Last time:

Radiative transport: Slow outward diffusion of radiative flux transports energy to surface

Derived 4th eq. of stellar structure

$$\frac{dT}{dR} = \frac{-3}{4ac} \frac{\kappa \rho}{T^3} \frac{L(R)}{4\pi R^2}$$

∴ Luminosity of the star determined by the temperature gradient $\frac{dT}{dR}$, which is set by the global structure of the star, <u>not</u> by L_{nuc}

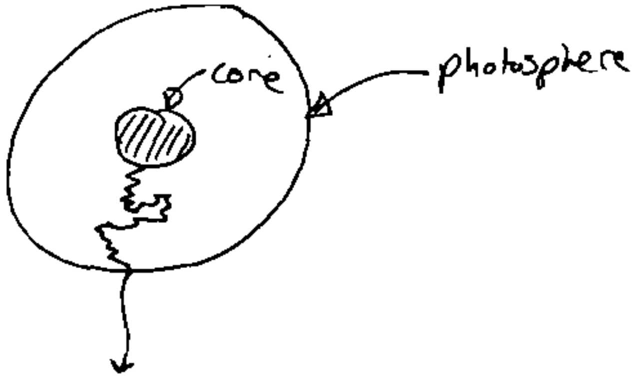
We also saw the stability of the stars output. ie: nuclear energy output balances the radiative output.

Simple view seen so far:

a) Inner hot core of star generates energy via thermonuclear reactions

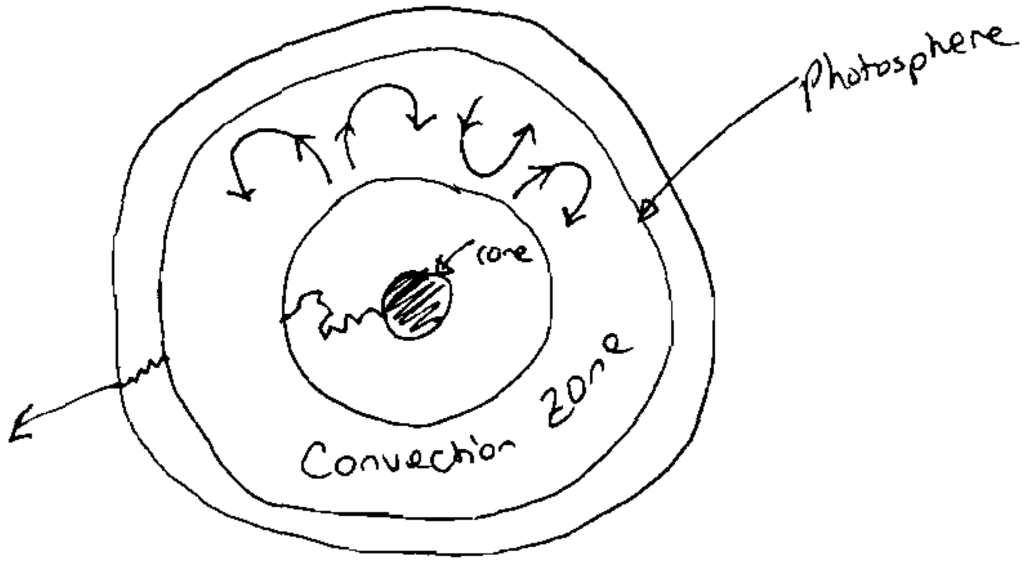
b) Energy slowly transported outward by diffusion.
- The mean free path slowly increases until photons reach photosphere where gas ahead is optically thin and photon escapes freely

Simple picture



But it is more complicated: The above picture neglects the zone *below* the photosphere in which energy is transported by Mechanica motions rather than radiation:

<u>The convection zone</u>



<u>Convection:</u> The cyclic motion of hot gas bubles that carries the excess energy outward and the cool elements inward.

Think of a pot of boiling water.

