PHYSICS 1B – Fall 2009

Electricity
&
Magnetism

Professor Brian Keating

SERF Building. Room 333

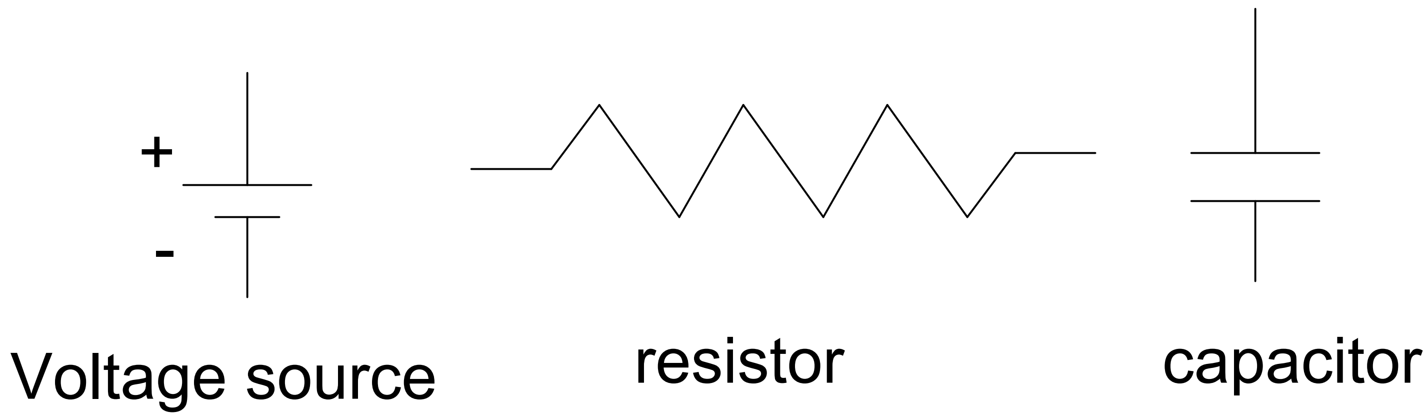
UCSD

UCSD

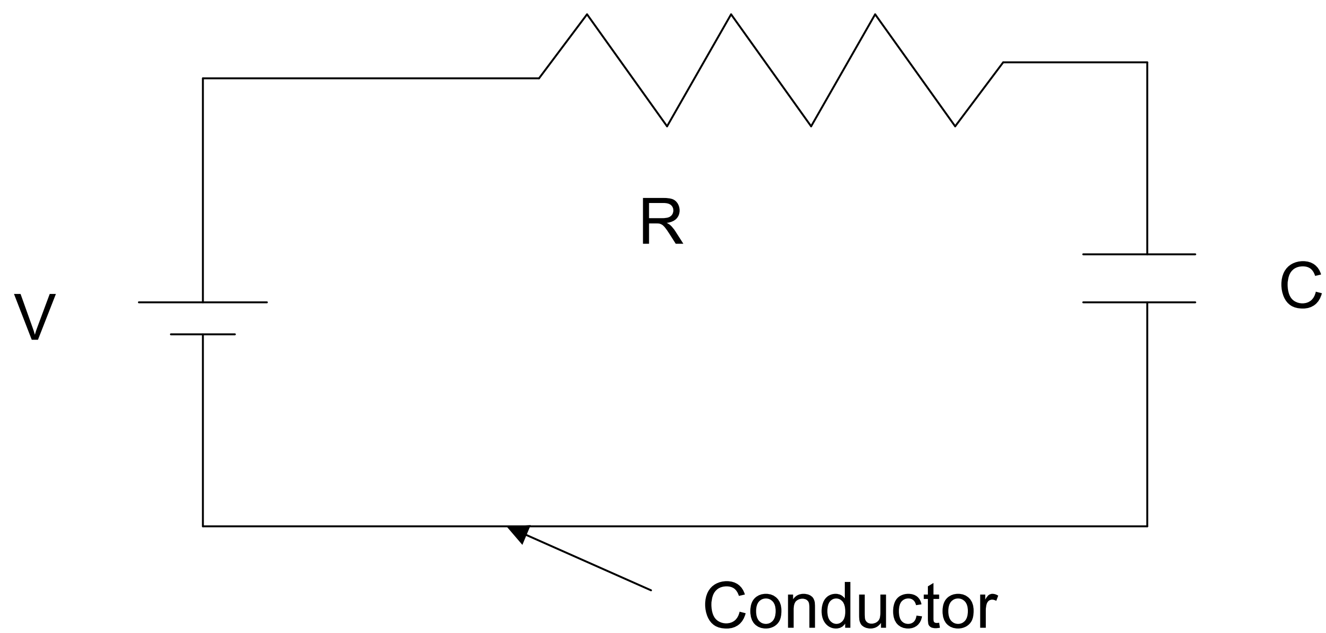
Capacitor combinations

Capacitors connected in series and parallel

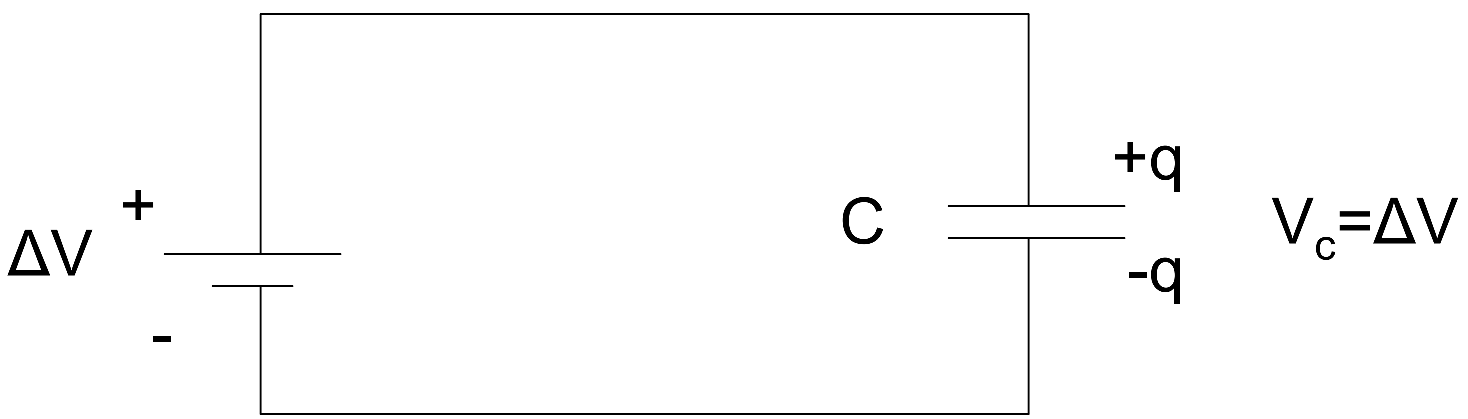
Electrical circuit elements



Circuit diagram

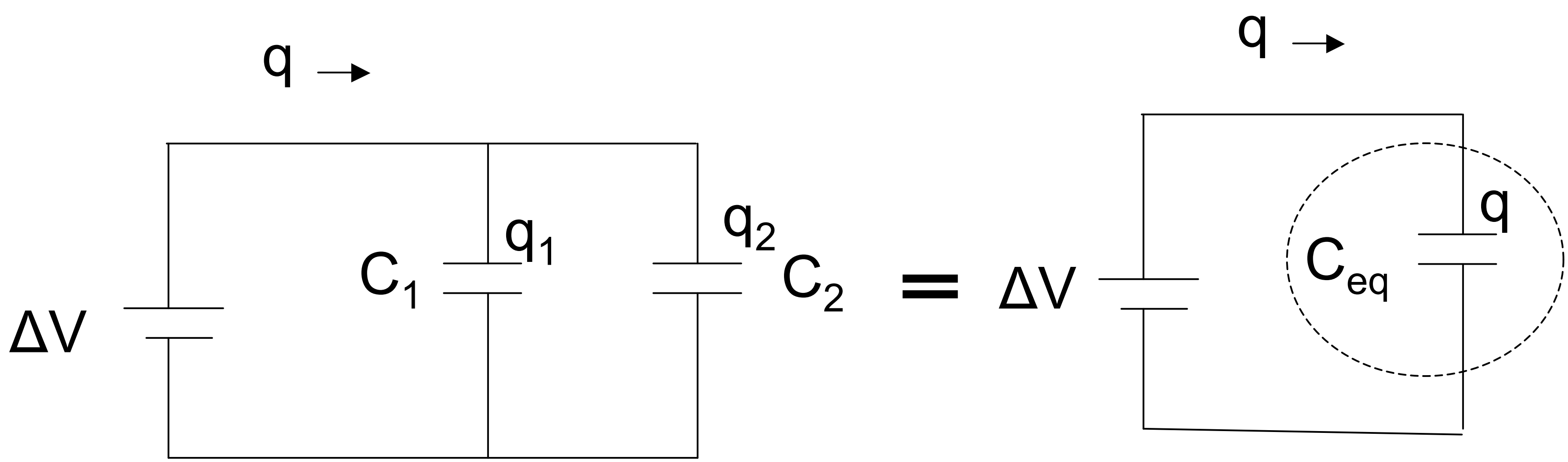


one capacitor



$$C = \frac{q}{\Delta V}$$

Two Capacitors in Parallel



$$C_{eq} = \frac{q}{\Delta V}$$

equivalent capacitance

What single capacitor has the same properties as two capacitors in parallel?

