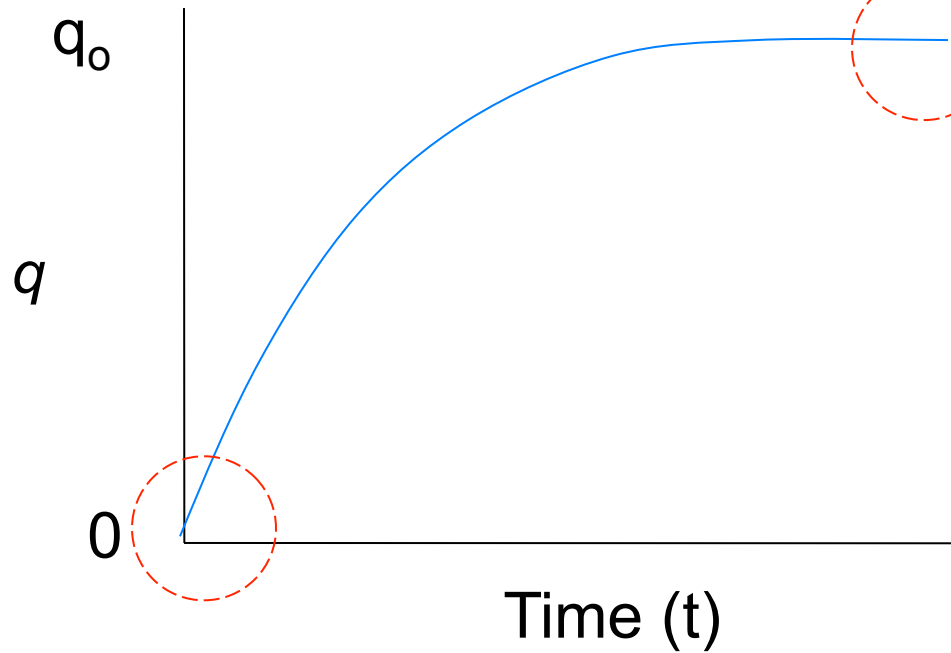
$$\Delta V_C = \approx 0$$

$$I = \approx \frac{\Delta V_0}{R}$$

$$q = q_0$$

$$\Delta V_C = \Delta V_0$$

# Charging Capacitor



$$\Delta V_0 = IR + \frac{q}{C}$$

short times

intermediate  
times

long times

$$q = \sim 0$$

$$\Delta V_C = \approx 0$$

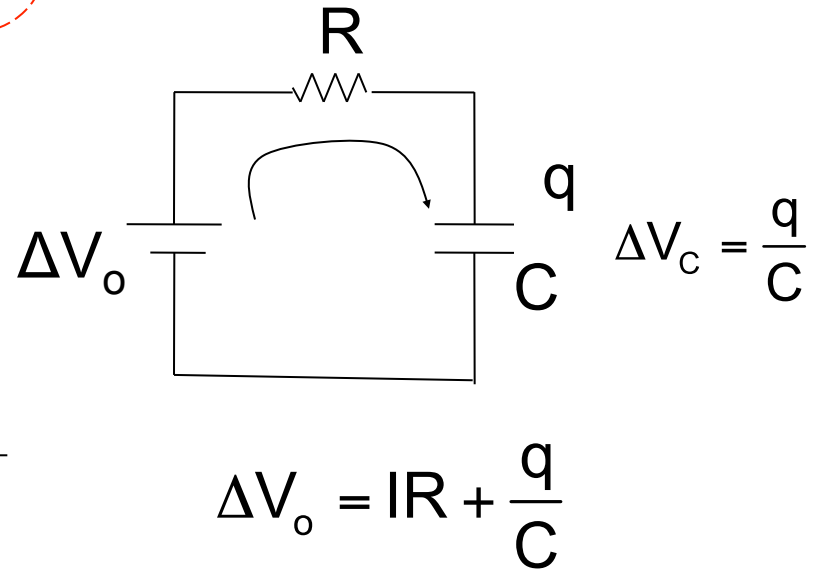
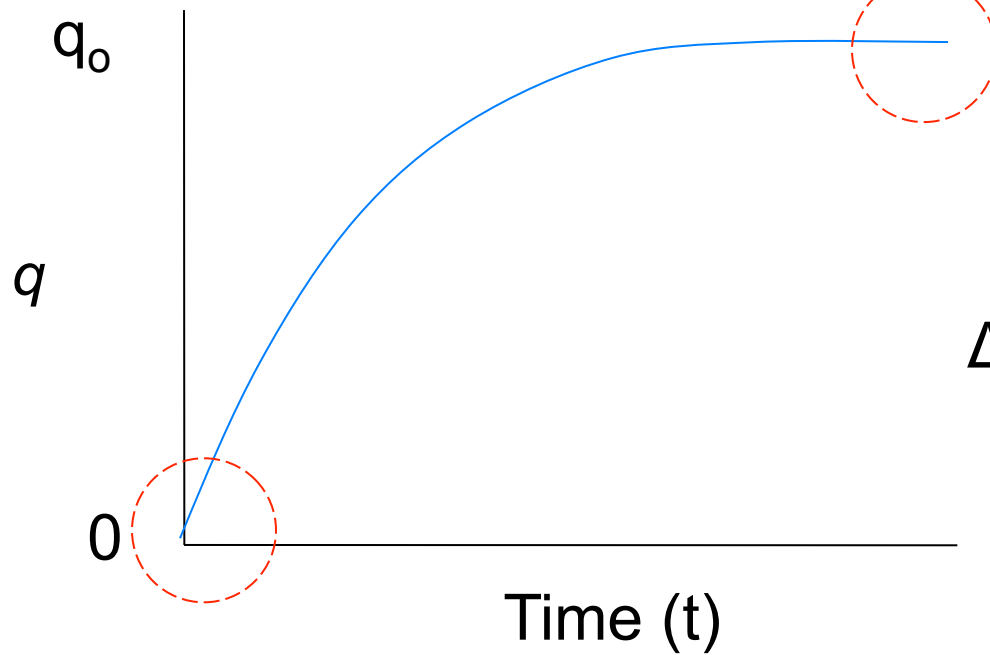
$$I = \approx \frac{\Delta V_0}{R}$$

$$q = q_0$$

$$\Delta V_C = \Delta V_0$$

$$I = 0$$

# Charging Capacitor



$$\Delta V_o = IR + \frac{q}{C}$$

short times

intermediate times

long times

$$q = \sim 0$$

$$q = q_o(1 - e^{-\frac{t}{\tau}})$$

$$q = q_o$$

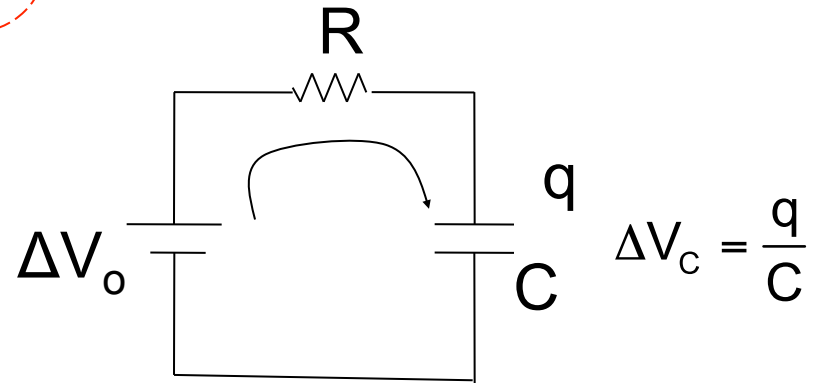
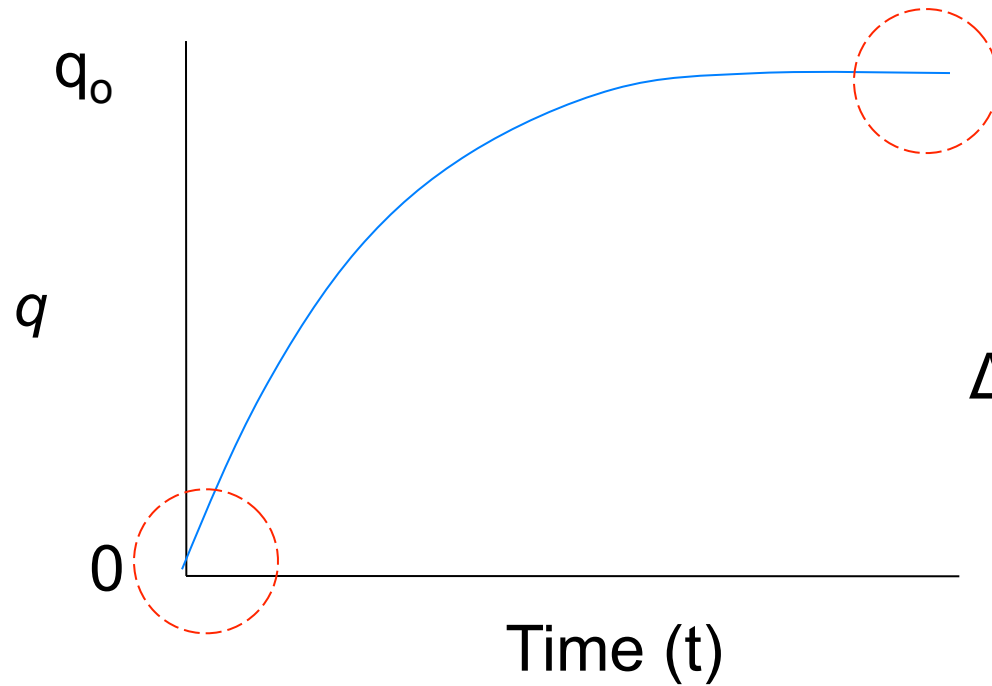
$$\Delta V_c = \approx 0$$

$$\Delta V_c = \Delta V_o$$

$$I = \approx \frac{\Delta V_o}{R}$$

$$I = 0$$

# Charging Capacitor



$$\Delta V_o = IR + \frac{q}{C}$$

short times

intermediate  
times

long times

$$q = \sim 0$$

$$q = q_o(1 - e^{-\left(\frac{t}{\tau}\right)})$$

$$q = q_o$$

$$\Delta V_c = \approx 0$$

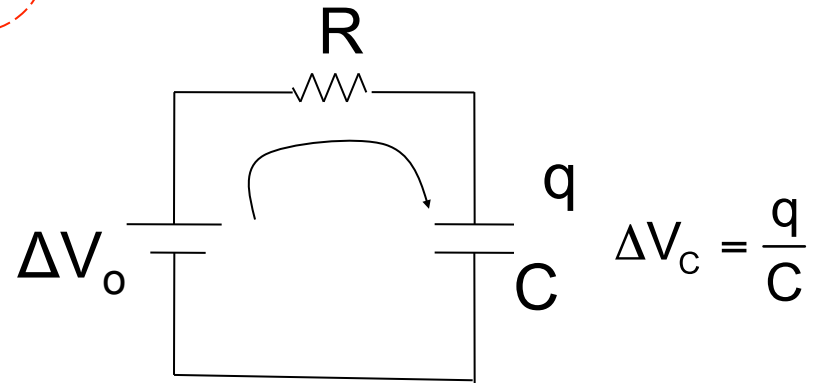
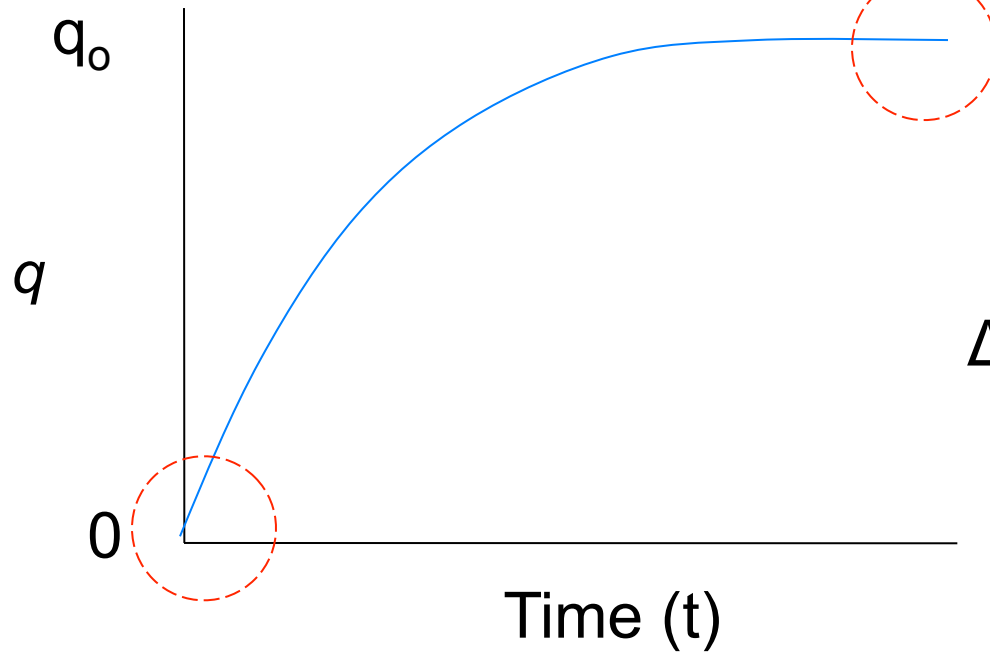
$$\Delta V_c = V_o(1 - e^{-\left(\frac{t}{\tau}\right)})$$

$$\Delta V_c = \Delta V_o$$

$$I = \approx \frac{\Delta V_o}{R}$$

$$I = 0$$

# Charging Capacitor



$$\Delta V_c = \frac{q}{C}$$

$$\Delta V_o = IR + \frac{q}{C}$$

short times

intermediate  
times

long times

$$q = \sim 0$$

$$q = q_o(1 - e^{-\left(\frac{t}{\tau}\right)})$$

$$q = q_o$$

$$\Delta V_c = \approx 0$$

$$\Delta V_c = V_o(1 - e^{-\left(\frac{t}{\tau}\right)})$$

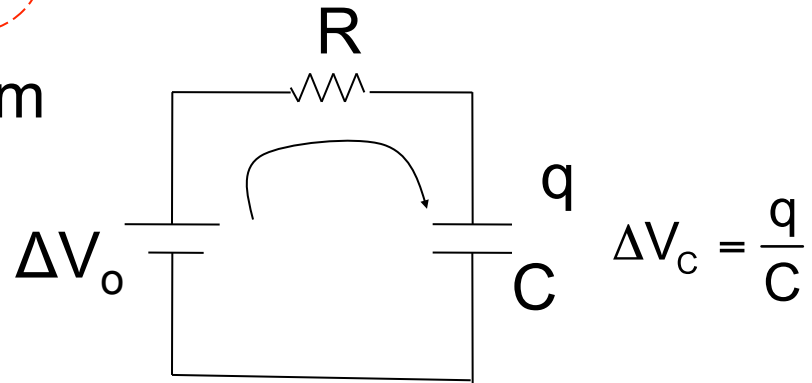
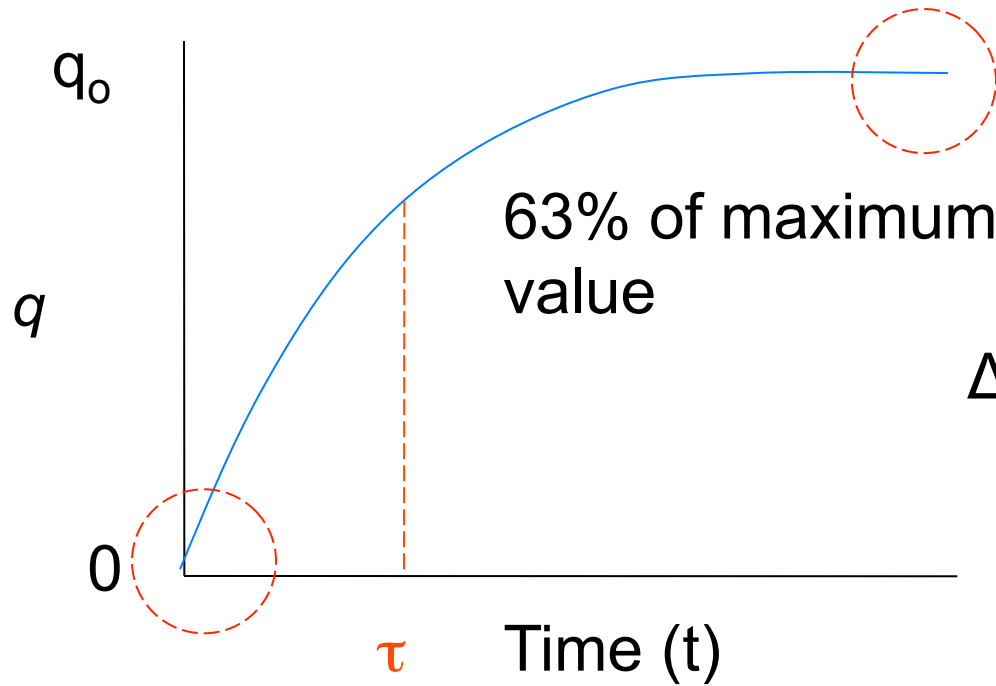
$$\Delta V_c = \Delta V_o$$

$$I = \approx \frac{\Delta V_o}{R}$$

$$I = \frac{\Delta V_o}{R} e^{-\left(\frac{t}{\tau}\right)}$$

$$I = 0$$

# Charging Capacitor



$$\Delta V_o = IR + \frac{q}{C}$$

short times

intermediate times

long times

$$q = \sim 0$$

$$q = q_o(1 - e^{-\left(\frac{t}{\tau}\right)})$$

$$q = q_o$$

$$\Delta V_c = \approx 0$$

$$\Delta V_c = V_o(1 - e^{-\left(\frac{t}{\tau}\right)})$$

$$\Delta V_c = \Delta V_o$$

$$I = \approx \frac{\Delta V_o}{R}$$

$$I = \frac{\Delta V_o}{R} e^{-\left(\frac{t}{\tau}\right)}$$

$$\tau = RC$$

$$I = 0$$



# PHYSICS 1B – Fall 2009



## Electricity & Magnetism



Professor Brian Keating  
SERF Building. Room 333

## Time Constant

$$\tau = RC$$

Dimensional analysis

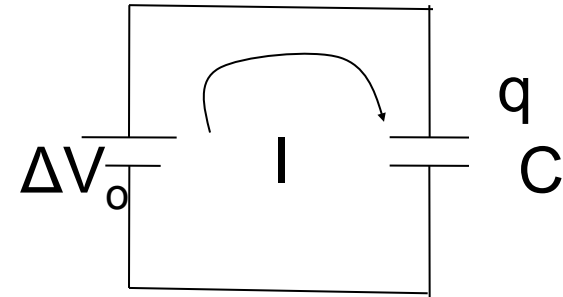
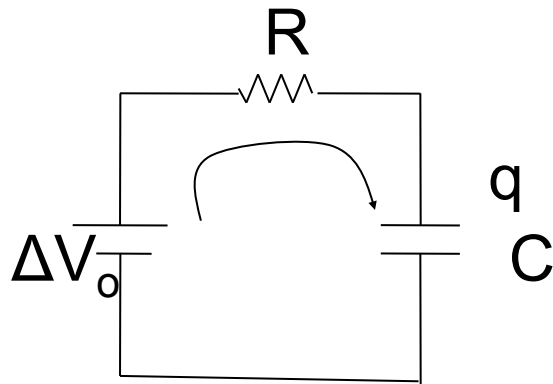
$$RC = \frac{V}{I} \frac{q}{V} = \frac{q}{I} = \frac{q}{q/t} = t$$