So detailed picture:

a) Energy generated in core
b) Transported from core to inner edge of convection zone by radiation
c) Energy transported across convection zone by bubbles of gas which dump energy in photosphere
d) Thereafter radiation caries energy through photosphere & then away from the Sun

Why does a convection zone exist?

Convection occurs when the magnitude of the Temperature gradient, $|dT/dR|$, is very steep.

Since $$\frac{dT}{dR} = \frac{-3}{4ac} \frac{\kappa \rho}{T^3} \frac{L(R)}{4\pi R^2}$$

We expect to get a steep gradient if the opacity is very large, or the Luminosity is very large. ([redacted]

For example: The presence of H^- makes the opacity very large. (H^- is the negative ion of H containing 2 bound electrons)

This would make convection dominate radiative transport.

Criteria for convection:

a) $$\boxed{\left|\frac{dT}{dR}\right| > \left|\left(\frac{\gamma-1}{\gamma}\right)\frac{dP}{dR}\,\frac{T}{P}\right|}$$

$\gamma \Rightarrow$ ratio of specific heats (C)

$\gamma \equiv \frac{C_p}{C_v}$ $C_p \rightarrow$ C at constant pressure
$C_V \rightarrow$ C at constant volume

Note: C is the amount of heat required to raise the temperature of a unit mass of a material by a unit temperature interval.

ie: $$C_p \equiv \left.\frac{\partial Q}{\partial T}\right|_P$$

Monoatomic gas $\gamma = 5/3$

As gas ionized $\gamma \longrightarrow 1$

b) Alternatively, another way to say this is

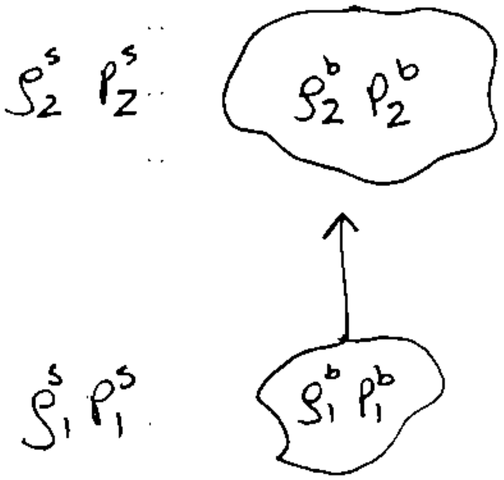
$$\boxed{\left|\frac{d\ln T}{d\ln P}\right| > \frac{\gamma-1}{\gamma}}$$

These are results $\rightarrow$ derivations in the book pgs 315-325

Lets try to understand criteria for convection

To get convection, need a hot bubble of gas to continue to rise, rather than sink back down after being displaced.

Lets consider a blob of gas at $P_1^{(b)}$ & $\rho_1^{(b)}$ in surrounding gas $P_1^{(s)}$, $\rho_1^{(s)}$



a) Displace blob upward by dr

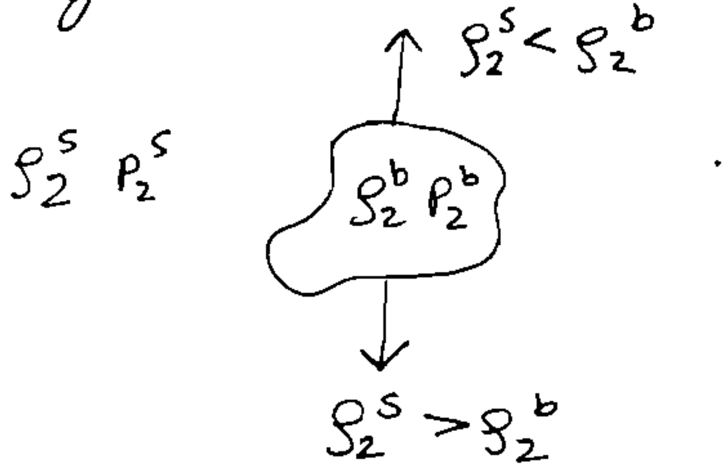
b) Process fast, so <u>NO</u> energy exchange with surrounding material: blob is <u>adiabatic</u>