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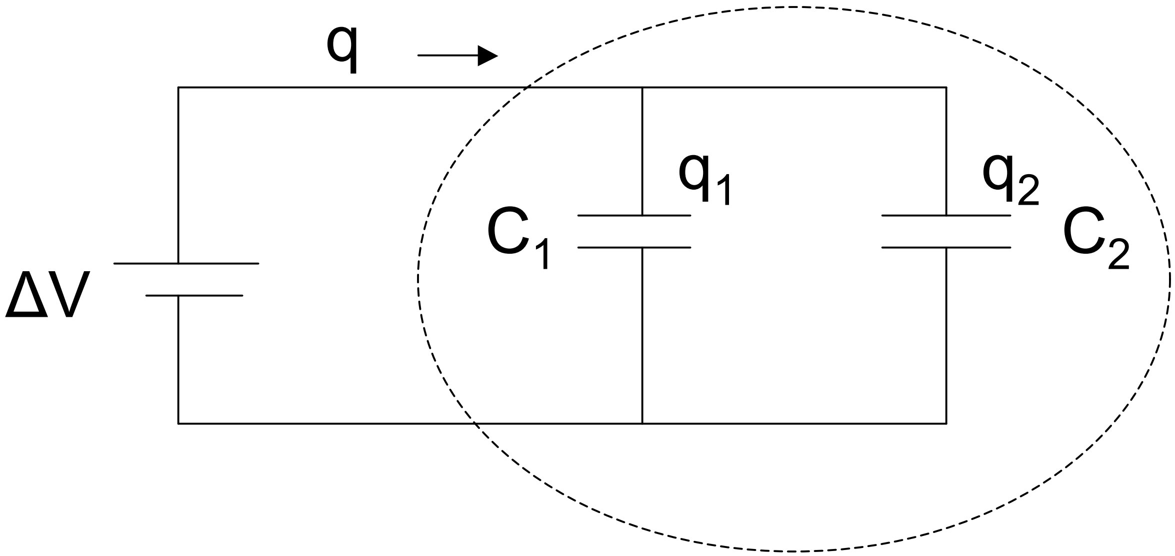
Capacitors