RC has units of time

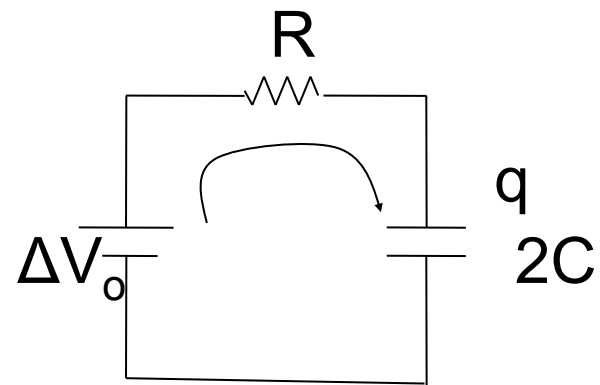
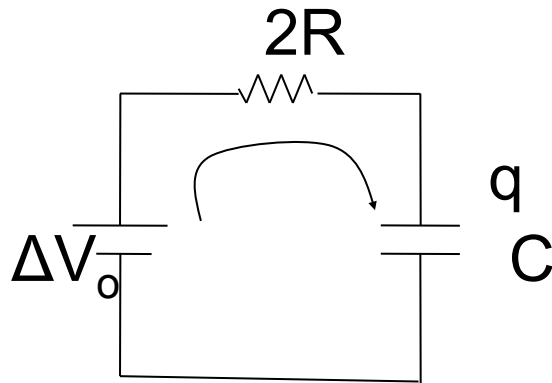
Time required to charge the capacitor

- increases with R – lower current flow
- Increases with C - more charge on capacitor

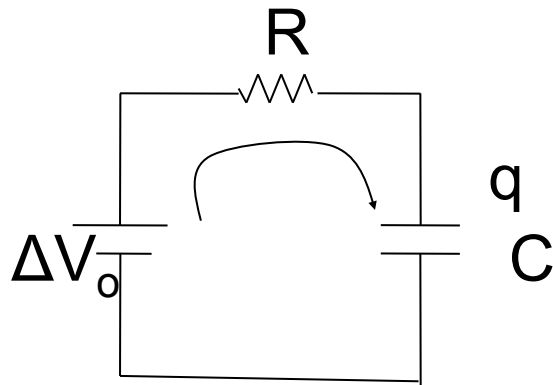
How does the time to charge the capacitor depend on R and C



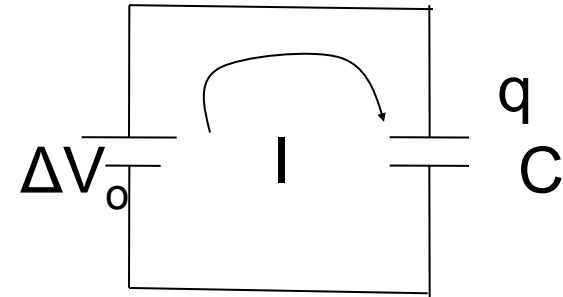
Charging time  $\tau_0$



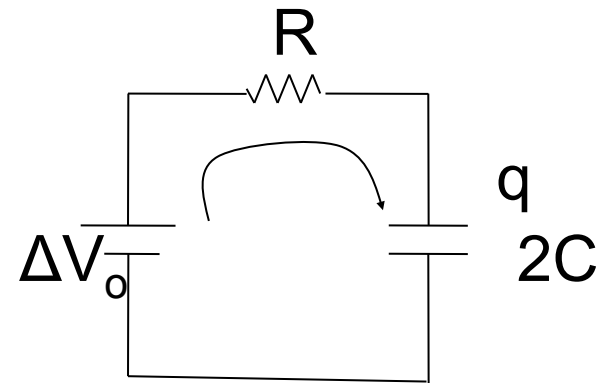
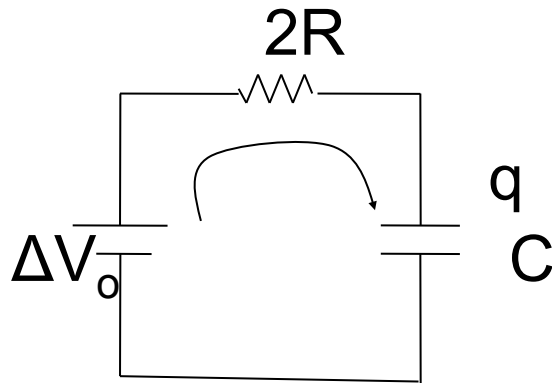
How does the time to charge the capacitor depend on R and C



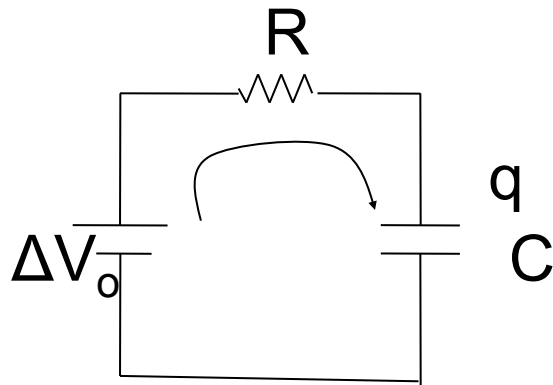
Charging time  $\tau_0$



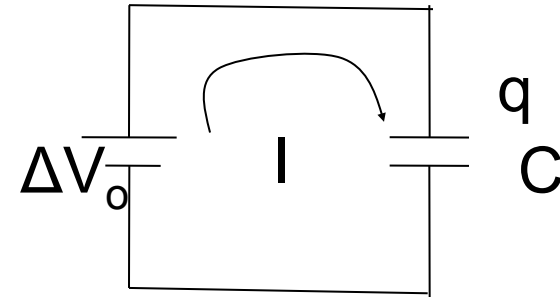
shorter than  $\tau_0$  because the current is larger



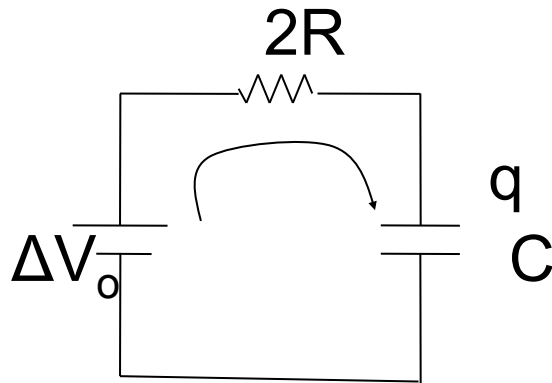
How does the time to charge the capacitor depend on R and C



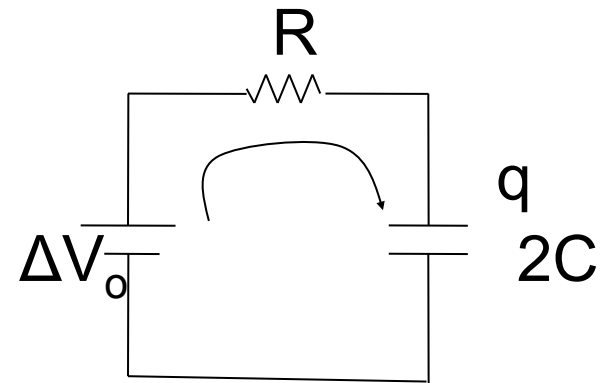
Charging time  $\tau_0$



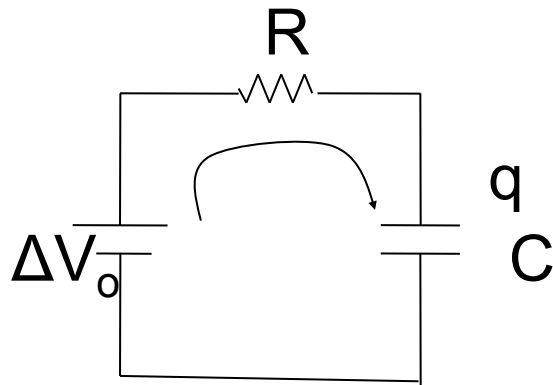
shorter than  $\tau_0$  because the current is larger



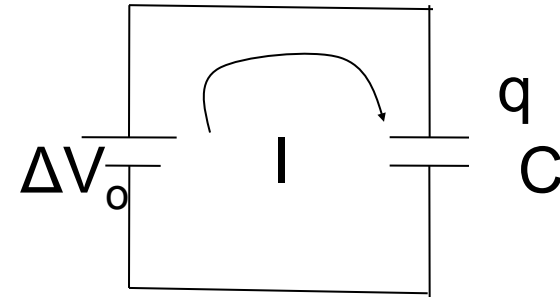
longer than  $\tau_0$   
the current is smaller



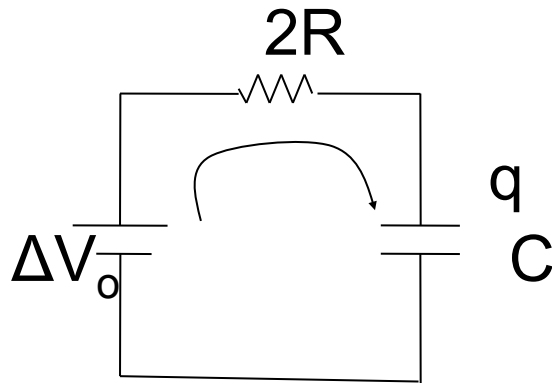
How does the time to charge the capacitor depend on R and C



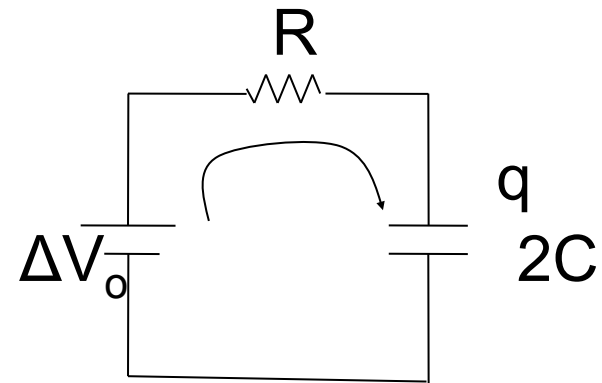
Charging time  $\tau_0$



shorter than  $\tau_0$  because the current is larger

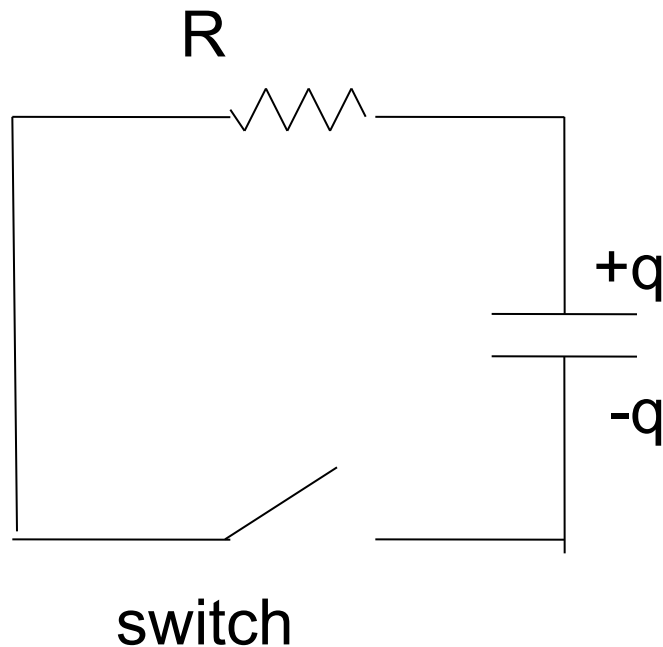


longer than  $\tau_0$   
the current is smaller



longer than  $\tau_0$   
more charge is transferred

# Discharging



Switch off

Capacitor charged

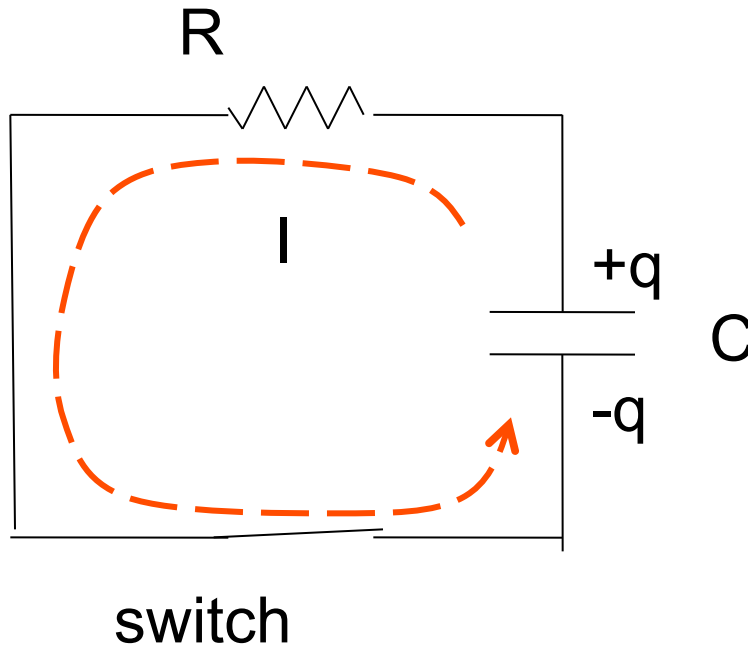
$$C \quad \Delta V_c = \frac{q}{C}$$

When the switch is closed to discharge the capacitor the capacitor has a maximum charge of  $q_0$  and maximum voltage  $V_0$ .

As the capacitor discharges the charge and voltage decrease with time.

The current will also decrease with time.

# Discharge



Switch on  
Current flows

$$\Delta V_R = IR = \Delta V_C = \frac{q}{C}$$

$$I = -\frac{\Delta q}{\Delta t} = \frac{q}{RC}$$

$$q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

The charge decays exponentially with time

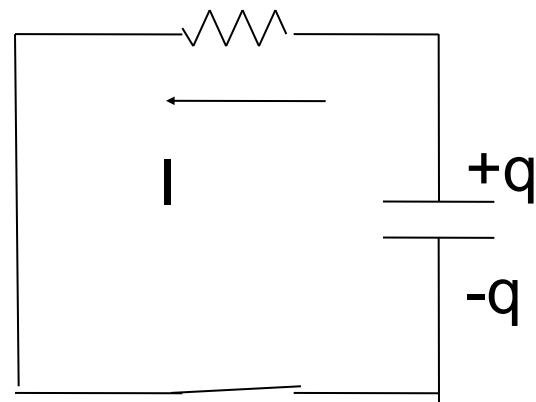


$q_0$

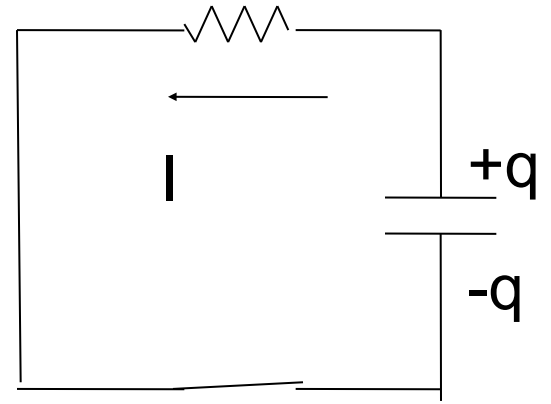
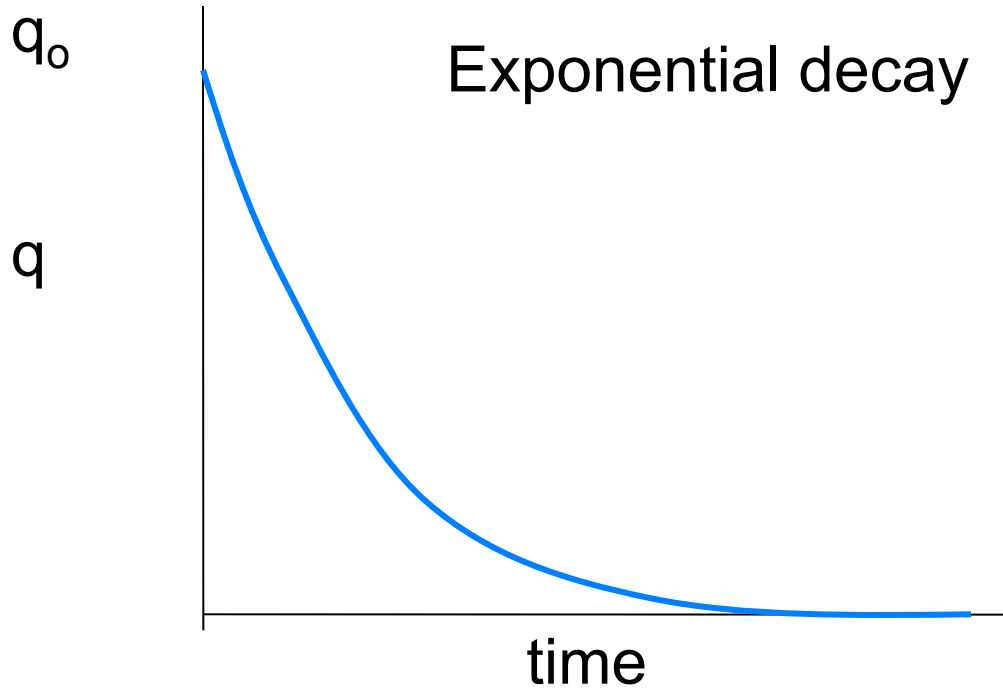
Exponential decay