c) After moving upward by dr, blob of gas has $P_2^{(b)}$ & $\rho_2^{(b)}$ in surrounding gas $P_2^{(s)}$ & $\rho_2^{(s)}$

d) Since blob moved upward, the pressure of the star is less than it was before at that point ie: $P_2^s < P_1^s$, therefore the blob will expand until internal and external pressures in balance.
since (b), it does so adiabatically.

<u>IF</u>
1) $\rho_2^s > \rho_2^b$: blob moves back to initial position
This is stable
2) $\rho_2^s < \rho_2^b$: blob continues to move outward

Archimedes' principle: if the initial density of a blob is less than the surroundings, it will go in the less dense direction.



So lets revisit when convection occurs

$$\frac{d \ln T}{d \ln P} = \frac{\frac{1}{T}\, dT/dR}{\frac{1}{P}\, dP/dR}$$

$$\frac{dT}{dR} = -\frac{3}{4ac}\,\frac{\rho \kappa}{T^3}\,\frac{L(R)}{4\pi R^2}$$

$$\frac{dP}{dR} = -\frac{G M(R)\, \rho}{R^2}$$

$$\therefore \quad \frac{d \ln T}{d \ln P} = \frac{\left[\frac{-3}{4ac}\,\frac{\rho \kappa}{T^4}\,\frac{L(R)}{4\pi \cancel{R^2}}\right]}{\left[-\frac{G M(R)\, \cancel{\rho}}{\cancel{R^2}\, P}\right]}$$

$$\frac{d\ln T}{d\ln P} = \frac{3}{16\pi acG} \frac{\kappa P}{T^4} \frac{L}{m}$$

So $\left|\frac{d\ln T}{d\ln P}\right| > \frac{\gamma-1}{\gamma}$ if

1) Large stellar opacity, ie: H^-

2) Large L/m, such as in cores of massive stars, since $\frac{L}{m} \propto m^2$

3) Region exists where ionization is occuring $(\gamma \to 1)$

Note: $\left|\frac{d\ln T}{d\ln P}\right| > \frac{\gamma-1}{\gamma}$ is derived from

The analysis of the gas blob. So the criteria after the blob has adiabatically expanded $(\rho_2^s < \rho_2^b)$ or $(\rho_2^s > \rho_2^b)$ directly results in the condition for convection

Back to nuclear reactions

We talked about The PPI, PPII, & PPIII chains and the CNO cycle. These all produce ${}_2He^4$, meaning that sun like stars develope ${}_2He^4$ cores.
∴ The mean molecular weight of the core increases.

$$\mu \approx 0.6 \rightarrow \mu \approx 1.33$$

But, we know $P = \frac{R \rho T}{\mu}$ $\left(P = \frac{\rho k T}{\mu m_H} \right)$

If ρ, & T stay the same, P has to decrease
But: P at a given radius must be maintained (to balance gravitational pull of interior matter) But μ bigger, P decreased...
→ So ∴ the core must contract to get P to support the star.

If the core contracts, the virial theorem results that the Temperature increases.
(gas "falls", energy goes to heat gas)

Eventually, The temperature high enough that ${}_2He^4$ nuclei can "overcome" their coulomb barriers and start to interact.