DANGER
HIGH VOLTAGE


DANGER
HIGH VOLTAGE
DISCHARGE CAPACITOR
BEFORE SERVICING

Two Parallel Capacitors



equivalent capacitance

$$C_{eq} = \frac{q}{\Delta V}$$

$$\Delta V = \Delta V_1 = \Delta V_2$$

$$q = q_1 + q_2 = C_1\Delta V + C_2\Delta V$$

$$C_{eq} = \frac{q}{\Delta V} = \frac{C_1\Delta V + C_2\Delta V}{\Delta V}$$

$$C_{eq} = C_1 + C_2$$

Parallel Capacitors

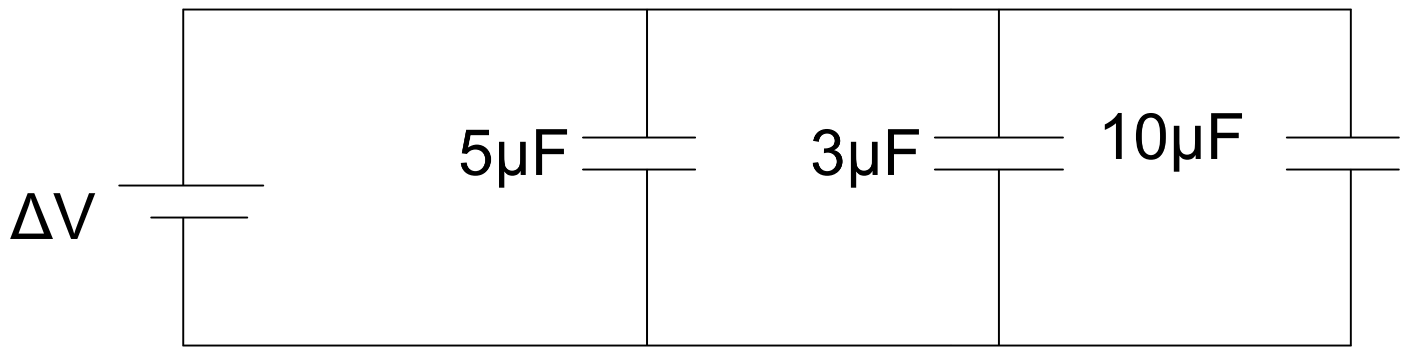
For N capacitors in parallel

$$C_{eq} = C_1 + C_2 + \ldots\ldots\ldots C_N$$

C_{eq} is the sum of capacitances

Like a larger capacitor, larger area

Find the equivalent capacitance



C_{eq} =

C_{eq} =

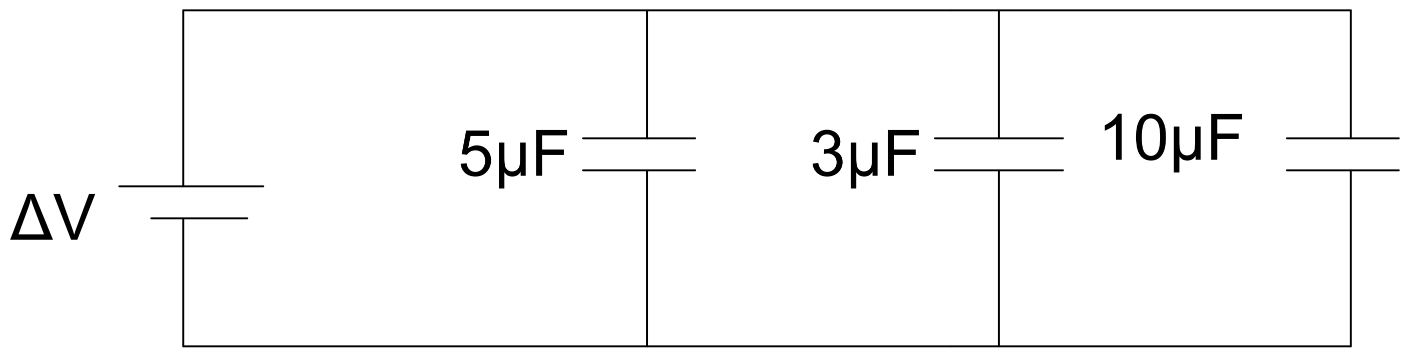
A. 15 uF

B. 17 uF

C. 18 uF

D. 20 uF

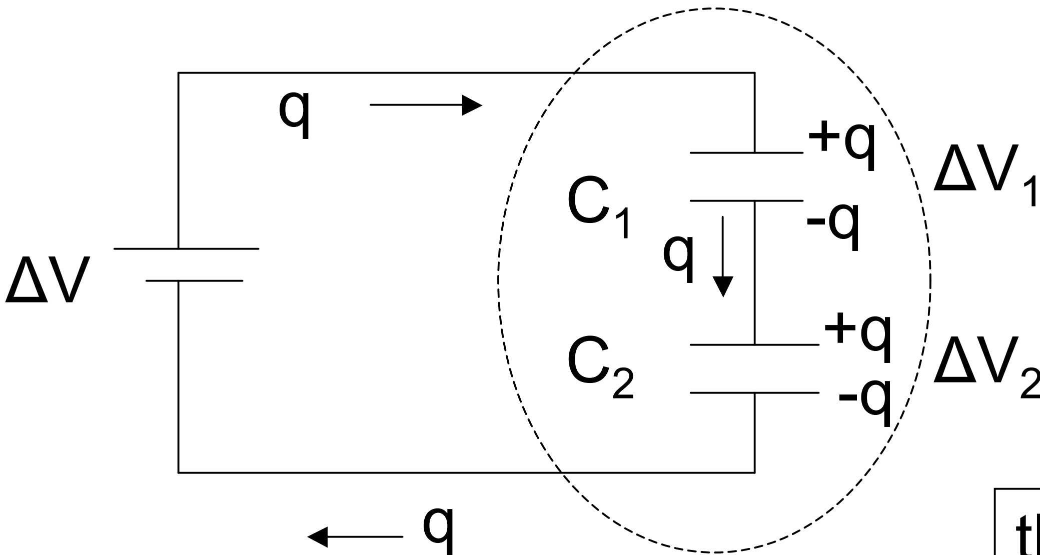
Find the equivalent capacitance



$$C_{eq} = C_1 + C_2 + C_3$$

$$C_{eq} = 5 + 3 + 10 = 18\mu F$$

Two Capacitors in Series

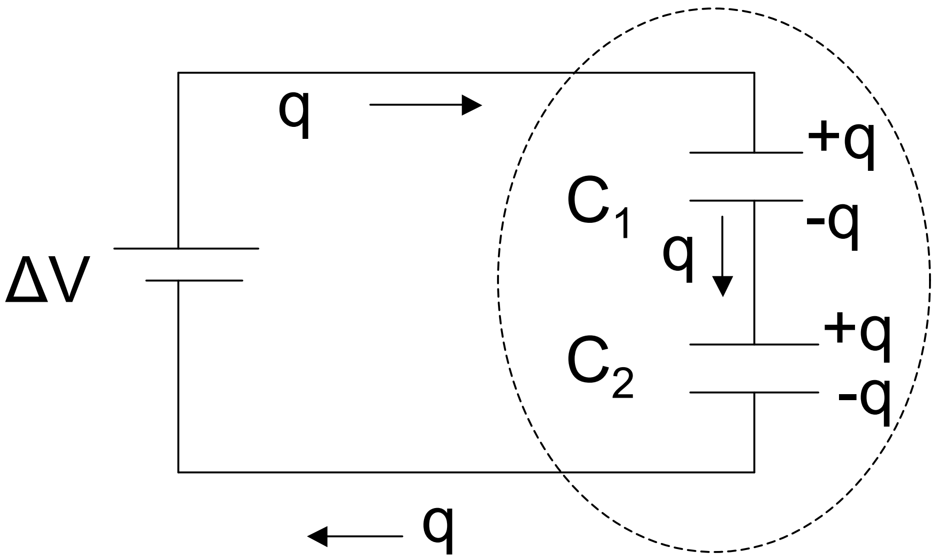


What is the equivalent capacitance?

$$C_{eq} = \frac{q}{\Delta V}$$

the charge on both capacitors in series is q

Two Capacitors in Series



$$C_{eq} = \frac{q}{\Delta V}$$

$$q = q_1 = q_2$$

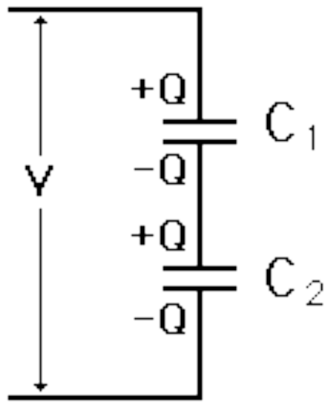
$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{q}{C_1} + \frac{q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{\Delta V}{q} = \frac{1}{q}\left(\frac{q}{C_1} + \frac{q}{C_2}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

For N capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots\dots\dots \frac{1}{C_N}$$

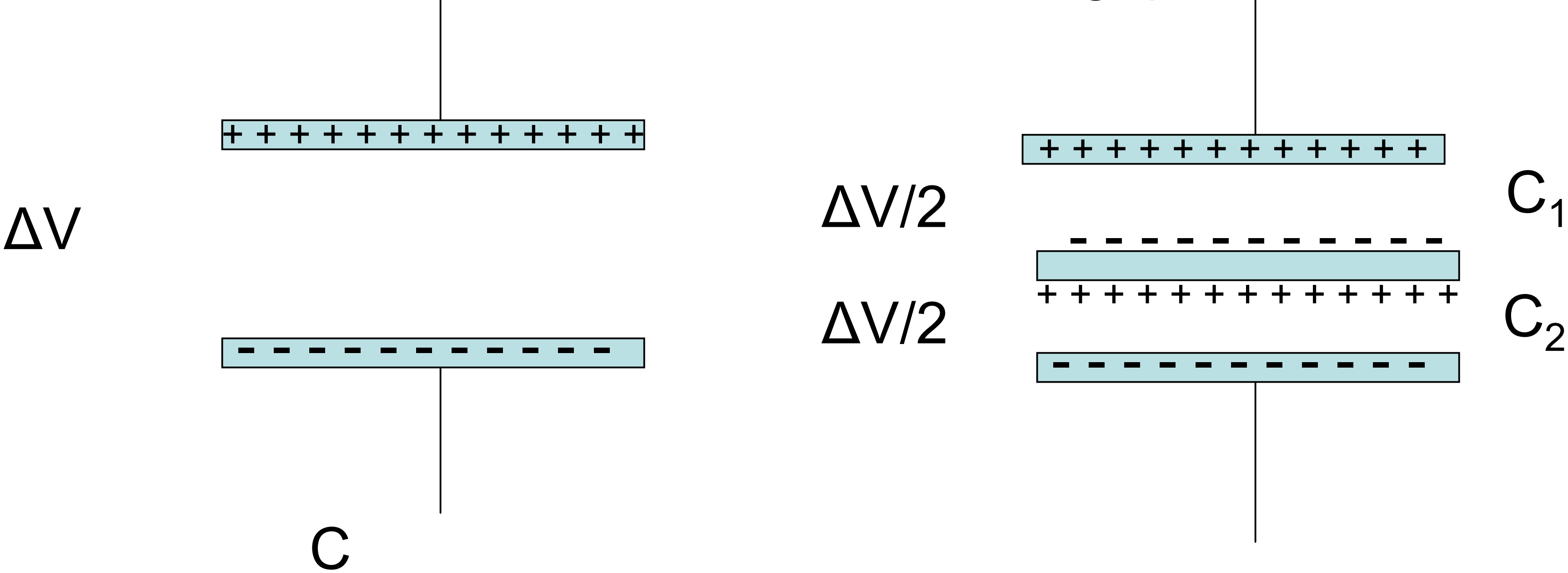


Capacitors in series

C_{eq} is smaller than the smallest capacitance. You store less charge on series capacitors than you would on either one of them alone with the same voltage!

Physical Argument

Take a parallel plate capacitor and place a thin metal plate with the same area in the middle of the gap.