$q$

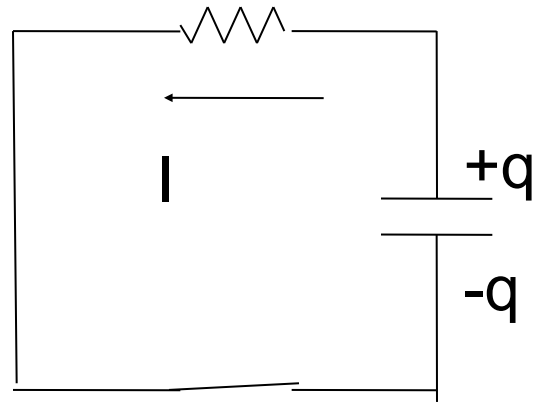
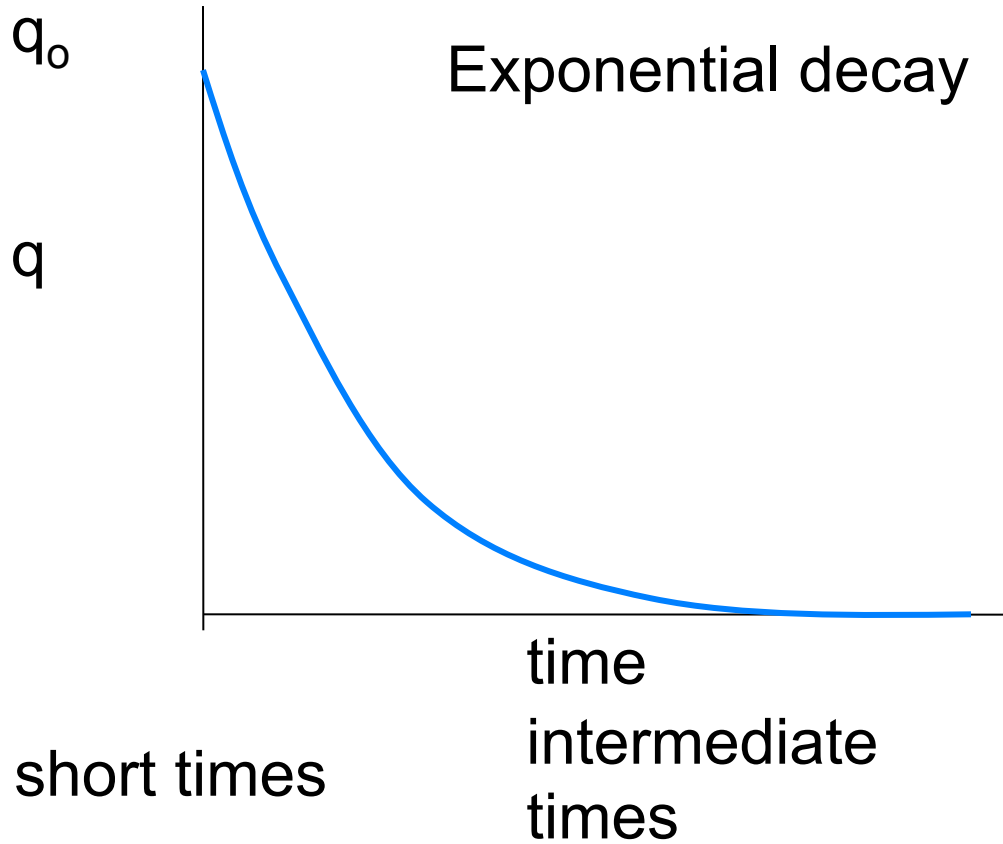
time



$$\Delta V_C - \Delta V_R = 0$$

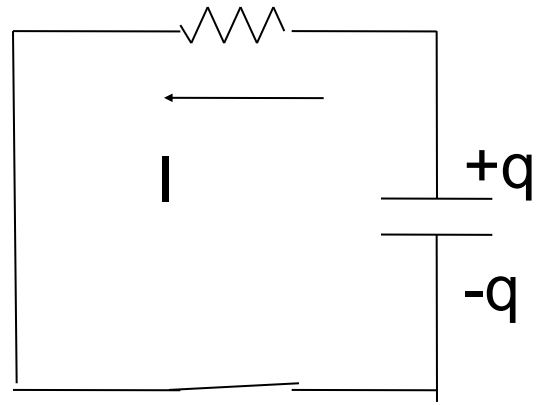
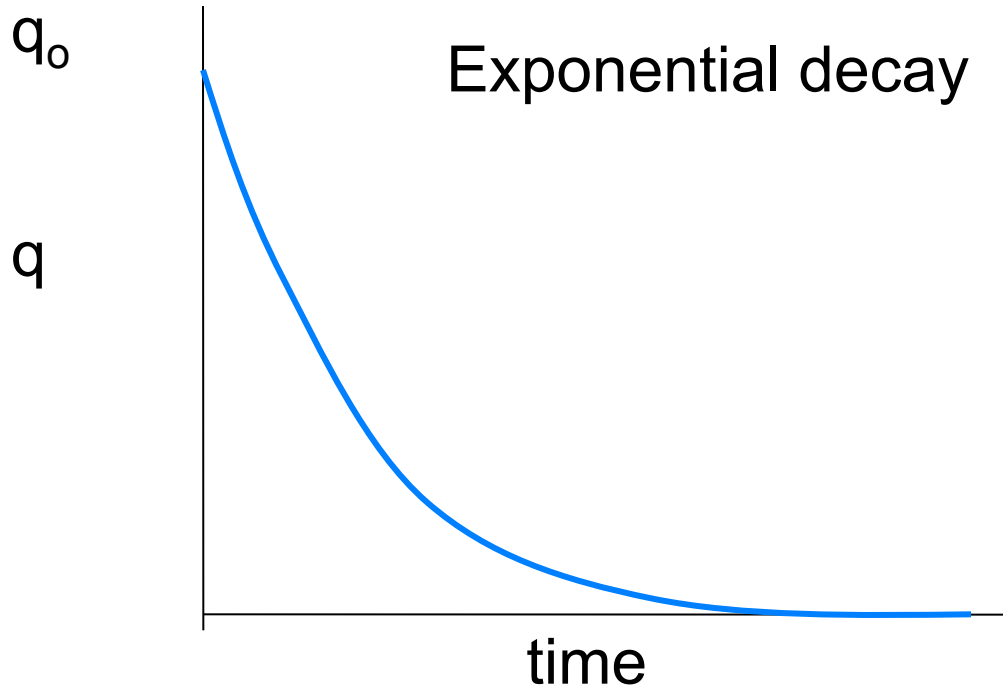


$$\Delta V_C - \Delta V_R = 0$$



$$\Delta V_C - \Delta V_R = 0$$

q =  
 $\Delta V_C =$   
 I =



$$\Delta V_C - \Delta V_R = 0$$

short times

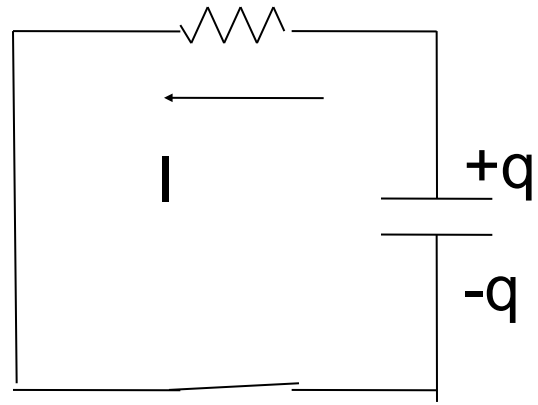
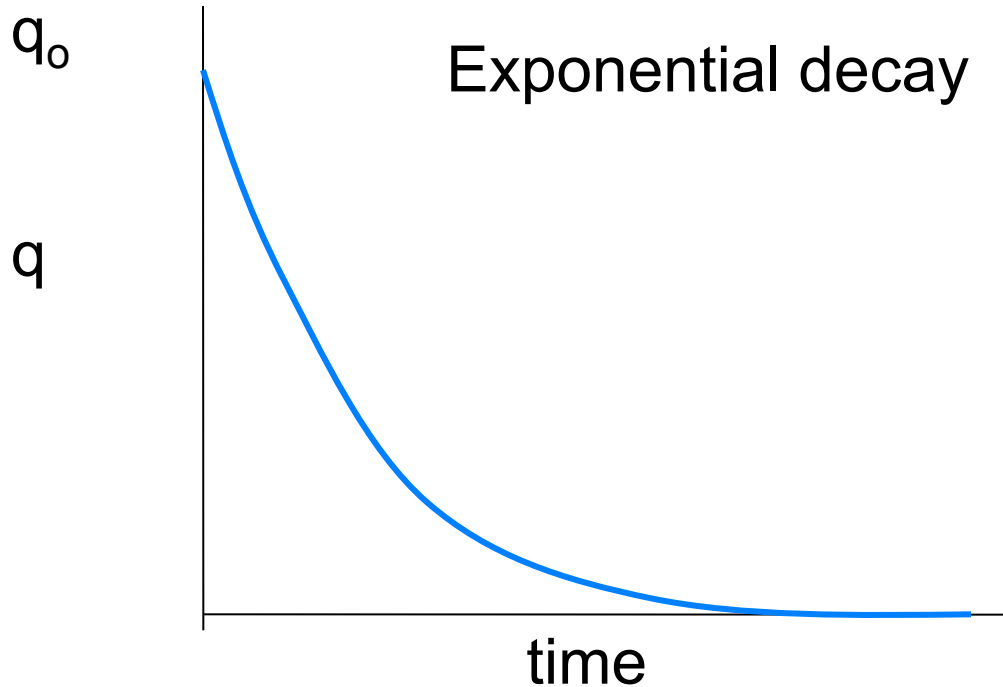
intermediate times

long times

$$q = q_0$$

$$\Delta V_C =$$

$$I =$$



$$\Delta V_C - \Delta V_R = 0$$

short times

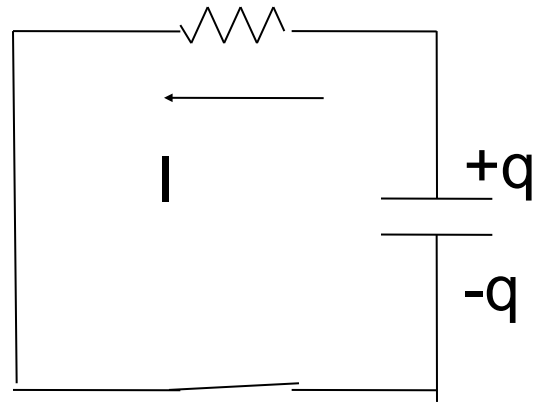
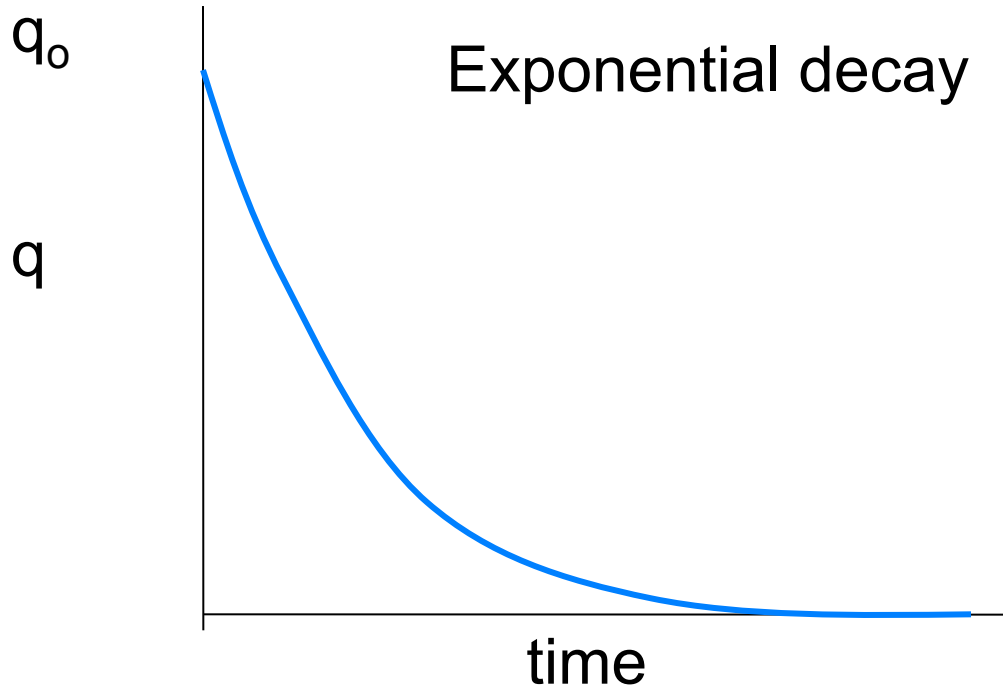
intermediate  
times

long times

$$q = q_0$$

$$\Delta V_C = \Delta V_0$$

$$I =$$



$$\Delta V_C - \Delta V_R = 0$$

short times

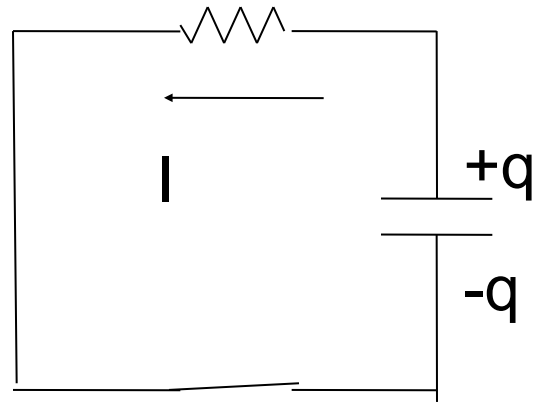
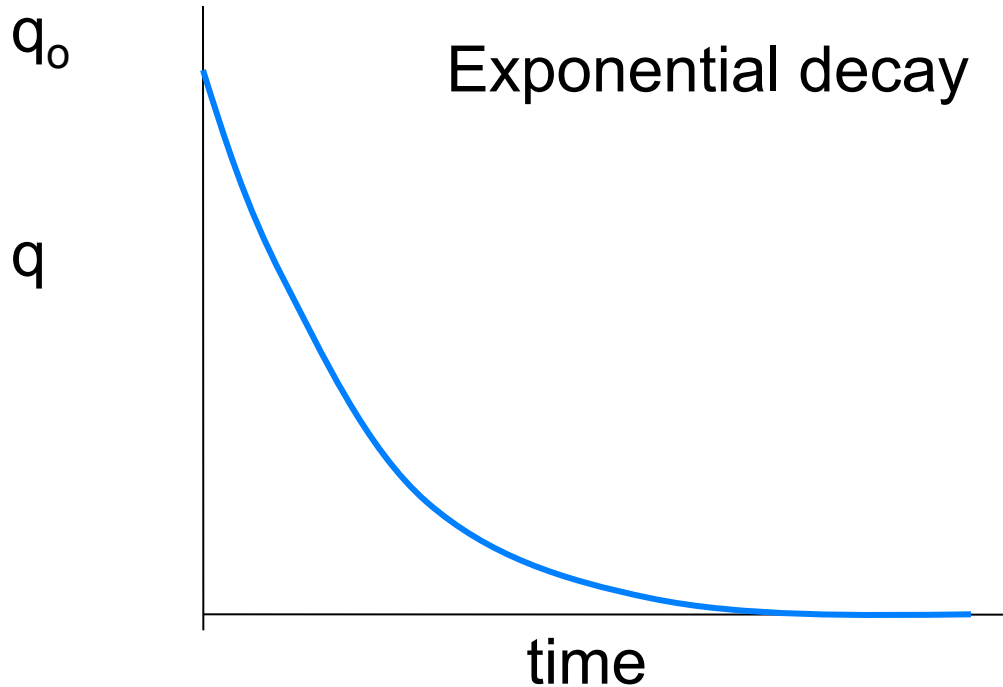
intermediate times

long times

$$q = q_0$$

$$\Delta V_C = \Delta V_0$$

$$I = \frac{\Delta V_0}{R}$$



$$\Delta V_C - \Delta V_R = 0$$

short times

intermediate times

long times

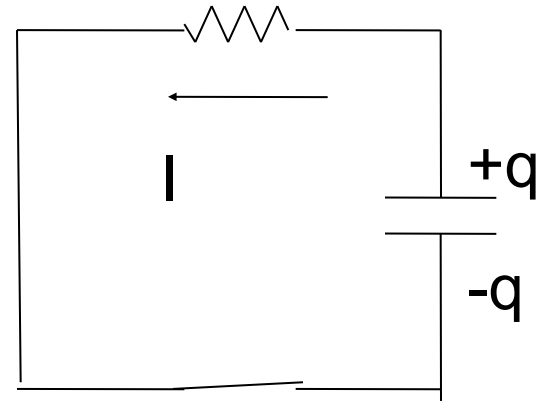
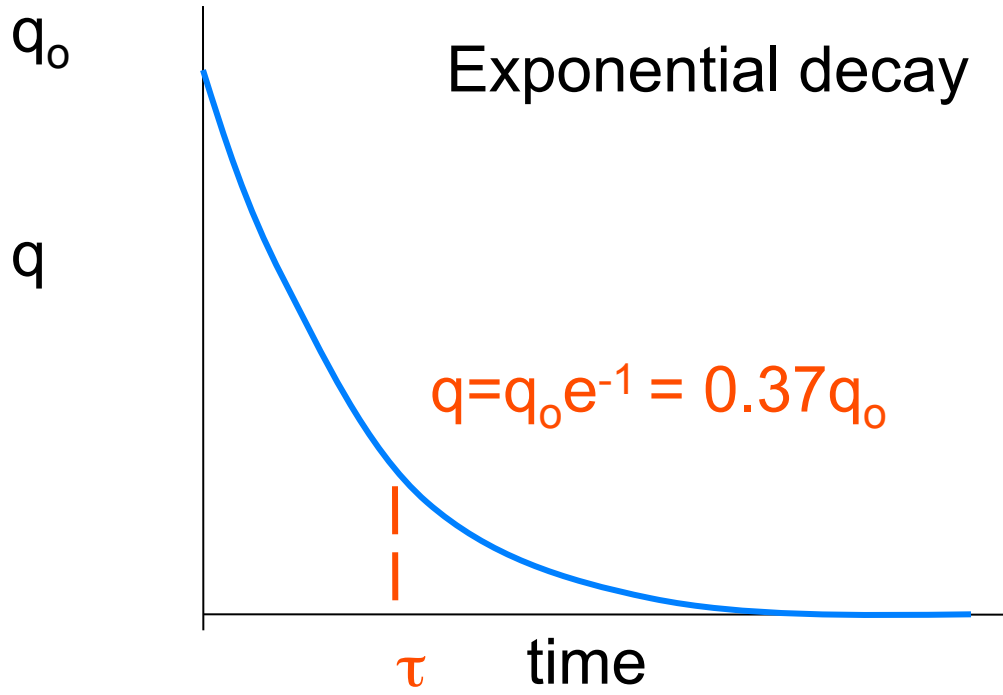
$$q = q_0$$