Result: Tripple Alpha Process

Tripple Alpha Process

Reaction in which helium is converted into Carbon

\# of nucleons

1) ${}_2He^4 + {}_2He^4 \rightarrow {}_4Be^8$

\# of protons

2) ${}_4Be^8 + {}_2He^4 \rightarrow {}_6C^{12} + \gamma$

Reaction 1 is endothermic, although only slightly

$$Q = 2M_{He}c^2 - M_{Be}c^2 = -92\,keV$$

(occurs at $T \sim 10^8 K$ & $KT \sim 100\,keV$)

Since reaction 1 is endothermic, ${}_4Be^8$ wants to decay back into 2 ${}_2He^4$ ~~#~~ nuclei

∴ For reaction to occur, need to be struck by another ${}_2He^4$ (alpha particle)

So really, think of it as a 3-body interaction, hence name Tripple alpha Process.

$$Q_{3\alpha} = 3M_{He}c^2 - M_C c^2 = 7.275\ MeV$$

or 5.8×10^{17} erg/g

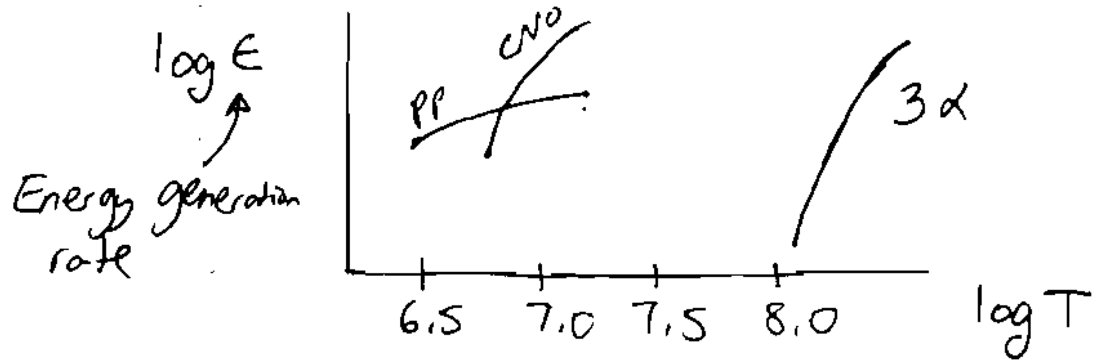
Note: $\epsilon_{3\alpha} \propto T_8^{41}$ (Nuclear energy generation rate)

$\sim 10\%$ of energy generated by fusion of H to He

<u>Net result</u>: Significant amounts of $_6C^{12}$ is produced. Most of the Carbon in the Universe is produced in this manner.

This is why we should think of ourselves as star stuff!

Temperature at which $_2He^4$ burning occurs, is higher than $H \rightarrow He$ burning because of higher Coulomb barrier. Energy production rates is more sensitive to temperature



$\epsilon_{3\alpha} \propto T^{41}$

<u>Further Synthesis</u>

Carbon & Oxygen burning

After sufficient $_6C^{12}$ is produced by 3α process, there are enough $_6C^{12}$ nuclei to capture $_2He^4$ nuclei and produce $_8O^{16}$, and some $_8O^{16}$ nuclei produce $_{10}Ne^{20}$.

Note: Need the high Temperatures as for 3α.

$${}_6C^{12} + {}_2He^4 \rightarrow {}_8O^{16} + \gamma$$

$${}_8O^{16} + {}_2He^4 \rightarrow {}_{10}Ne^{20} + \gamma$$

But as the number of protons (Z) increase, further burning at ${}_2He^4$ core temperatures becomes difficult due to increased height of Coulomb barrier.

More Massive Stars ($M \gtrsim 8 M_\odot$)

If stars are sufficiently massive, temperatures higher than $\sim 10^8$ K of ${}_2He^4$ core are attainable.