the component capacitances are larger than the total

$C_1 = C_2 = \; 2C$

$C_{eq} = \; C$

The equivalent capacitance is less than the component capacitances

$C_{eq} < C_1$ or C_2

34. Find the equivalent capacitance.

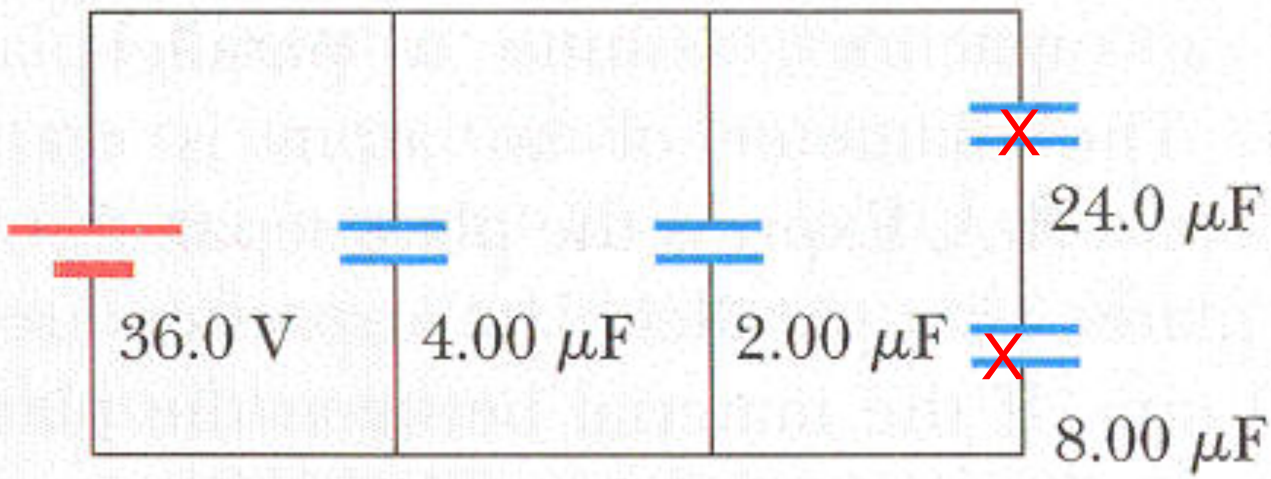


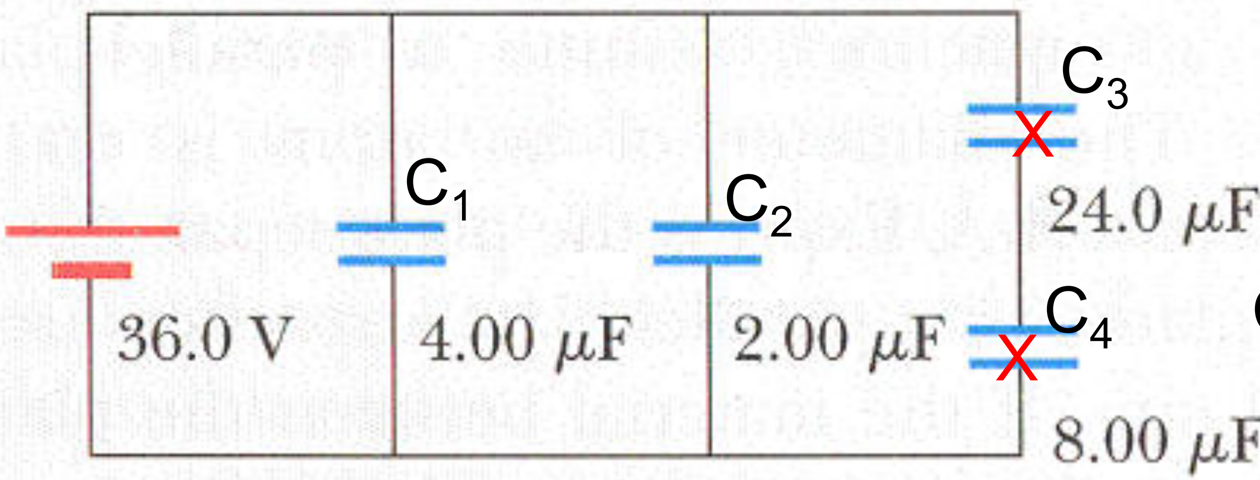
FIGURE P16.34

C_{series}=6μF

$$\frac{1}{C_{series}} = \frac{1}{24} + \frac{1}{8} = \frac{4}{24} = \frac{1}{6}$$

C_{eq}= 4.00+2.00+6.00=12.00 μF

34. Find the charge on each capacitor.



C_{series} =6μF

FIGURE P16.34

$$q = C\Delta V$$

$q_1=C_1\Delta V=4x10^{-6}(36)=1.44x10^{-4}$ C

$q_2 =C_1\Delta V=2x10^{-6}(36)=0.72x10^{-4}$ C

$q_3=q_4=C_{series}\Delta V=6x10^{-6}(36)=2.16x10^{-4}C$

34. Find the voltage drop across each capacitor.

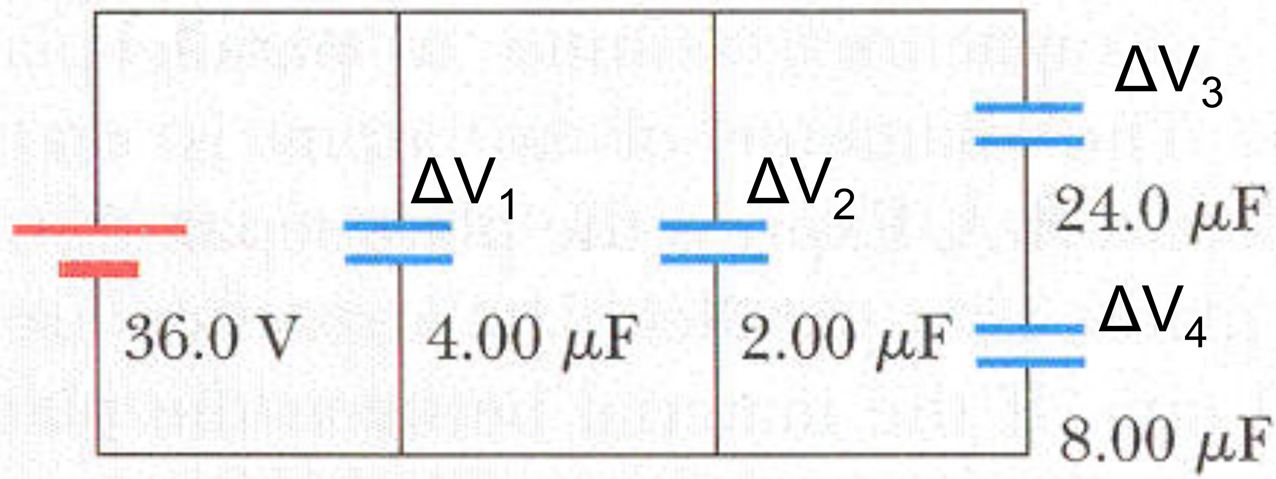


FIGURE P16.34

$$\Delta V = \frac{q}{C}$$

$\Delta V_1 = \Delta V_2 = 36V$

$$\Delta V_3 = \frac{q}{C_3} = \frac{2.16x10^{-4}}{24x10^{-6}} = 9.0V$$

$$\Delta V_4 = \frac{q}{C_4} = \frac{2.16x10^{-4}}{8x10^{-6}} = 27V$$

series capacitors
The larger C has the smaller voltage drop

16.9 Dielectrics, Energy

Dielectric constant-effect on capacitance

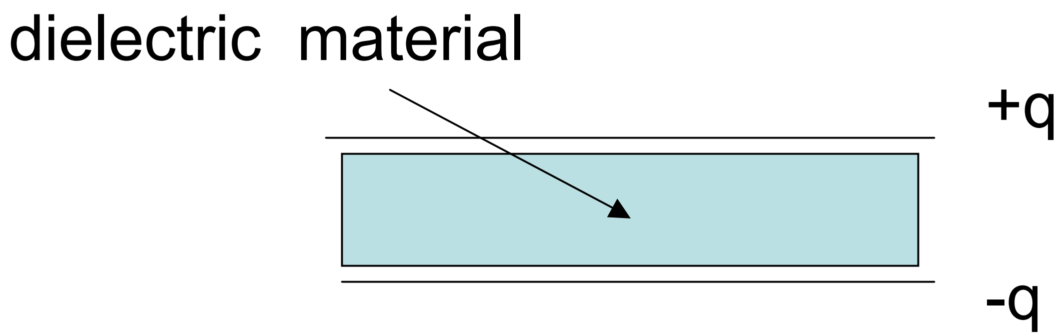
Energy stored in a capacitor

Energy density (depends on E^2)

Biological Membranes

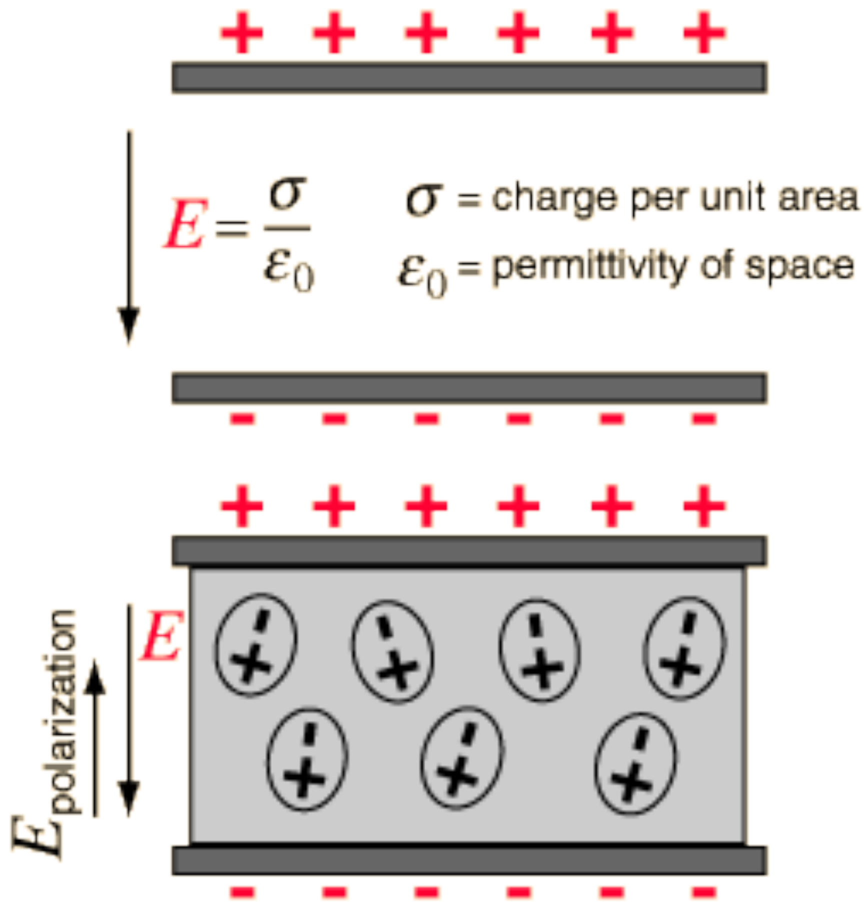
Dielectric material – insulators such as paper, glass plastic, ceramic.   Used in the gap in capacitors.