$$q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$\Delta V_C = \Delta V_0$$

$$I = \frac{\Delta V_0}{R}$$

# Exponential decay



$$\Delta V_C - \Delta V_R = 0$$

short times

intermediate  
times

long times

$$q = q_0 \qquad q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

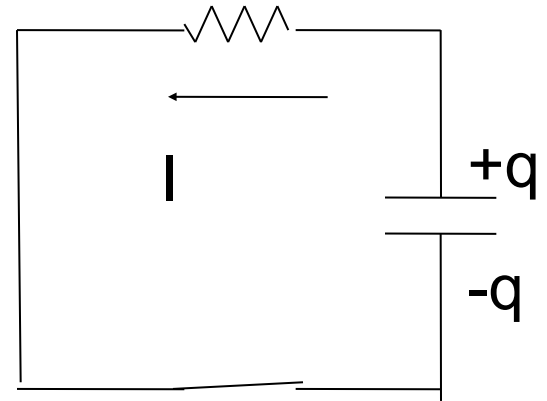
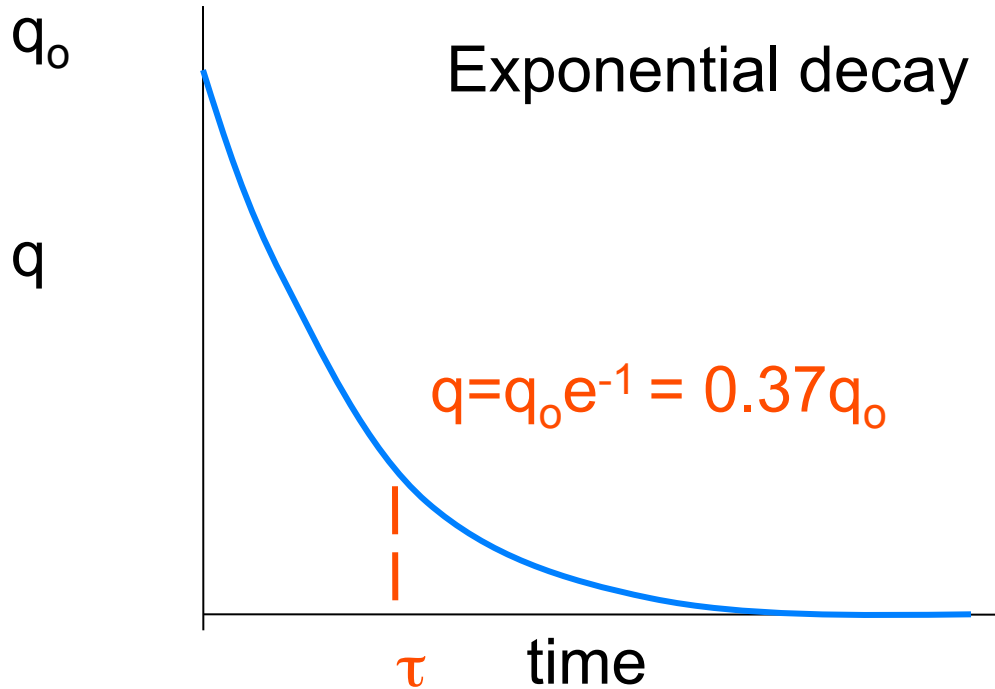
$$\Delta V_C = \Delta V_0$$

$$I = \frac{\Delta V_0}{R}$$

$$\tau = RC$$



# Exponential decay



$$\Delta V_C - \Delta V_R = 0$$

short times

intermediate times

long times

$$q = q_0$$

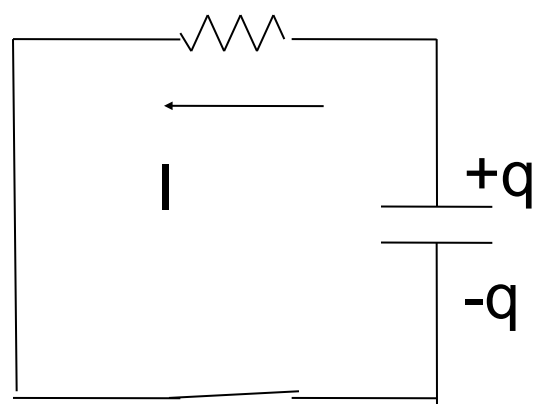
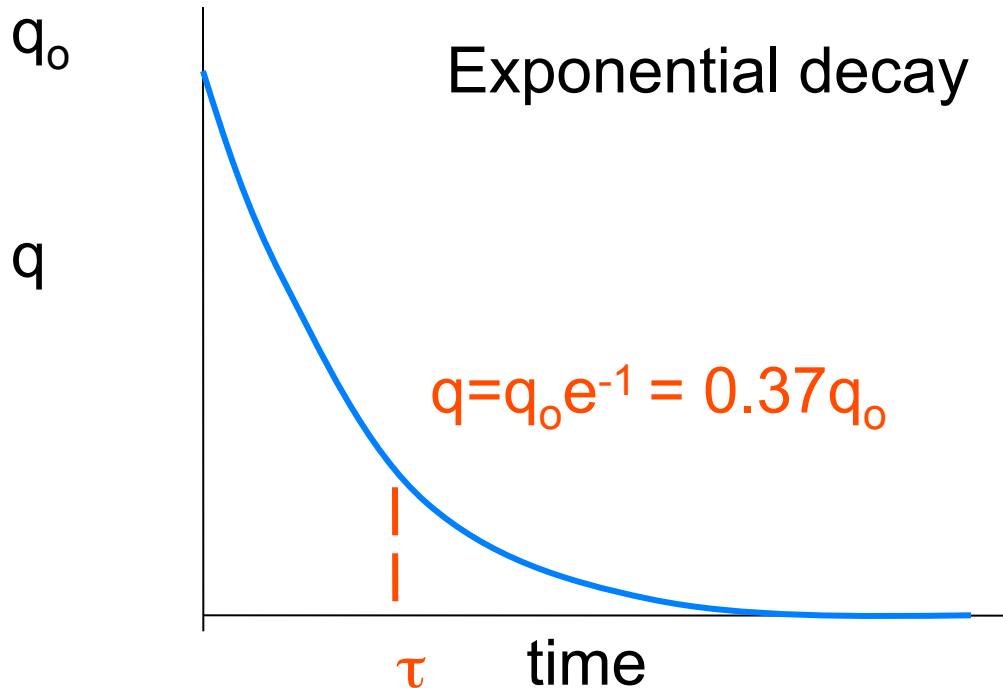
$$q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$\Delta V_C = \Delta V_0$$

$$\Delta V_C = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$I = \frac{\Delta V_0}{R}$$

$$\tau = RC$$



$$\Delta V_C - \Delta V_R = 0$$

short times

intermediate times

long times

$$q = q_0$$

$$q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$\Delta V_C = \Delta V_0$$

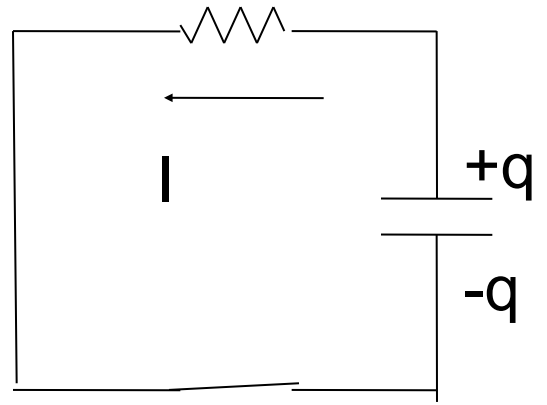
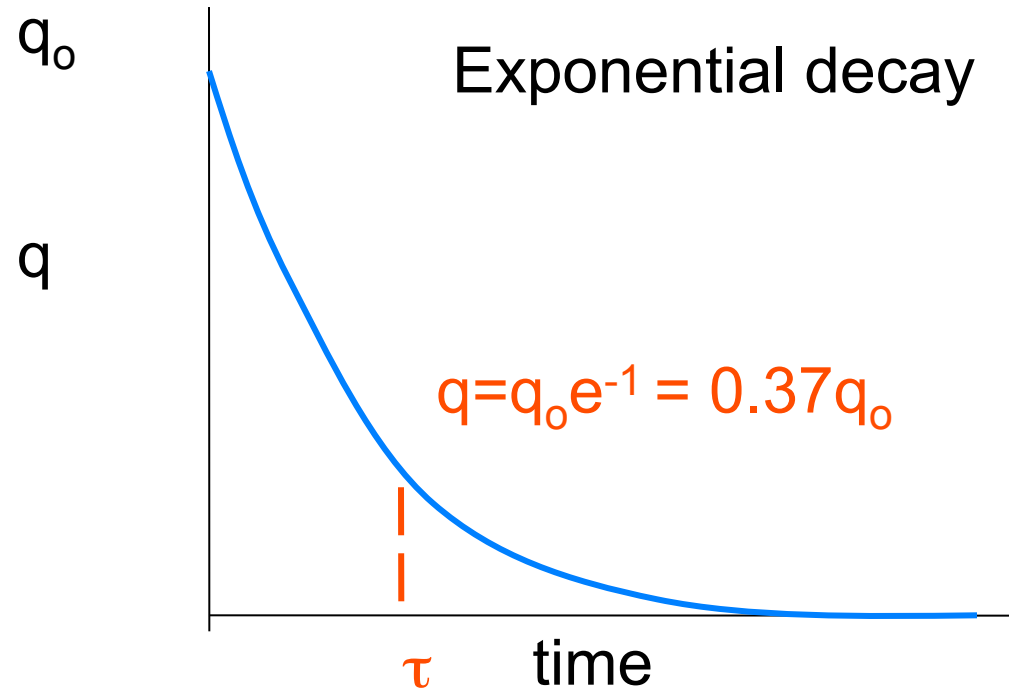
$$\Delta V_C = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$I = \frac{\Delta V_0}{R}$$

$$I = \frac{\Delta V_0}{R} e^{-\left(\frac{t}{\tau}\right)}$$

$$\tau = RC$$

# Exponential decay



$$\Delta V_C - \Delta V_R = 0$$

short times

intermediate times

long times

$$q = q_0$$

$$q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$0$$

$$\Delta V_C = \Delta V_0$$

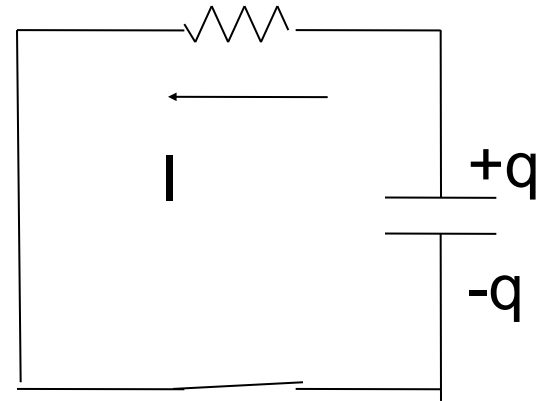
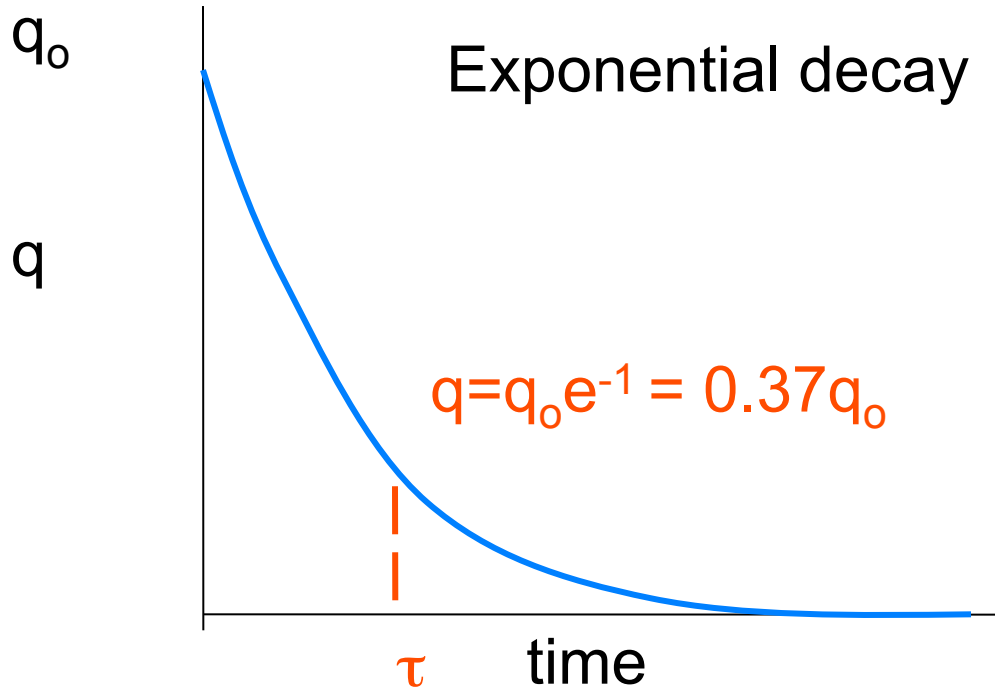
$$\Delta V_C = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$I = \frac{\Delta V_0}{R}$$

$$I = \frac{\Delta V_0}{R} e^{-\left(\frac{t}{\tau}\right)}$$

$$\tau = RC$$

# Exponential decay



$$\Delta V_C - \Delta V_R = 0$$

short times

intermediate times

long times

$$q = q_0$$

$$q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

0

$$\Delta V_C = \Delta V_0$$

$$\Delta V_C = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

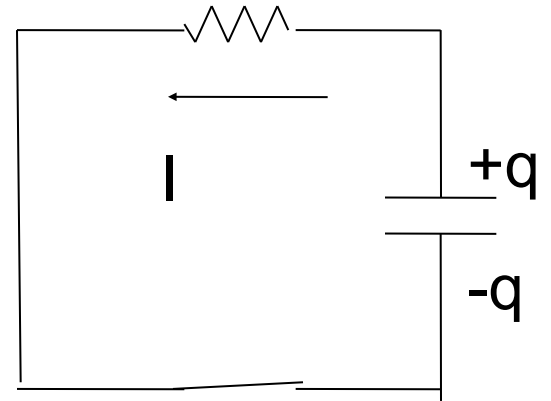
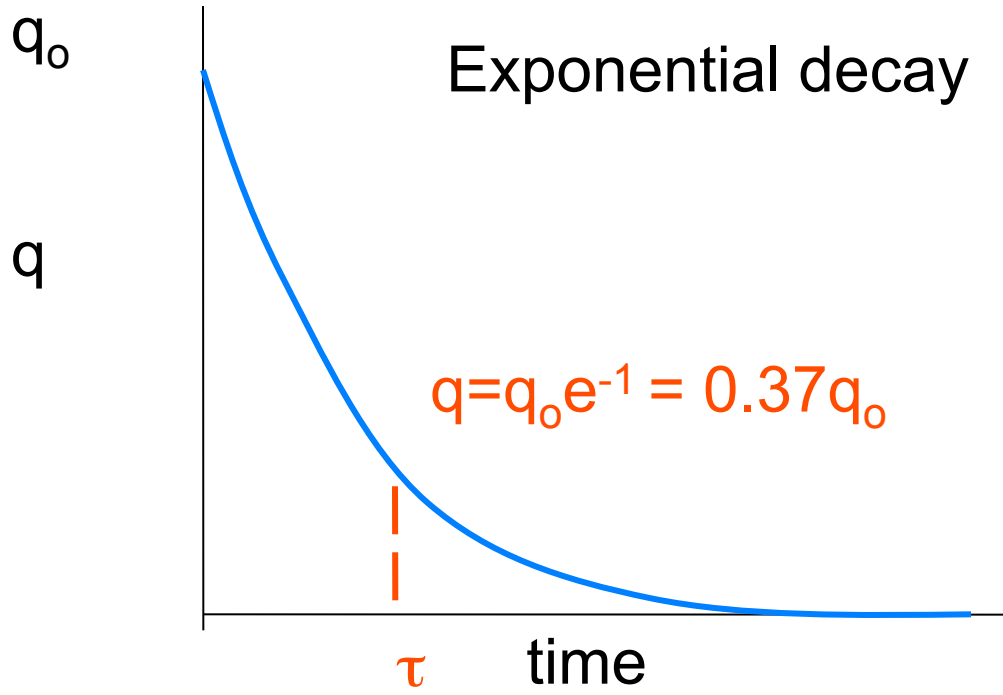
0

$$I = \frac{\Delta V_0}{R}$$

$$I = \frac{\Delta V_0}{R} e^{-\left(\frac{t}{\tau}\right)}$$

$$\tau = RC$$

# Exponential decay



$$\Delta V_C - \Delta V_R = 0$$

short times

intermediate times

long times

$$q = q_0$$

$$q = q_0 e^{-\left(\frac{t}{\tau}\right)}$$

0

$$\Delta V_C = \Delta V_0$$

$$\Delta V_C = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

0

$$I = \frac{\Delta V_0}{R}$$

$$I = \frac{\Delta V_0}{R} e^{-\left(\frac{t}{\tau}\right)}$$

0

$$\tau = RC$$

# Exponential decay

Found in many other systems-  
Chemical reaction, nuclear decay



When the rate of decay of a species is proportional to the amount of the species

$$\frac{\Delta A}{\Delta t} = -\frac{A}{\tau}$$

The result is exponential decay

$$A = A_0 e^{-\left(\frac{t}{\tau}\right)}$$

$\tau$  is a constant

A 12  $\mu$ farad capacitor is discharged through a 2 k $\Omega$  resistor. How long does it take for the voltage to decay to 5% of the initial voltage.