Carbon burning: For $T \sim 6 \times 10^8$ K

$${}_6C^{12} + {}_6C^{12} \rightarrow \begin{cases} {}_8O^{16} + 2\,{}_2He^4 & \text{(endothermic} \\ {}_{10}Ne^{20} + {}_2He^4 & \\ {}_{11}Na^{23} + {}_1H^1 & \\ {}_{12}Mg^{23} + n & \text{(endothermic)} \\ {}_{12}Mg^{24} + \gamma & \end{cases}$$

Oxygen burning: For $T \sim 10^9$ K

$${}_8O^{16} + {}_8O^{16} \rightarrow \begin{cases} {}_{12}Mg^{24} + 2\,{}_2He^4 & \text{(endothermic)} \\ {}_{14}Si^{28} + {}_2He^4 & \\ {}_{15}P^{31} + {}_1H^1 & \\ {}_{16}S^{31} + n & \\ {}_{16}S^{32} + \gamma & \end{cases}$$

Even higher temperatures lead to more reactions. For instance

$$\left.\begin{array}{l} {}_{14}Si^{28} + {}_{14}Si^{28} \longrightarrow {}_{26}Fe^{54} + {}_{1}H^{1} + {}_{1}H^{1} \\ \text{or} \\ {}_{14}Si^{28} + {}_{2}He^{4} \rightleftarrows {}_{16}S^{32} + \gamma \\ \text{or} \\ {}_{16}S^{32} + {}_{2}He^{4} \rightleftarrows {}_{18}Ar^{36} + \gamma \end{array}\right\} \text{Examples. MANY others exist.}$$

Notice I put arrows both ways on the last two examples.

Massive particles interact with energetic photons, disintegrating nuclei.
$\Rightarrow$ photodisintegration.

The reactions go both ways, and an <u>equilibrium</u> is achieved.

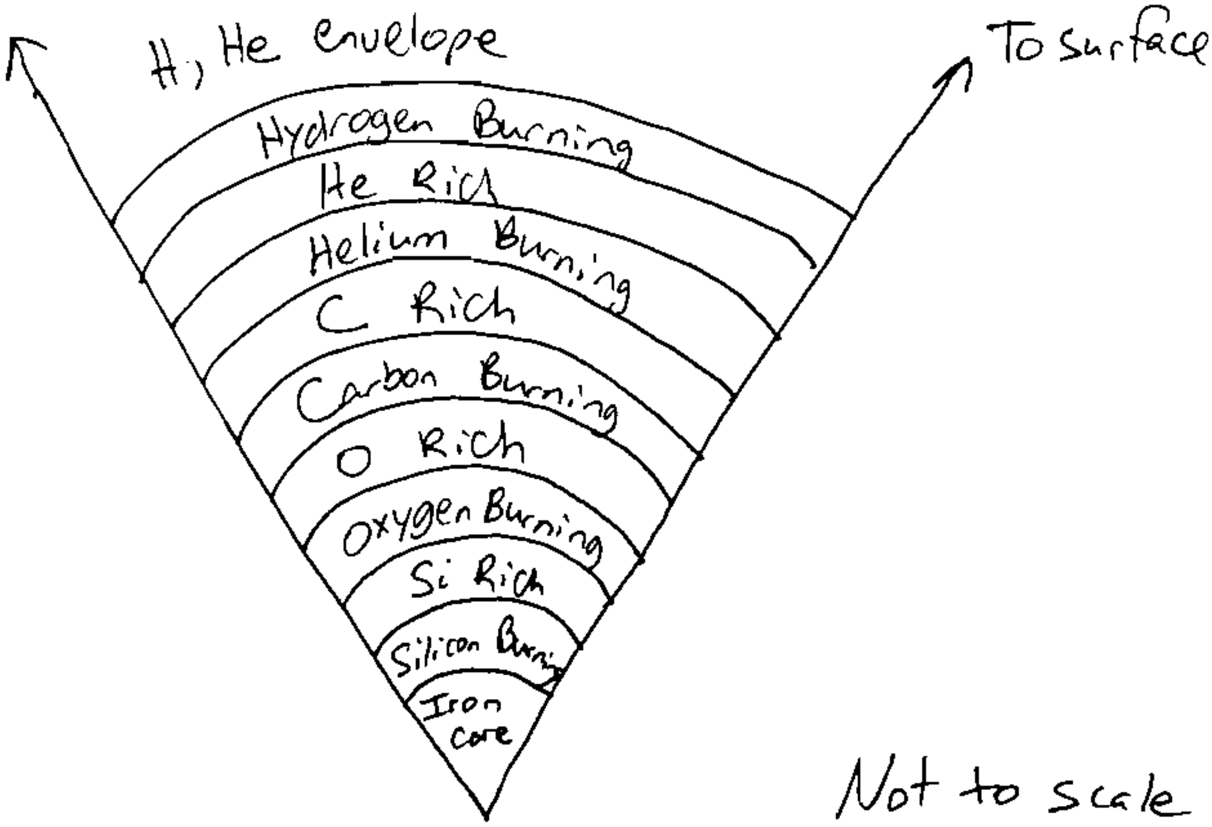
The resulting "<u>Nuclear Statistical equilibrium</u>" is not perfect: a leakage occurs toward the stable iron group nuclei (Fe, Co, Ni) which resist photodisintegration until temperatures $\sim 7 \times 10^9$ K.