"Dielectric Strength" - is the electric field at which conduction occurs through the material

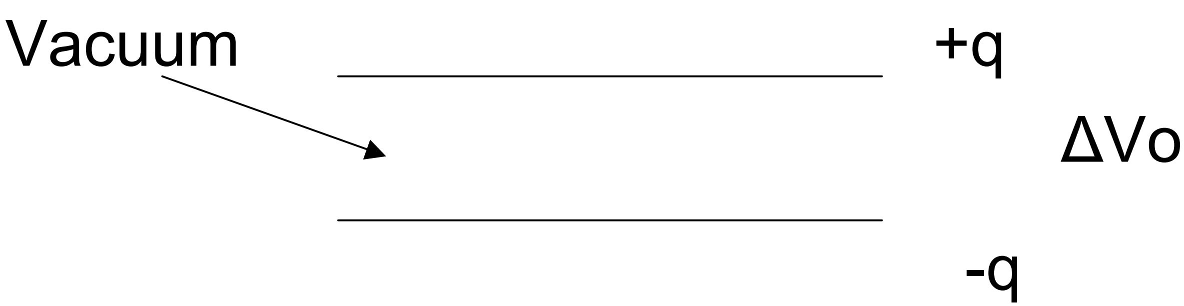


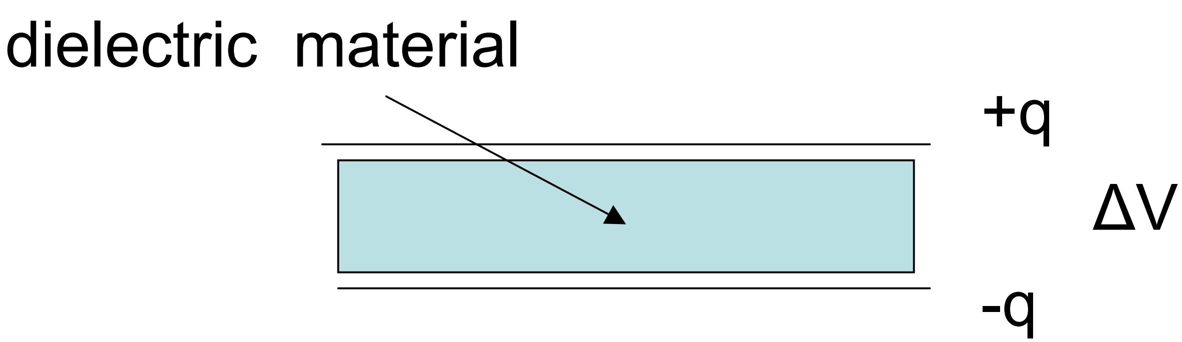
Electric Fields in Dielectric Filled Capacitors

$$E_{effective} = E - E_{polarization} = \frac{\sigma}{k\varepsilon_0}$$



Effects of a dielectric material inserted into a capacitor, with charge q





$$\Delta V = \frac{\Delta V_o}{\kappa}$$

Potential due to charge q decreases by κ

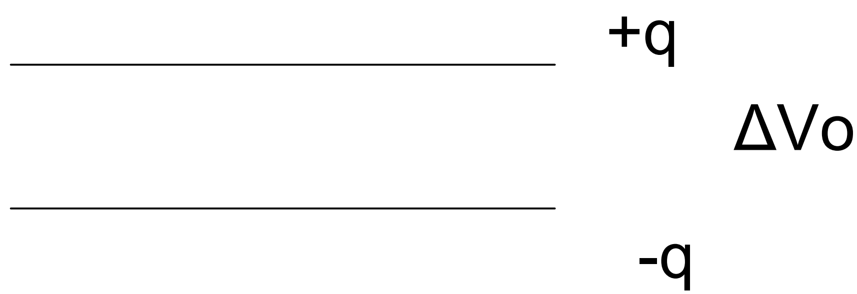
κ= dielectric constant (dimensionless)

Dielectric Properties of Selected Materials

Material	dielectric constant, κ	Dielectric Strength (Volt/m)
Vacuum	1.000000	-----
Air	1.00059	2×10^6
Polystyrene	2.3	24×10^6
Paper	3.4	16×10^6
Pyrex	5.6	14×10^6
Water	80	-------

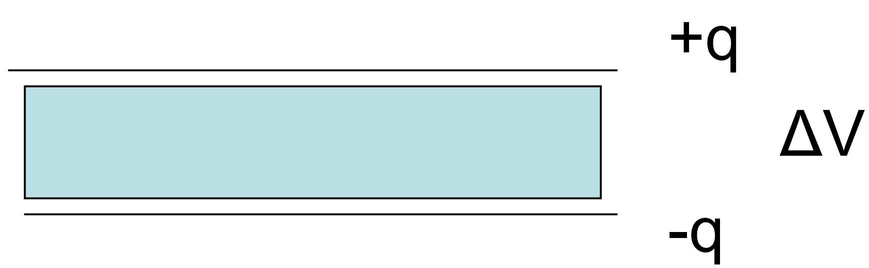
How does the capacitance change?

Originally



C_o

Add dielectric

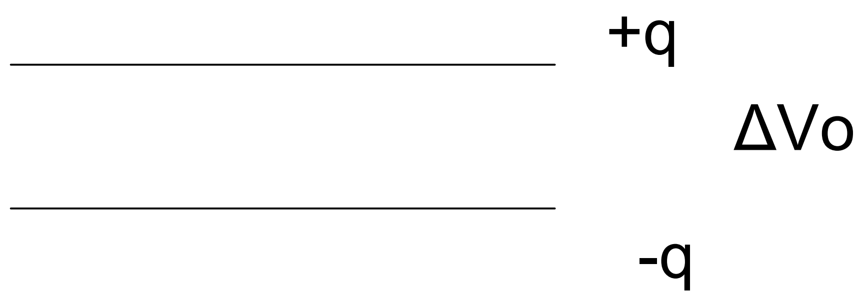


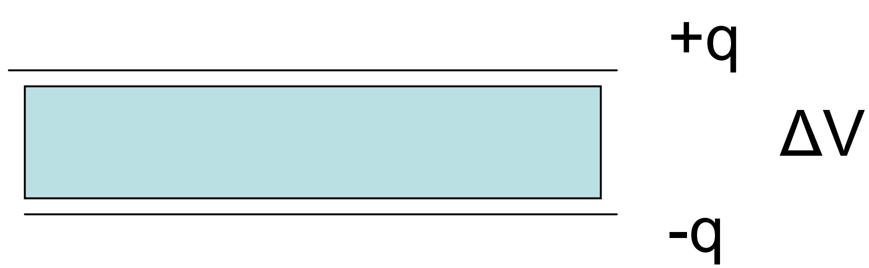
$$C = \frac{q}{\Delta V} = \frac{\kappa q}{\Delta V_o}$$

$$C = \kappa C_o$$

Capacitance increases

How does the E field change?



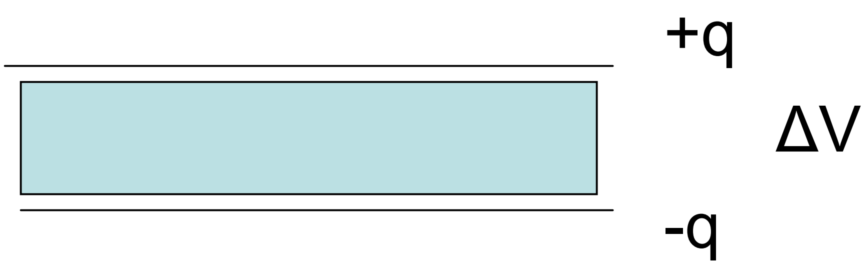


$$E = \frac{\Delta V}{d} = \frac{\Delta V_o}{\kappa d}$$

$$E = \frac{E_o}{\kappa}$$

Electric field decreases (when not connected to a battery)

$$E = \frac{q}{\kappa \varepsilon_o A} = \frac{q}{\varepsilon A}$$



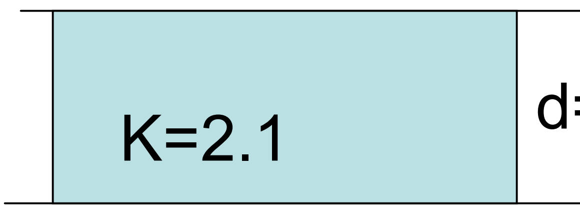
$$\varepsilon = \kappa \varepsilon_o$$

Permittivity is increased

Compared to vacuum

Example: A parallel plate capacitor consists of metal sheets ($A = 1.0 m^2$) separated by a Teflon sheet ($\kappa=2.1$) with a thickness of 0.005 mm. (a) find the capacitance. (b) Find the maximum voltage. The maximum electric field across Teflon is $60x10^6$ V/m. – this is its *dielectric strength*.