A 12  $\mu$ farad capacitor is discharged through a 2 k $\Omega$  resistor. How long does it take for the voltage to decay to 5% of the initial voltage.

$$\tau = RC = 2 \times 10^3 (12 \times 10^{-6}) = 24 \times 10^{-3} \text{ s} = 24 \text{ ms}$$



A 12  $\mu$ farad capacitor is discharged through a 2 k $\Omega$  resistor. How long does it take for the voltage to decay to 5% of the initial voltage.

$$\tau = RC = 2 \times 10^3 (12 \times 10^{-6}) = 24 \times 10^{-3} \text{ s} = 24 \text{ ms}$$

$$V = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

A 12  $\mu$ farad capacitor is discharged through a 2 k $\Omega$  resistor. How long does it take for the voltage to decay to 5% of the initial voltage.

$$\tau = RC = 2 \times 10^3 (12 \times 10^{-6}) = 24 \times 10^{-3} \text{ s} = 24 \text{ ms}$$

$$V = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$\frac{V}{V_0} = e^{-\left(\frac{t}{\tau}\right)}$$

A 12  $\mu$ farad capacitor is discharged through a 2 k $\Omega$  resistor. How long does it take for the voltage to decay to 5% of the initial voltage.

$$\tau = RC = 2 \times 10^3 (12 \times 10^{-6}) = 24 \times 10^{-3} \text{ s} = 24 \text{ ms}$$

$$V = V_0 e^{-\left(\frac{t}{\tau}\right)}$$

$$\frac{V}{V_0} = e^{-\left(\frac{t}{\tau}\right)}$$

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{\tau}$$

A 12  $\mu$ farad capacitor is discharged through a 2 k $\Omega$  resistor. How long does it take for the voltage to decay to 5% of the initial voltage.

$$\tau = RC = 2 \times 10^3 (12 \times 10^{-6}) = 24 \times 10^{-3} \text{ s} = 24 \text{ ms}$$

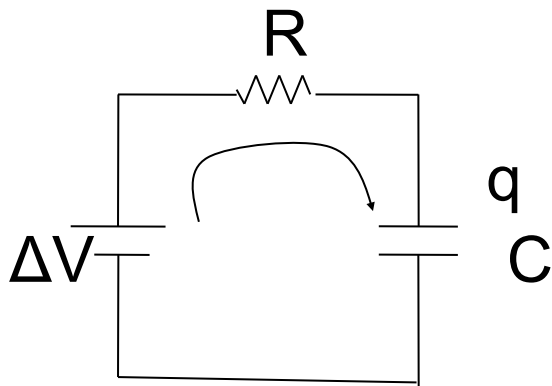
$$V = V_o e^{-\left(\frac{t}{\tau}\right)}$$

$$\frac{V}{V_o} = e^{-\left(\frac{t}{\tau}\right)}$$

$$\ln\left(\frac{V}{V_o}\right) = -\frac{t}{\tau}$$

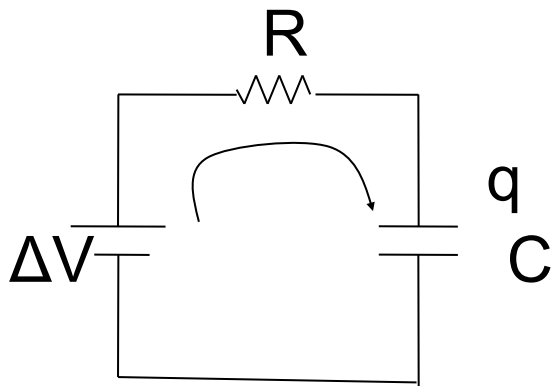
$$t = -\tau \ln\frac{V}{V_o} = -24 \times 10^{-3} (\ln(0.05)) = 7.2 \times 10^{-2} \text{ s}$$

33. Consider a series RC circuit for which  $R=1.0\text{ M}\Omega$ ,  $C=5.0\text{ }\mu\text{F}$  and  $\varepsilon=30\text{ V}$ . The capacitor is initially uncharged when the switch is open. (a) Find the charge on the capacitor 10 s after the switch is closed.



33. Consider a series RC circuit for which  $R=1.0\text{ M}\Omega$ ,  $C=5.0\text{ }\mu\text{F}$  and  $\varepsilon=30\text{ V}$ . The capacitor is initially uncharged when the switch is open. (a) Find the charge on the capacitor 10 s after the switch is closed.

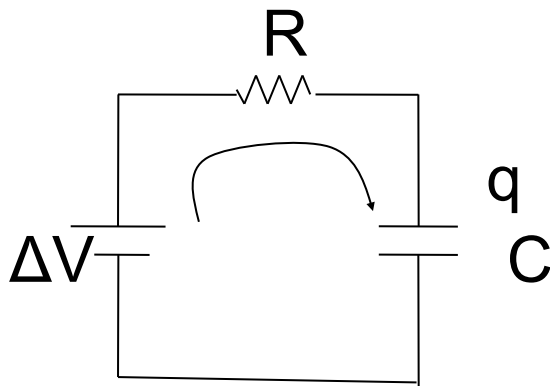
$$\tau = RC = 1 \times 10^6 (5 \times 10^{-6}) = 5.0\text{ s}$$



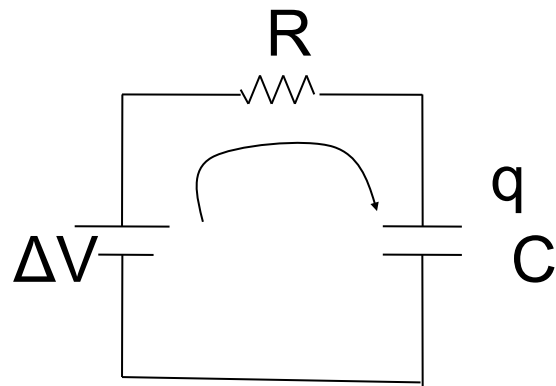
33. Consider a series RC circuit for which  $R=1.0\text{ M}\Omega$ ,  $C=5.0\text{ }\mu\text{F}$  and  $\varepsilon=30\text{ V}$ . The capacitor is initially uncharged when the switch is open. (a) Find the charge on the capacitor 10 s after the switch is closed.

$$\tau = RC = 1 \times 10^6 (5 \times 10^{-6}) = 5.0\text{ s}$$

$$q = q_o (1 - e^{-\frac{t}{RC}}) = q_o (1 - e^{-\frac{t}{\tau}})$$



33. Consider a series RC circuit for which  $R=1.0\text{ M}\Omega$ ,  $C=5.0\text{ }\mu\text{F}$  and  $\varepsilon=30\text{ V}$ . The capacitor is initially uncharged when the switch is open. (a) Find the charge on the capacitor 10 s after the switch is closed.



$$\tau = RC = 1 \times 10^6 (5 \times 10^{-6}) = 5.0 \text{ s}$$

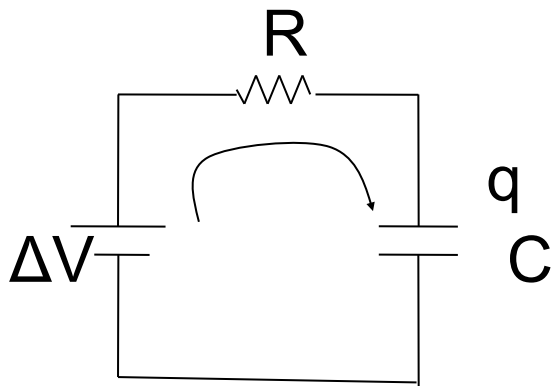
$$q = q_0 (1 - e^{-\frac{t}{RC}}) = q_0 (1 - e^{-\frac{t}{\tau}})$$

$$C = \frac{q}{\Delta V}$$

$$q_0 = \Delta V C = 30 (5 \times 10^{-6}) = 1.5 \times 10^{-4} \text{ C}$$



33. Consider a series RC circuit for which  $R=1.0\text{ M}\Omega$ ,  $C=5.0\text{ }\mu\text{F}$  and  $\varepsilon=30\text{ V}$ . The capacitor is initially uncharged when the switch is open. (a) Find the charge on the capacitor 10 s after the switch is closed.



$$\tau = RC = 1 \times 10^6 (5 \times 10^{-6}) = 5.0 \text{ s}$$

$$q = q_0 (1 - e^{-\frac{t}{RC}}) = q_0 (1 - e^{-\frac{t}{\tau}})$$

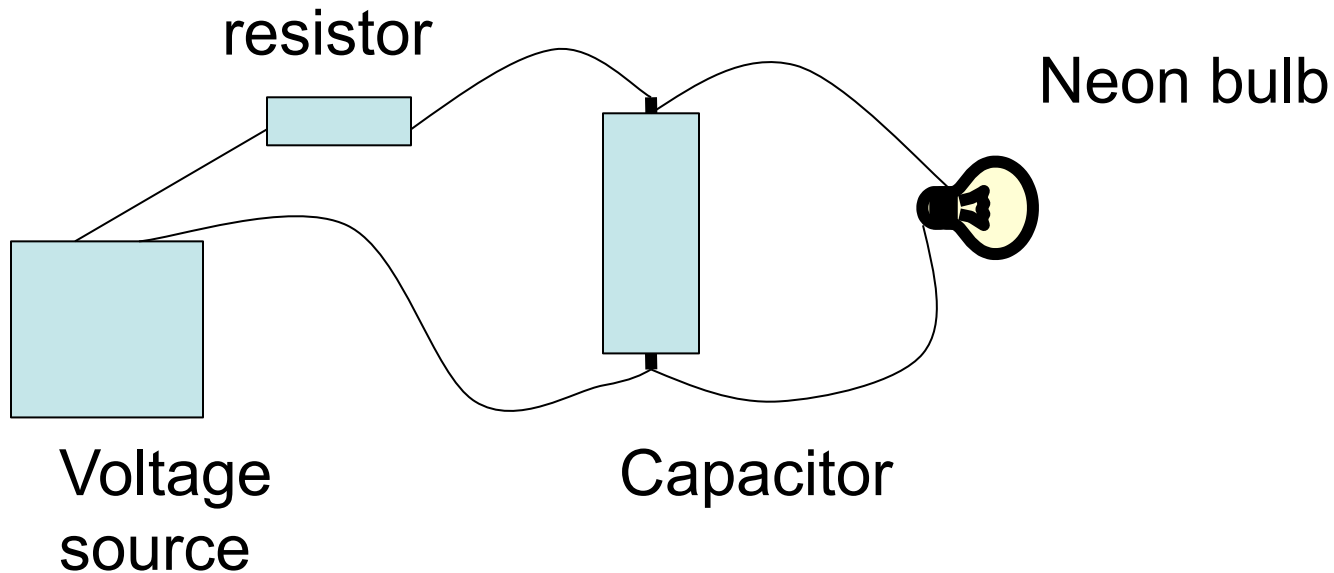
$$C = \frac{q}{\Delta V}$$

$$q_0 = \Delta V C = 30 (5 \times 10^{-6}) = 1.5 \times 10^{-4} \text{ C}$$

$$q = q_0 (1 - e^{-\frac{t}{RC}}) = 1.5 \times 10^{-4} (1 - e^{-\frac{10}{5}})$$

$$q = 1.3 \times 10^{-4} \text{ C}$$

You plan to make a flasher circuit that charges a capacitor through a resistor up to a voltage at which a neon bulb discharges (about 100V) about once every 5 sec. If you have a 10 microfarad capacitor what resistor do you need?

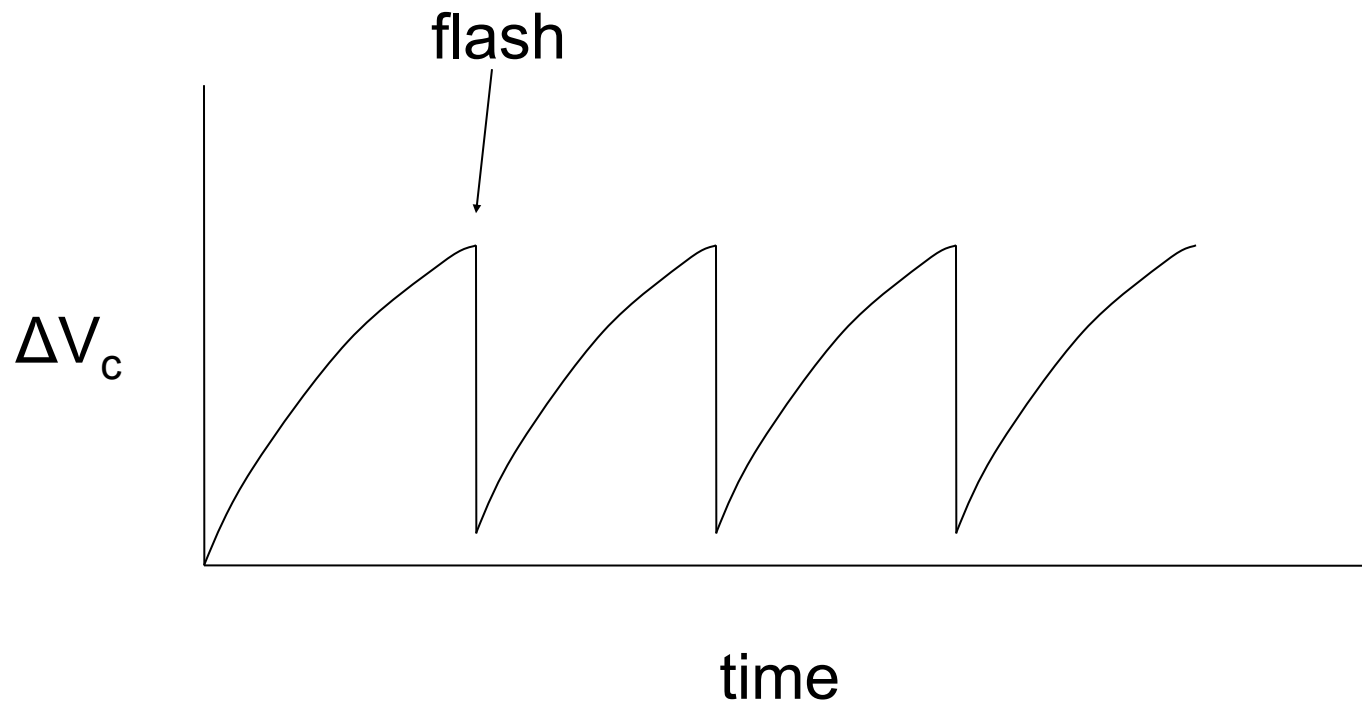


$$\tau = RC$$

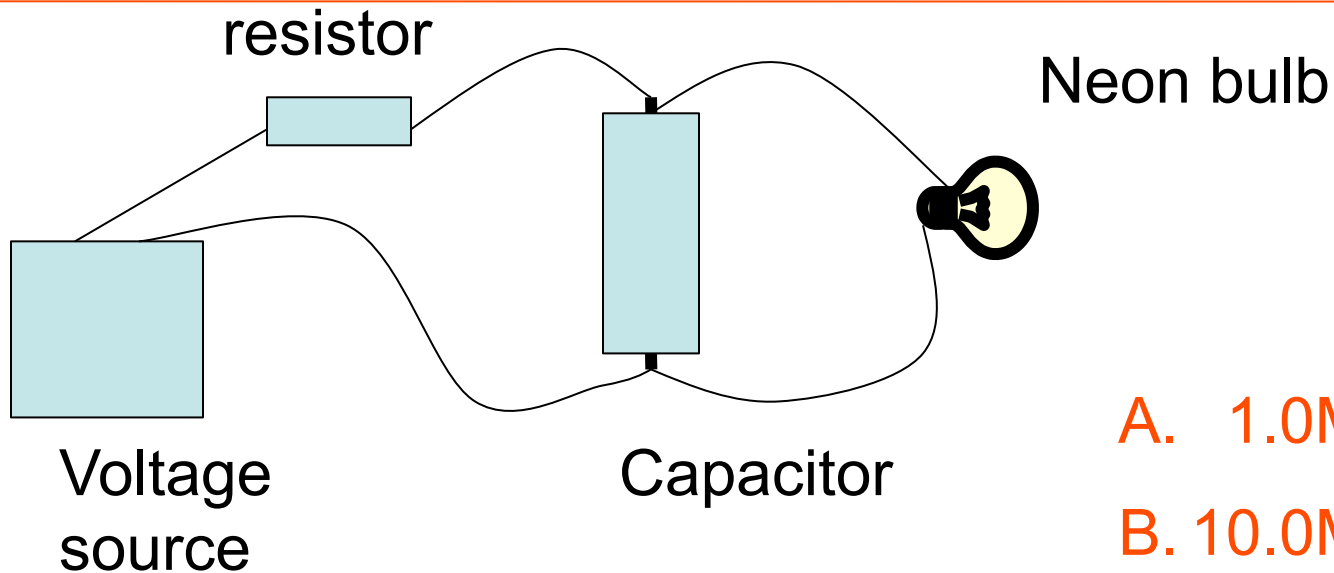
$$R = \frac{\tau}{C} = \frac{5}{10 \times 10^{-6}} = 0.5 \times 10^6 \Omega$$

About  
0.5M $\Omega$

# Charging



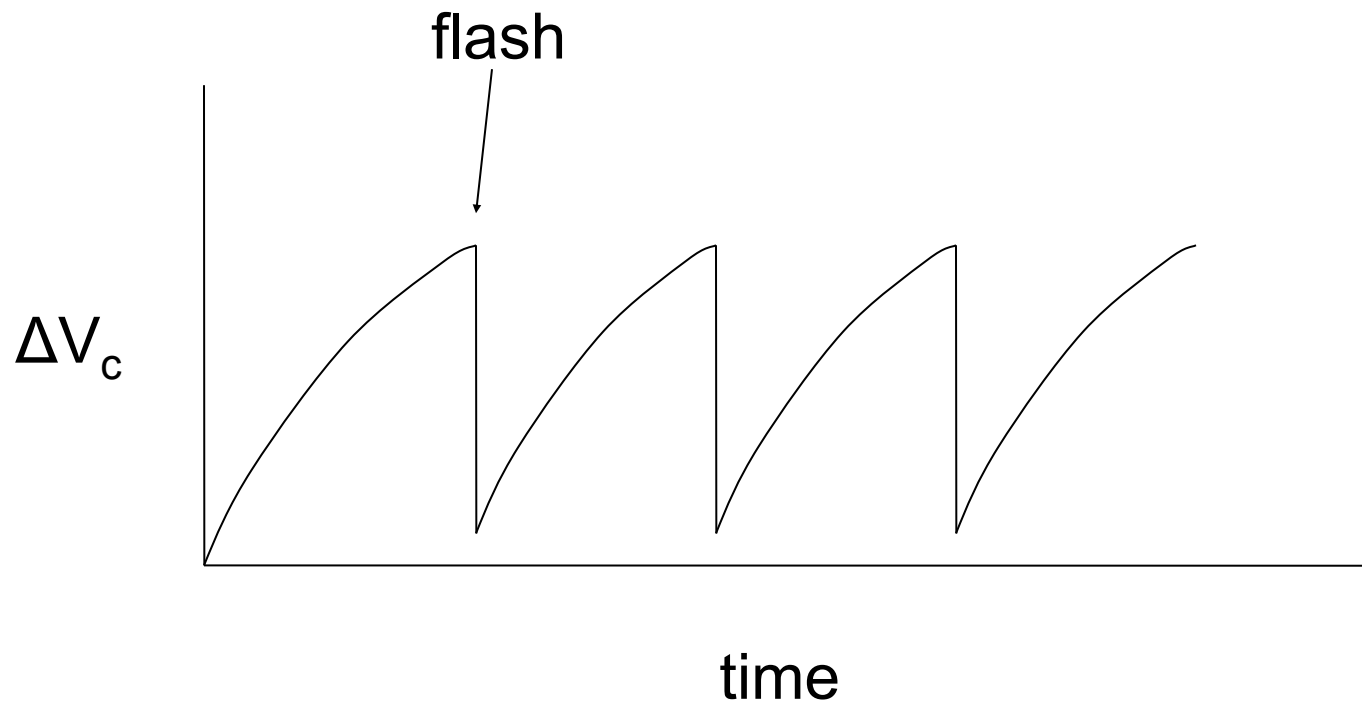
You plan to make a flasher circuit that charges a capacitor through a resistor up to a voltage at which a neon bulb discharges (about 100V) about once every 5 sec. If you have a 10 microfarad capacitor what resistor do you need?