Each successive reaction requires higher temperatures owing to the increase in the height of the coulomb barrier!

How do we get higher temperatures?

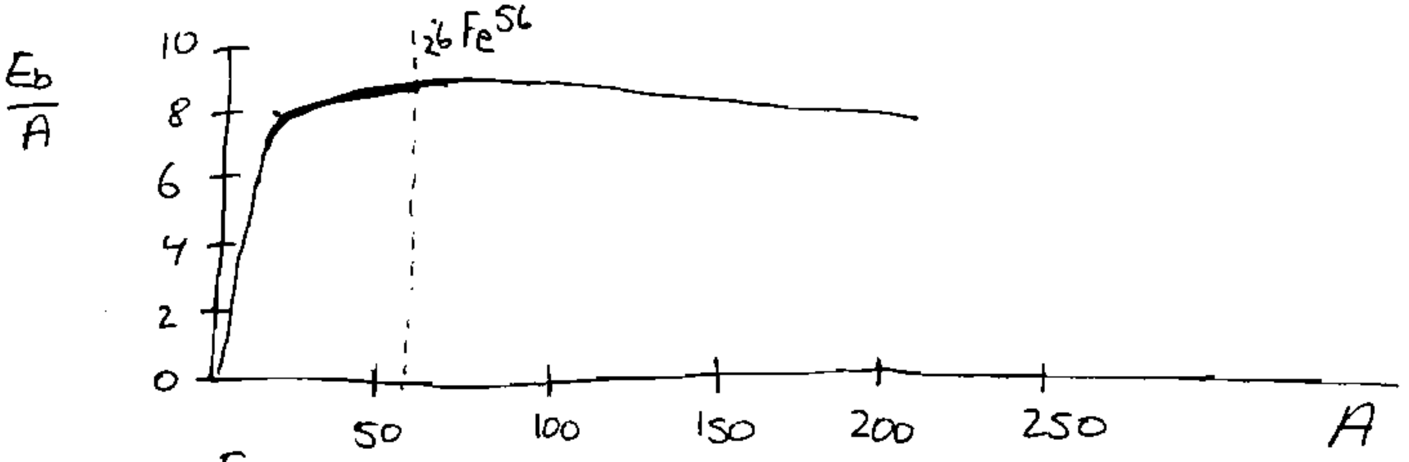
The Star must contract (quasi-statically) for the temperature to rise sufficiently so that QM tunneling can proceed at a reasonable rate

The result for massive stars is an onion like interior, with processed material sandwiched between nuclear burning Shells. These regions exist because the temperature and density are not sufficient to cause nuclear reactions for that composition.



Can this process of contraction, temperature rise, followed by burning of heavier species go on indefinately???

<u>No</u>: Fe is at the peak of the binding energy per nucleon curve. All reactions beyond $_{26}Fe^{56}$ are <u>very</u> endothermic and will not occur!