(a)



d=0.005m

$A=0.25m^2$

$$C = \frac{\kappa \varepsilon_o A}{d} = \frac{2.1(8.8x10^{-12})(1.0)}{0.005x10^{-3}}$$

$$C = 3.7x10^{-6} F$$

A parallel plate capacitor consists of metal sheets(A= 0.25m^2) separated by a Teflon sheet (κ=2.1) with a thickness of 0.005 mm. (a) find the capacitance. (b) Find the maximum voltage. The maximum electric field across Teflon is 60x10^6 V/m.(dielectric strength)

(b)

$$\Delta V_{\max} = E_{ds} d = 60x10^6 (0.005x10^{-3}) = 300V$$

Molecular basis for dielectric constant

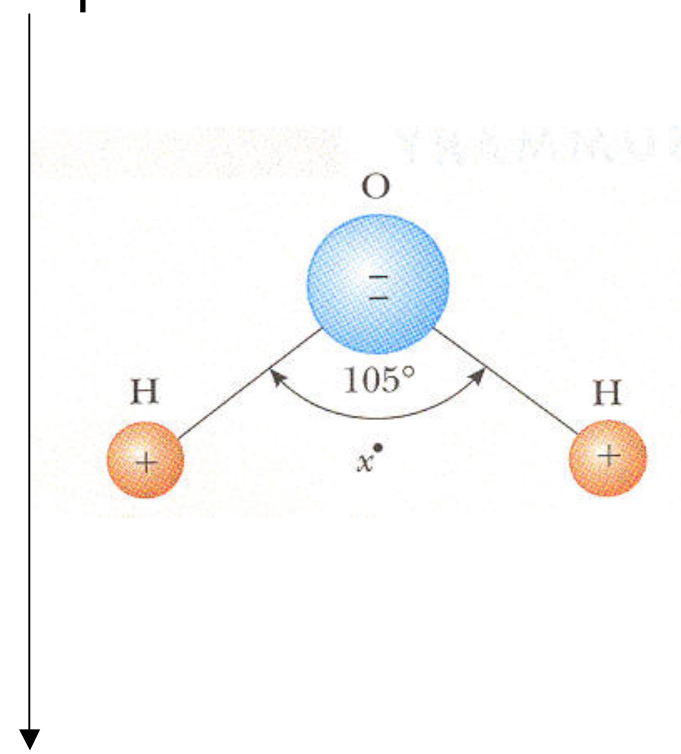
Dipolar molecules.

e.g. water

are oriented in
an E field

E

+



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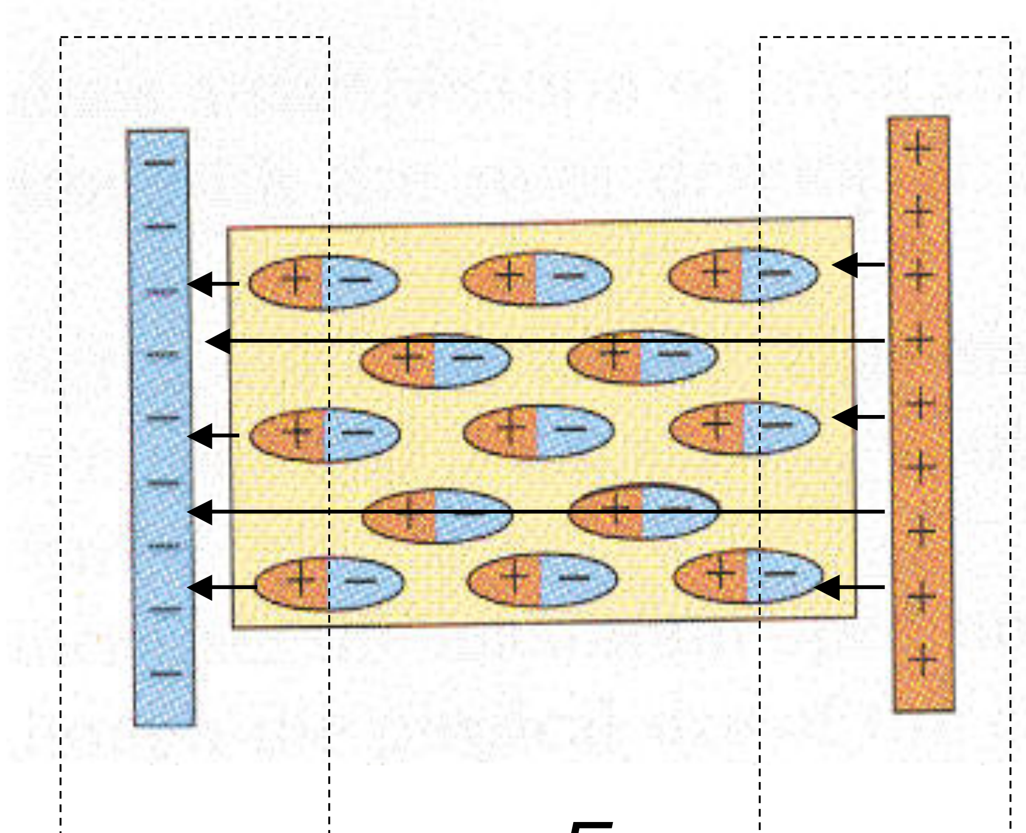
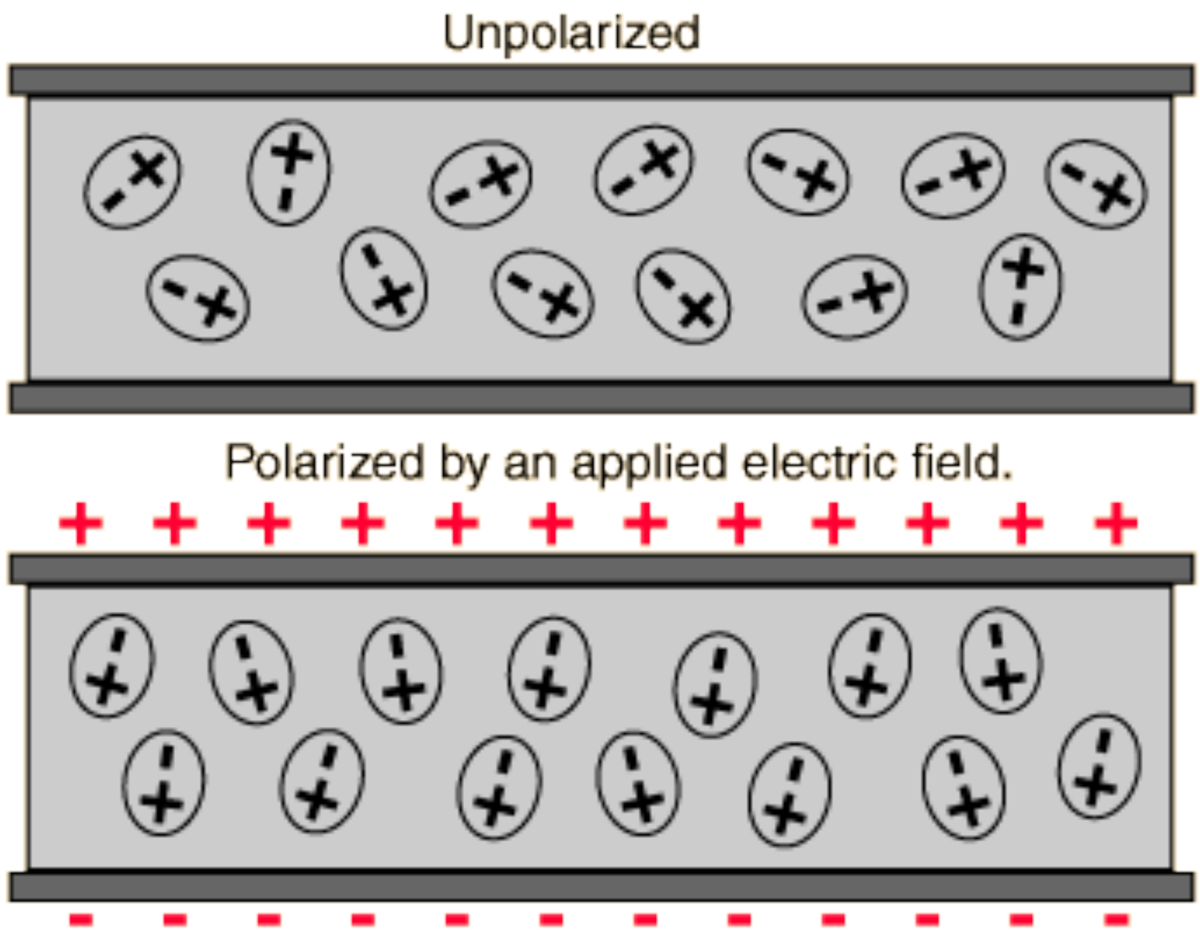
Oriented molecules decrease the net charge near the plates