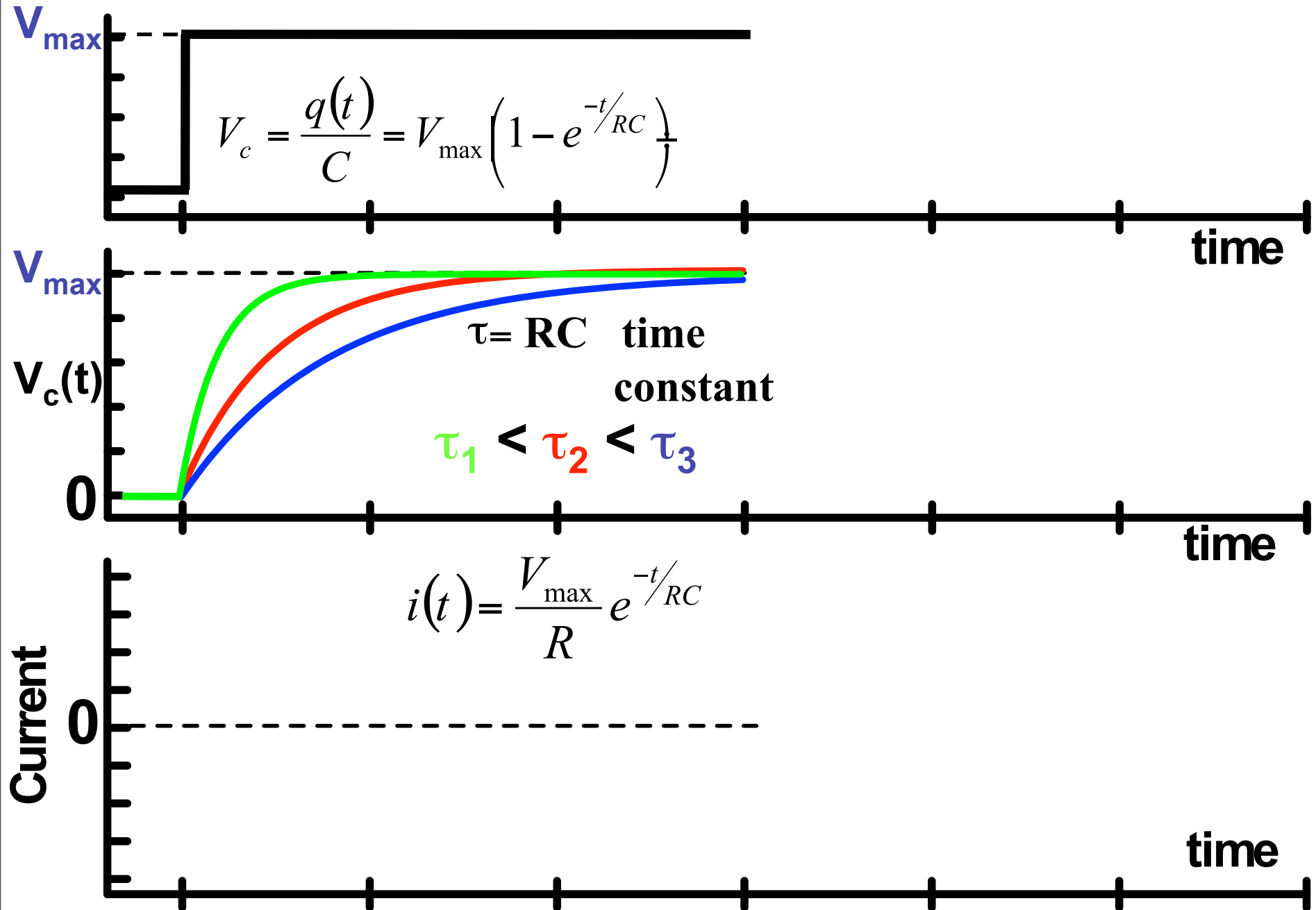
- A.  $1.0\text{M}\Omega$
- B.  $10.0\text{M}\Omega$
- C.  $5.0\text{M}\Omega$
- D.  $0.5\text{M}\Omega$



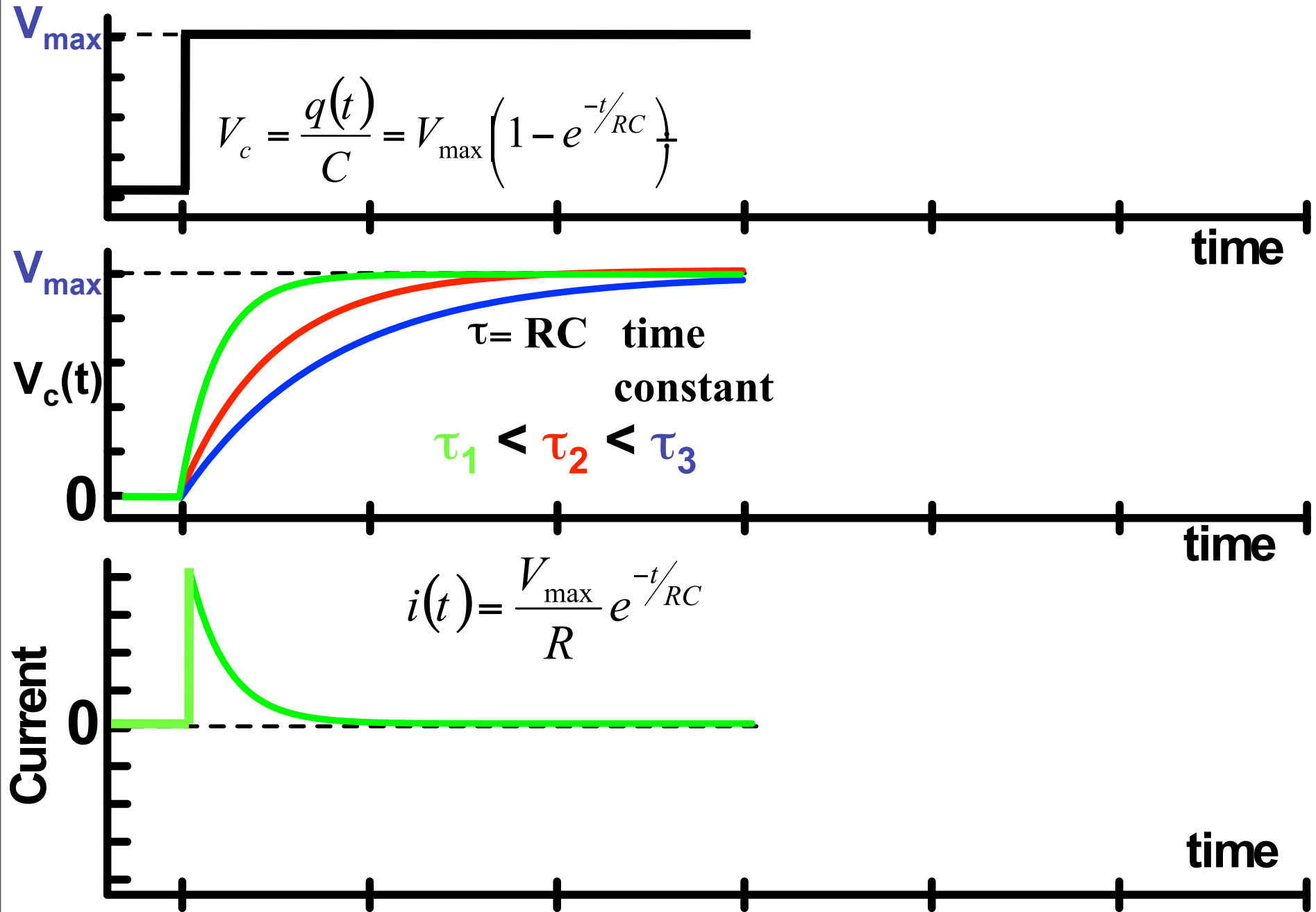
# Charging



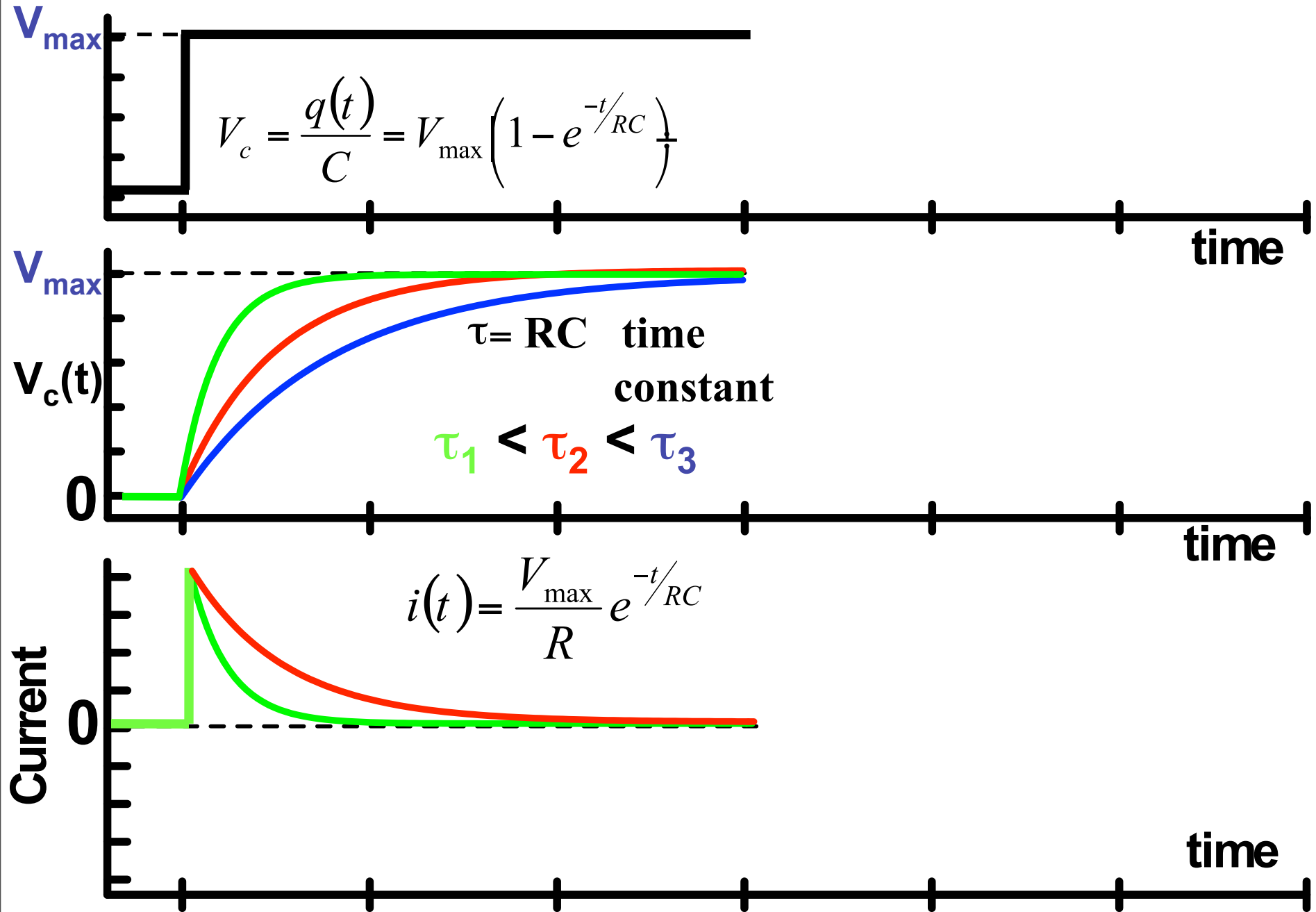
# RC: charging



# RC: charging

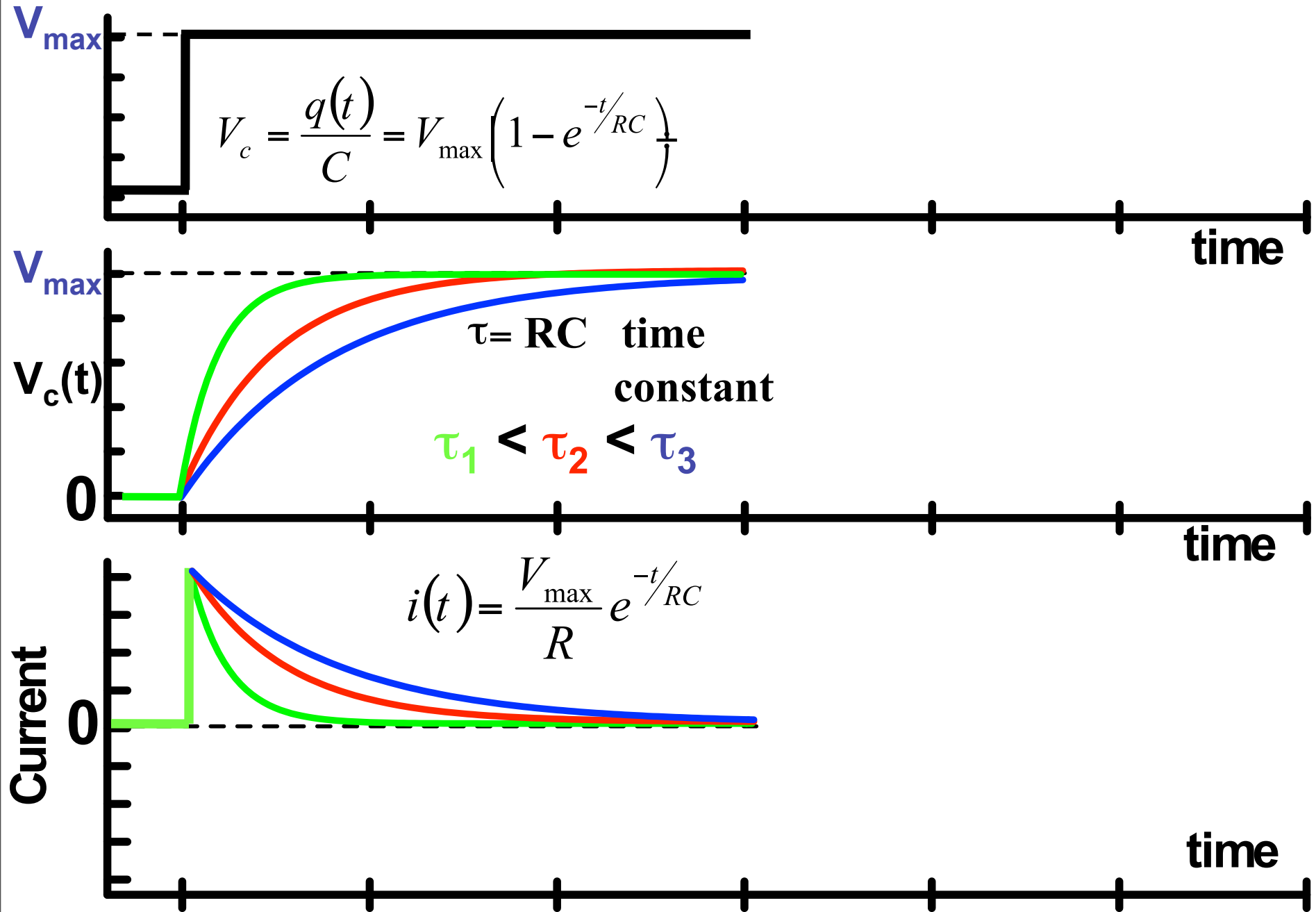


# RC: charging

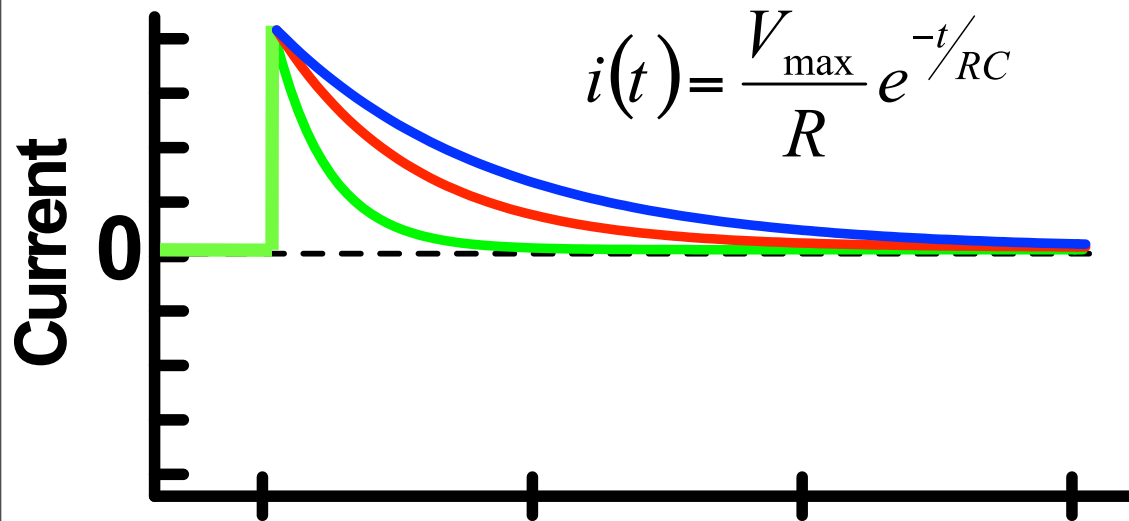
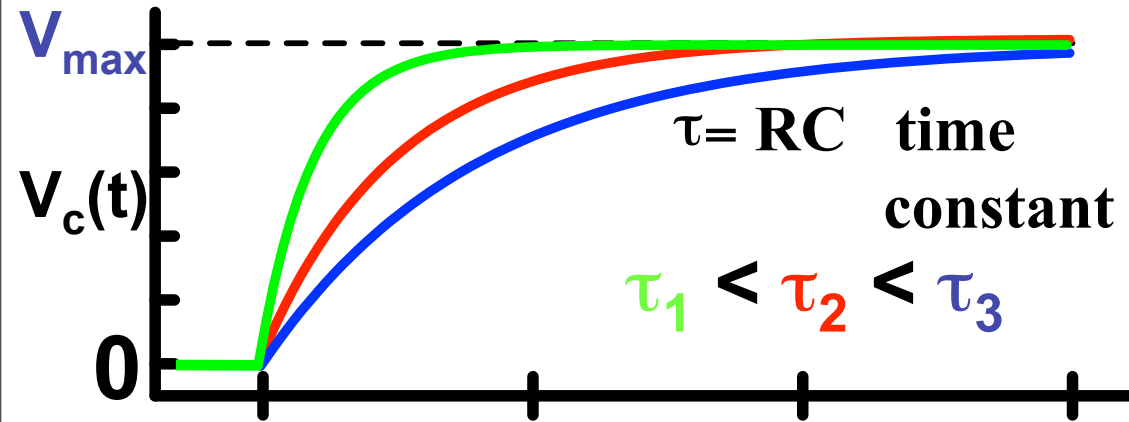
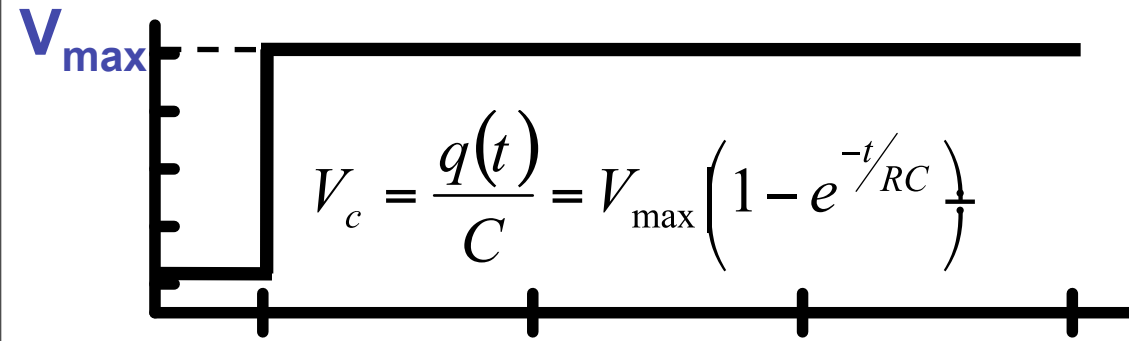




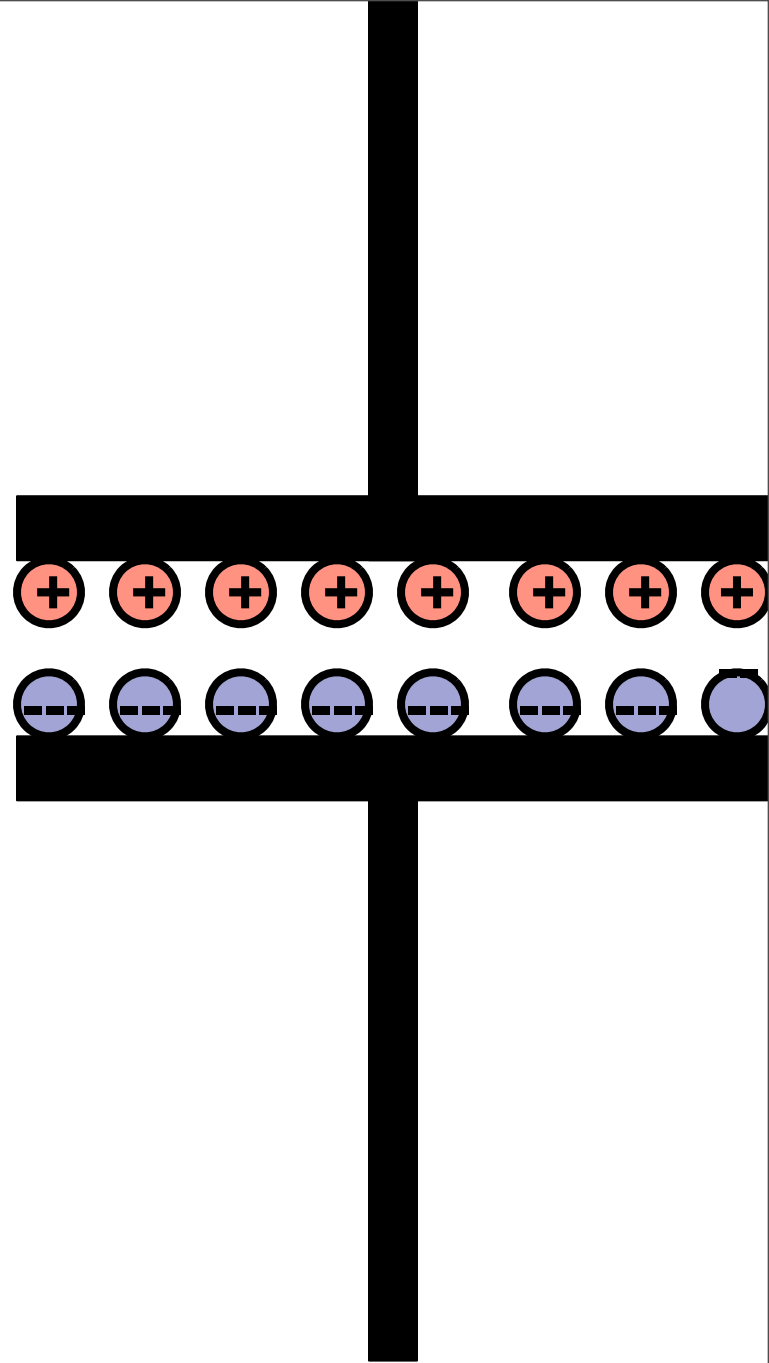
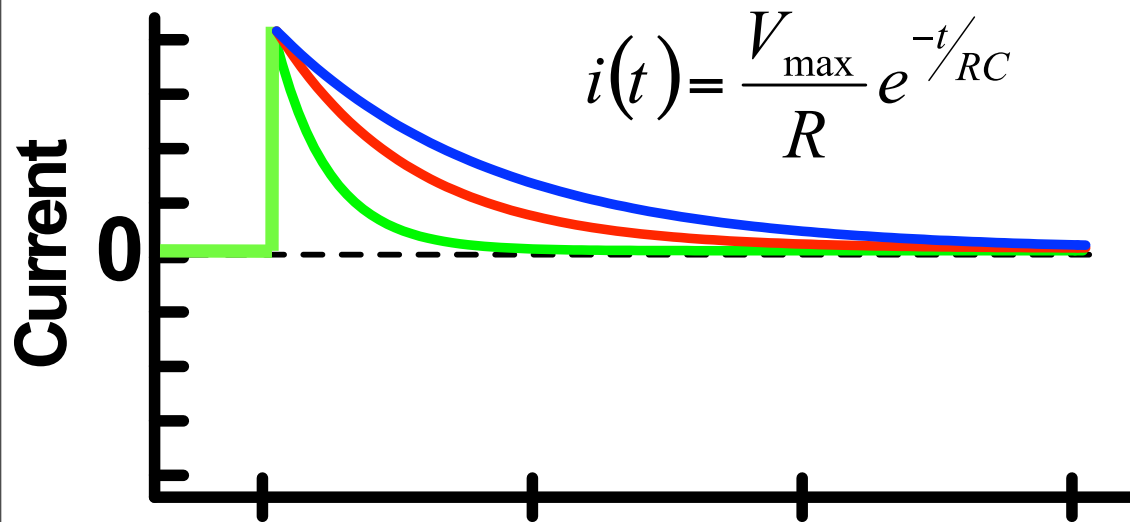
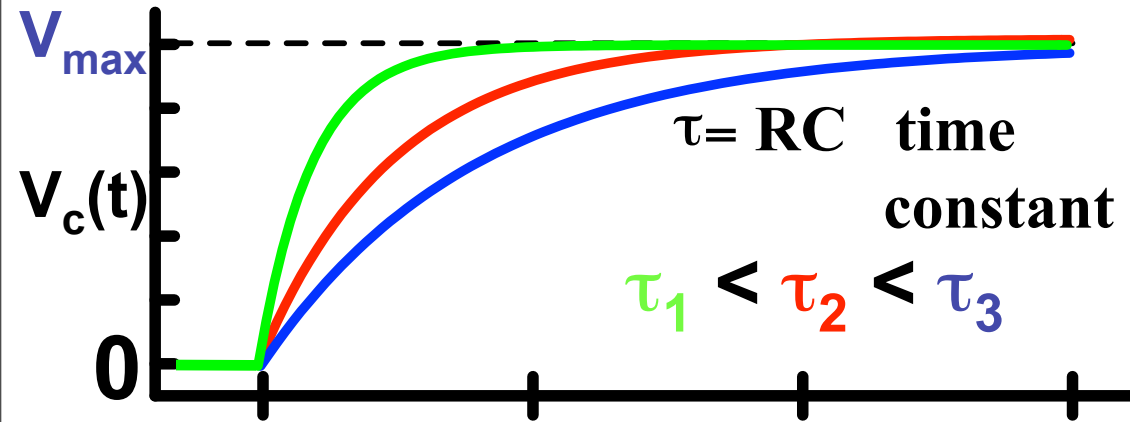
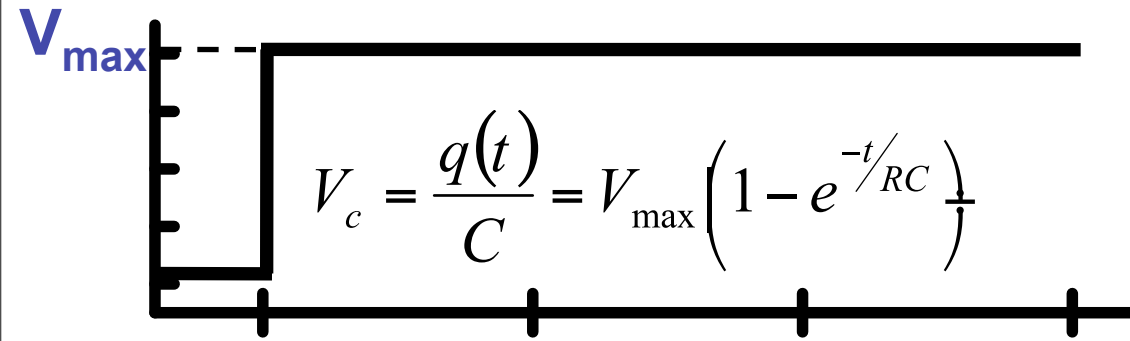
# RC: charging