The E field in the Capacitor is reduced



$$E = \frac{E_o}{\kappa}$$

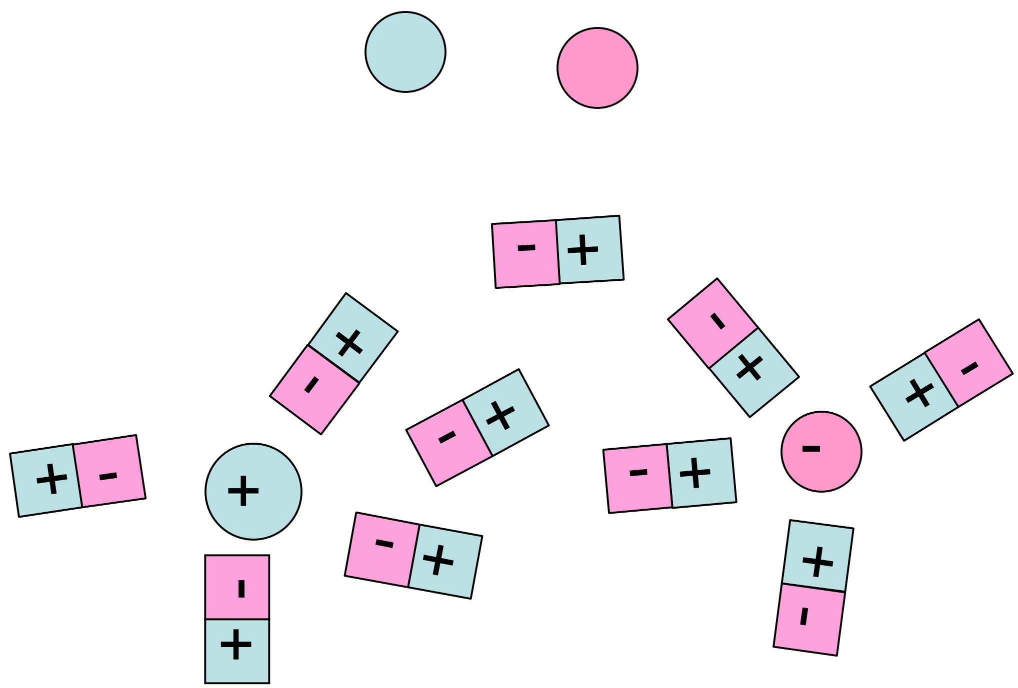
Polarization of Dielectric



Dielectric Screening

High dielectric constant of water allows ions to dissociate

Na^+ Cl^-



$$V = \frac{V_o}{\kappa} = \frac{V_o}{80}$$

Find the potential energy in electron volts for the interaction of Na^+ and Cl^- separated by 0.5 nm in water.

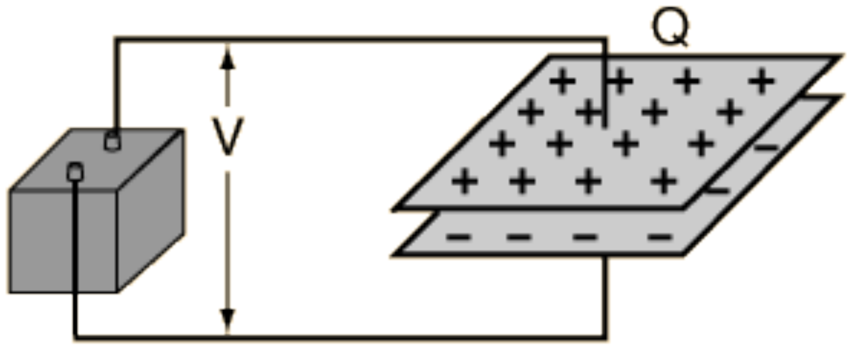
$$PE = \frac{q_{Na} q_{cl}}{4\pi\kappa\varepsilon_o r} = \frac{-(1.6x10^{-19})^2}{4\pi(80)(8.8x10^{-12})(0.5x10^{-9})}$$

$$PE = -5.8x10^{-21} Jx\left(\frac{1eV}{1.6x10^{-19} J}\right)$$

PE= -0.036 electron Volts

comparable to thermal energies (kinetic energy of the ion) about 0.025 eV at room temperature

Energy stored in a capacitor.



$C = \frac{Q}{V}$ Capacitor

$$Ed = \frac{Fd}{q} = \frac{W}{q} = \Delta V$$

For constant electric field.

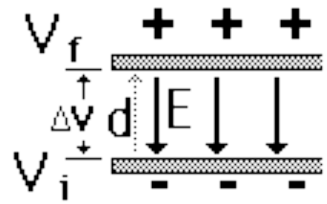
$E = \frac{F}{q}$ $W = q\Delta V$	General definition relationships

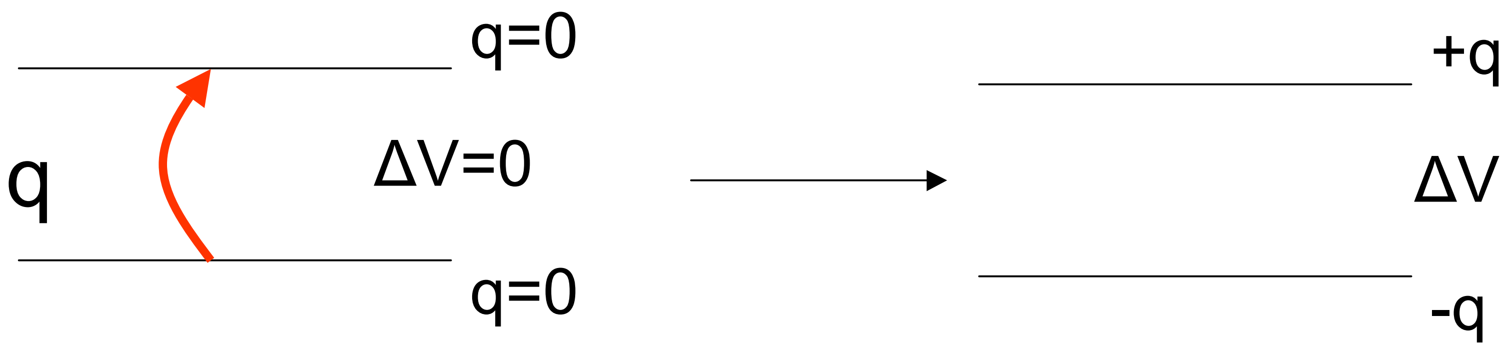
$E = \frac{V}{d}$ $V = Ed$	Constant field special case relationships

Work done to charge a capacitor

$$V_f - V_i = \frac{Fd}{q} = -Ed$$

Moving a charge from bottom to top plate requires work and raises voltage.





$$W = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}C\Delta V^2$$

So *work* depends on the *square* of q or ΔV

A parallel plate capacitor consists of metal sheets (A= 1m^2) separated by a teflon sheet (κ=2.1) with a thickness of 0.005 mm. Find the maximum energy that can be stored.