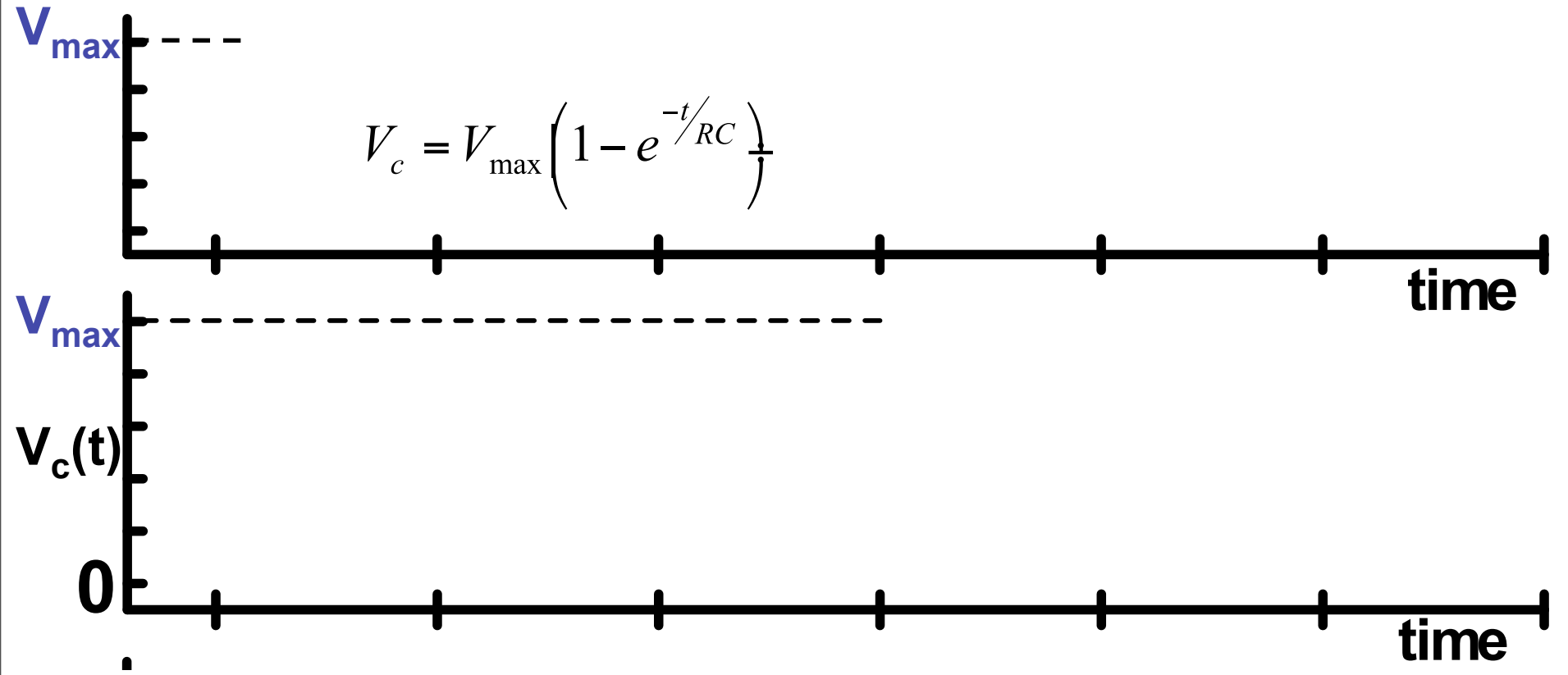
# RC: charging



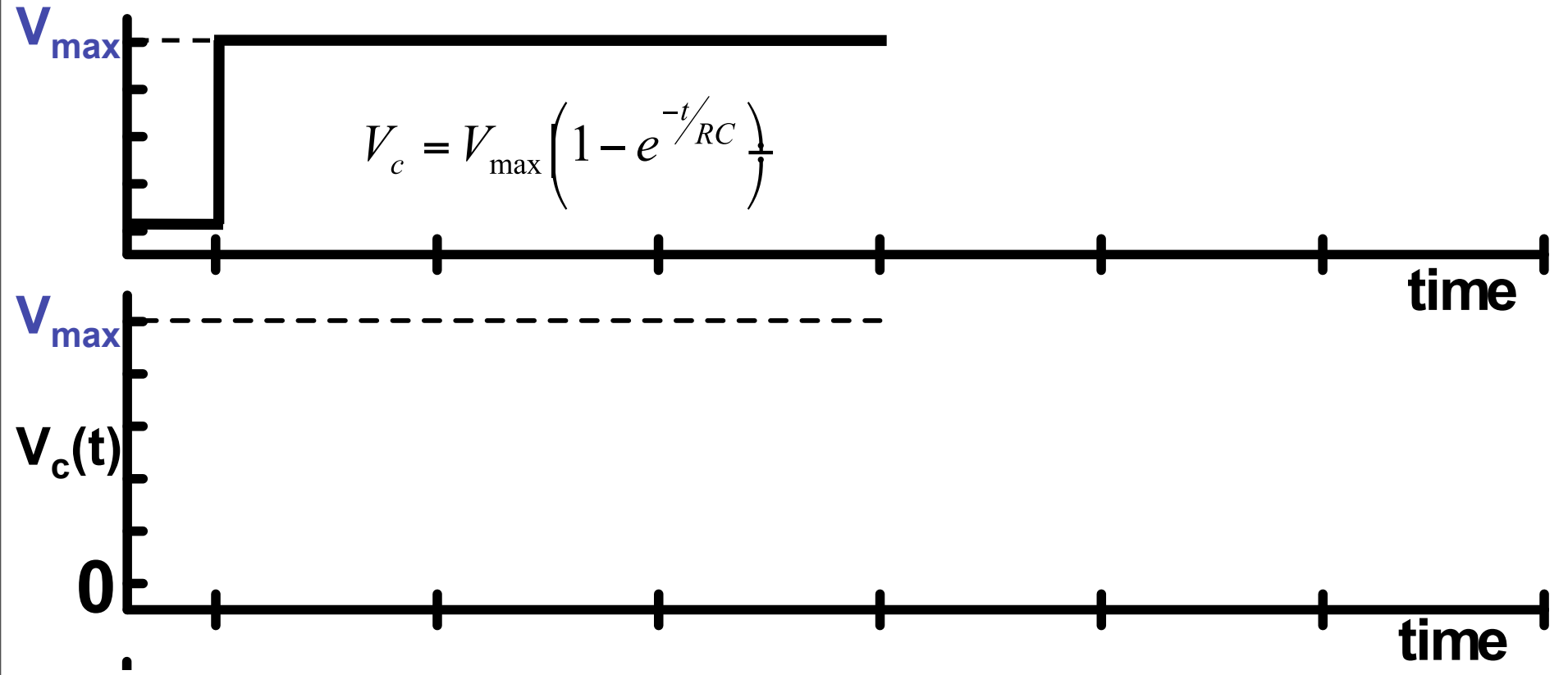
# RC: charging



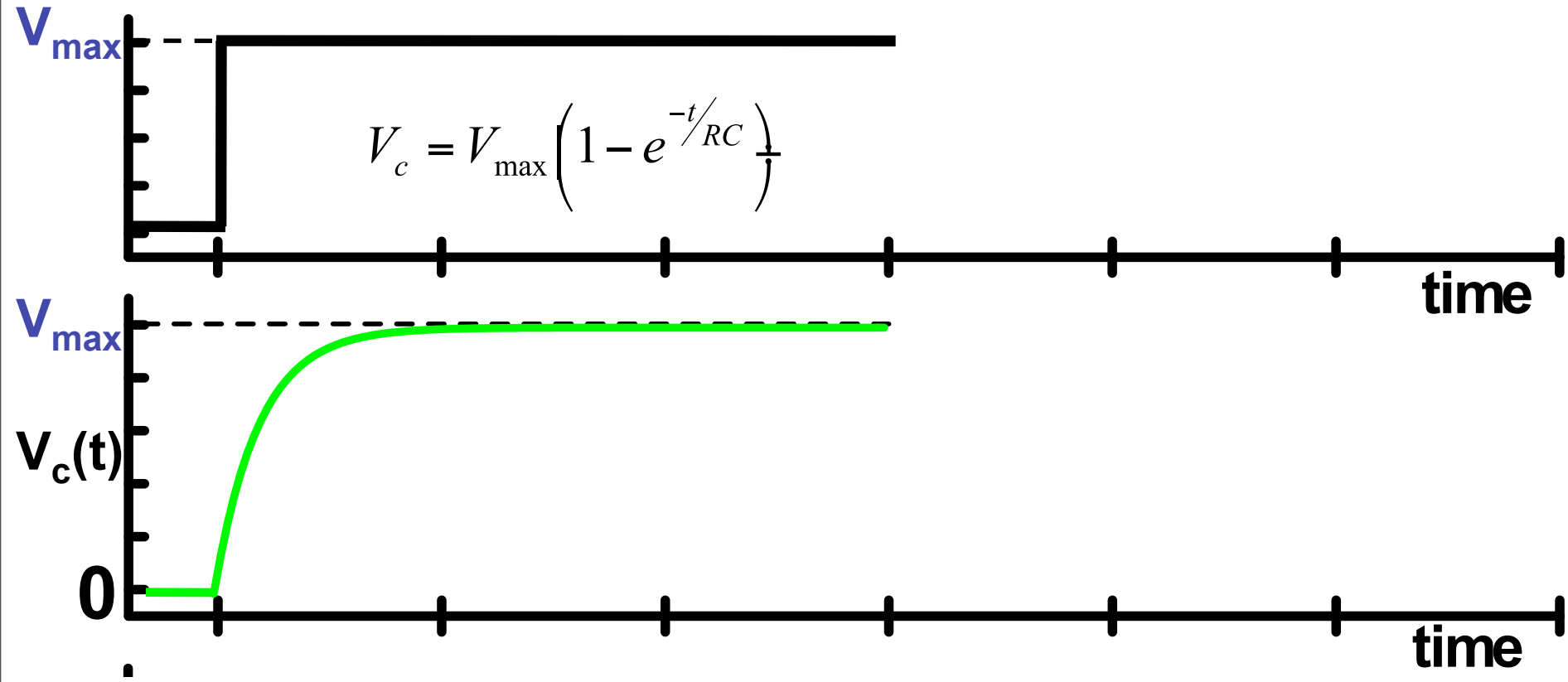
# RC: charging



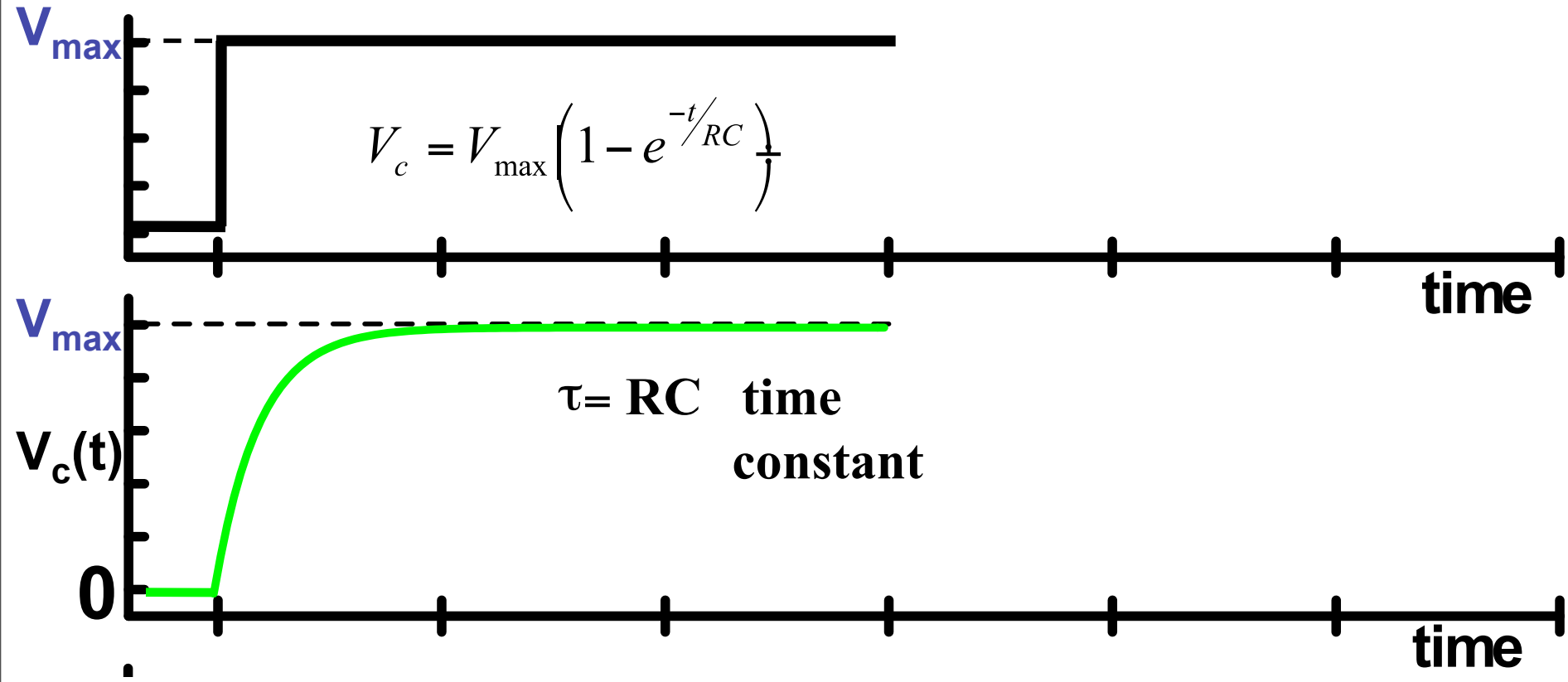
# RC: charging



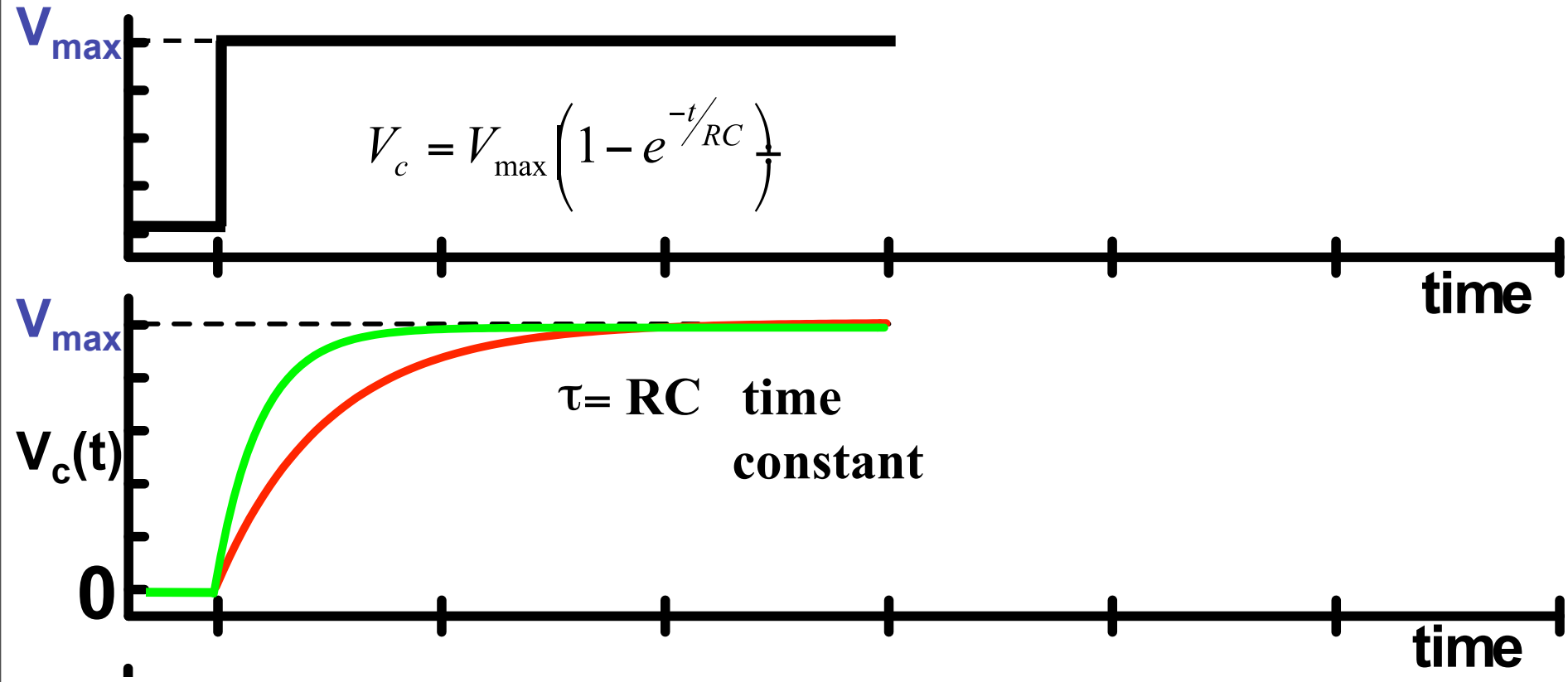
# RC: charging



# RC: charging

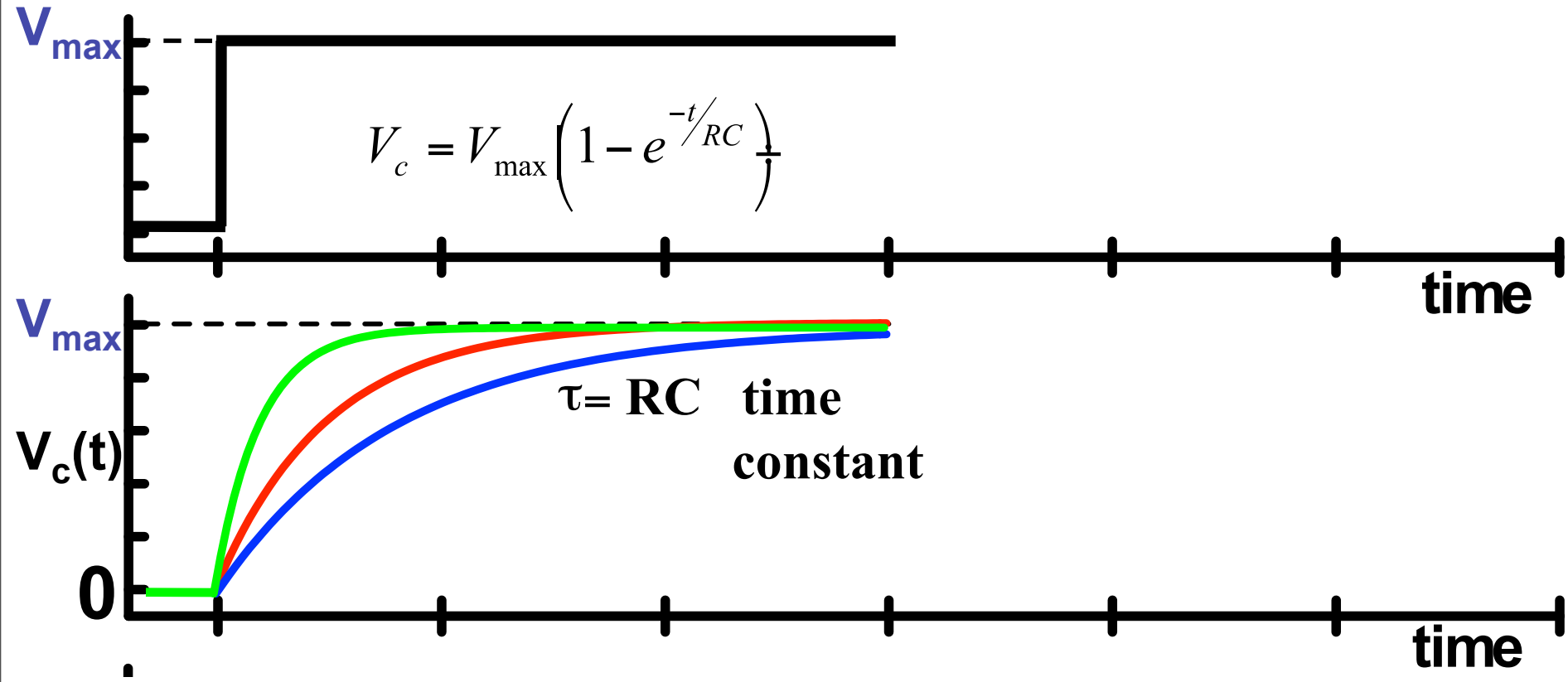


# RC: charging

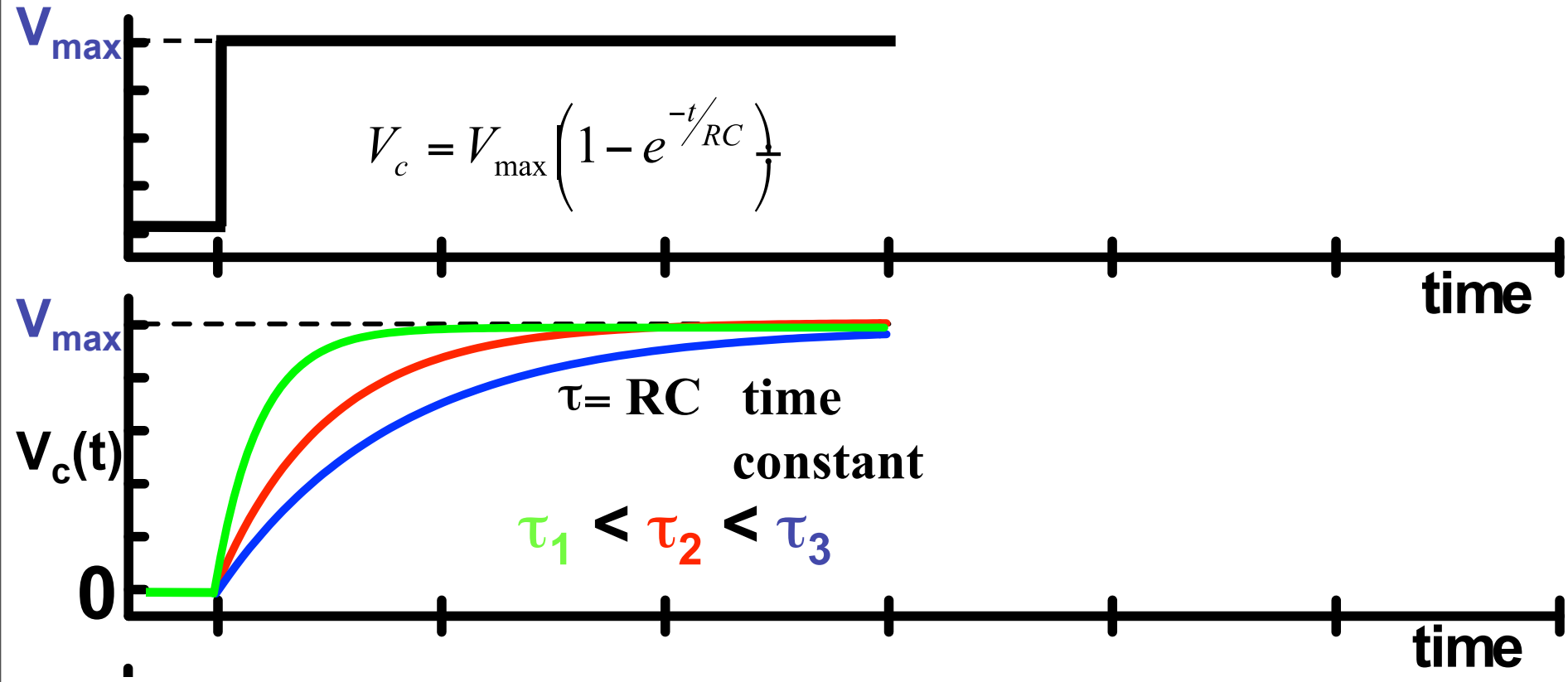




# RC: charging



# RC: charging



# HW – Clickers Out