$C=3.7 \times 10^{-6} F$ $\Delta V_{max}=300V$

C

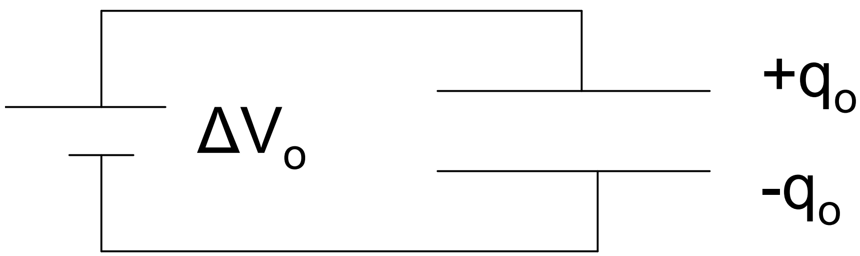
$$Energy = \frac{1}{2} C\Delta V^2 = \frac{1}{2}(3.7x10^{-6})(300)^2$$

$$Energy = 1.7x10^{-1} J$$

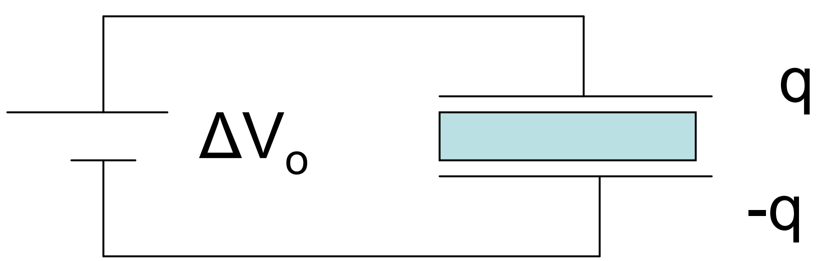
(quick quiz 16.6)

Insert the dielectric material with dielectric constant κ into the capacitor keeping the voltage source connected.

Find C,q,E, PE



C_o



C= κC_o

q= $CV = \kappa C_o V_o = \kappa q$

E= $\dfrac{\Delta V}{d} = \dfrac{\Delta V_o}{d} = E_o$

PE= $\dfrac{1}{2}C\Delta V^2 = \dfrac{1}{2}\kappa C\Delta V_o^{\,2} = \kappa PE_o$

Energy Density in a Capacitor

Suppose you wanted to store a large amount of energy in a capacitor with a given volume of 1m^3 using Teflon As the dielectric (dielectric strength of $60\text{x}10^6$ V/m). What is the maximum energy that could be stored?

$$Energy = \frac{1}{2}CV^2$$

$$C = \frac{A\kappa\varepsilon_o}{d}$$

$$Energy = \frac{1}{2}\frac{A\kappa\varepsilon_o}{d}V^2 = \frac{1}{2}\frac{Ad\kappa\varepsilon_o}{}\frac{V^2}{d^2} = \frac{1}{2}\kappa\varepsilon_o E^2(volume)$$

$$\frac{Energy}{volume} = \frac{1}{2}\kappa\varepsilon_o E^2$$

The energy density depends only on the E field squared.

For the maximum electric field = dielectric strength

$$Energy = \frac{1}{2}\kappa\varepsilon_o E^2 (volume)$$

$$Energy = \frac{1}{2}(2.1)(8.8x10^{-12})(60x10^6)^2(1)$$

$$Energy = 3.4x10^4 J$$

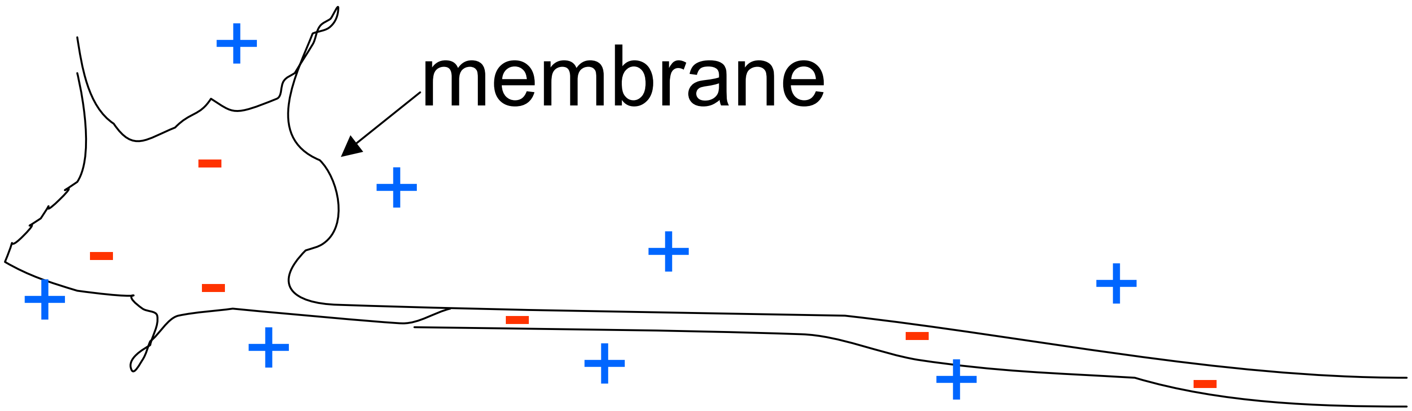
For a 1 m^3 capacitor at the maximum voltage.

For comparison the energy content of burning

1 gallon of gasoline is $1.3x10^8$ J

Chemical energy has a higher energy density.

Capacitance of Biological Membranes



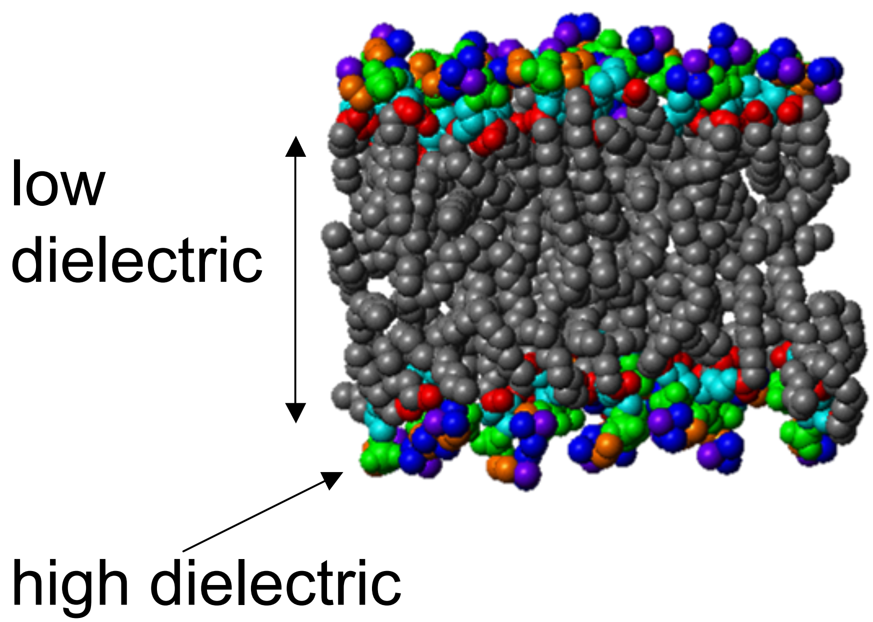
Axon - Nerve cells

Potential difference across the membrane

Nerve transmission – involves a discharge of membrane potential

Biological membranes –Capacitance

The low dielectric portion of a biological membrane has a thickness of 2.0 nm. Assume that it has a dielectric constant of 2.5 (silicone oil) find the capacitance of 1m^2 of membrane.



$$C = \frac{\varepsilon A}{d} = \frac{\kappa \varepsilon_o A}{d}$$

$$C = \frac{2.5(8.8x10^{-12})(1)}{2.0x10^{-9}}$$

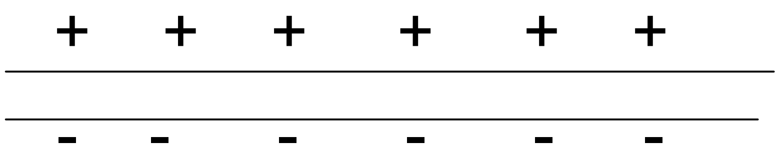
$$C = 1.1x10^{-2} F$$

High Capacitance

Compare to 3.7x10^{-6} F for 1m^2 the Teflon capacitor.

A nerve cell has a potential across it of 60 mV. Find the density of charges on the membrane that can give rise to this potential

$\Delta V = 60$ mV



$$E = \frac{\Delta V}{d} = \frac{60x10^{-3}}{2x10^{-9}} = 3x10^{7} V/m$$

$$E = \frac{q}{A\kappa\varepsilon_o} = \frac{\sigma}{\kappa\varepsilon_o}$$

This is close to the dielectric strength

$$\sigma = \kappa\varepsilon_o E = 2.5(8.8x10^{-12})(3x10^{7})$$

$$\sigma = 6.6x10^{-4} C/m^2$$

This corresponds to an ion density of $4.1x10^{-3}$ ions /nm^2 or a distance between ions of about 16 nm. A small number of excess charges.