E_b = binding energy

$\left[\frac{E_b}{A}\right]$ = MeV/nucleon

A -> mass number (# nucleons, ie: protons & neutro

The peak of E_b/A occur at $_{26}Fe^{56}$.
If $_{26}Fe^{56}$ core contracts, gas does not heat up because all reactions absorb massive amounts of energy. The result is a catastrophic loss of pressure. The core collapses, and we get a Supernova Explosion!

Where do elements heavier than $_{26}Fe^{56}$ come from?

Fusion of charged particles to successively heavier elements will <u>NOT</u> occur.

But, this limitation does not apply for free neutron capture.

Since neutron capture not limited by the Coulomb barrier, the only obstacle is the scarcity of neutrons.

The process is called the s- and r- processes. Someone will present this topic. (rapid & slow processes)

Physical Structure of the star again

At the end of the MS, we have a $_1H^1$ burning core of a star that looks like this:

r_{ic} - radius of inner core

This is a $_2He^4$ Core embedded in an $_1H^1$ envelope which exerts a surface pressure on it.

Thermal Energy of core : $E_{KE} = \frac{3}{2} N_{ic} KT$

$N = \frac{M_{ic}}{\mu_{ic} m_H}$ $\therefore$ $E_{KE} = \frac{3}{2} \frac{M_{ic} K T}{\mu_{ic} m_H}$

Note: ic means interior core

Gravitational Energy (Potential Energy)

$\Omega_G = -\frac{3}{5} \frac{G m^2}{r}$ (uniform sphere)

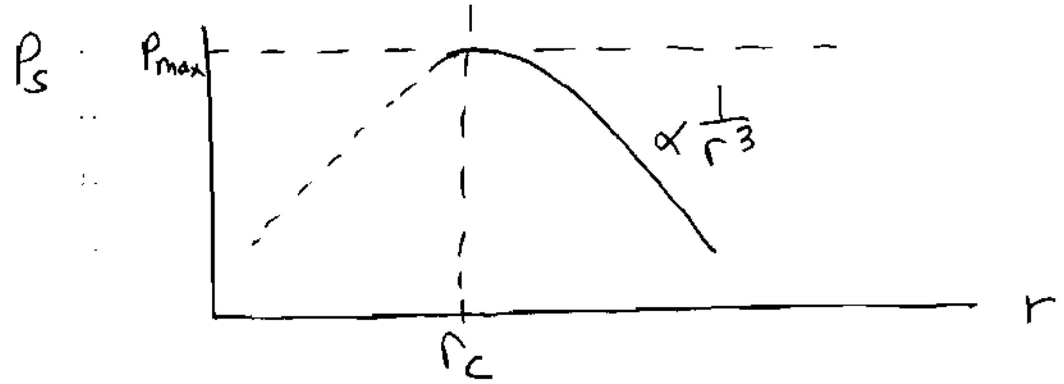
Virial theorem : $-2 E_{KE} = \Omega_G$

However, we don't integrate to the outer surface but to a specific radius which has a surface P.
$\therefore$ Virial Theorem with surface pressure :

$$4\pi r_{ic}^3 P_s = 2 E_{KE} + \Omega_G$$

$$\therefore \quad P_s = \frac{1}{4\pi r_{ic}^3}\left[\frac{3MkT_{ic}}{\mu_{ic} m_H} - \frac{3}{5}\frac{GM^2}{r_{ic}}\right]$$

$$P_s = \frac{1}{4\pi}\left[\frac{3MkT_{ic}}{\mu_{ic} m_H r_{ic}^3} - \frac{3}{5}\frac{GM^2}{r_{ic}^4}\right]$$



Hydrostatic equilibrium possible for $r > r_c$. But, as core shrinks, P_s increases until $P_s = P_{max}$. No stable configuration possible at $r < r_c$ since P decreases as r decreases and gravity overwhelms thermal energy.

Schönberg-Chandrasekhar limit

The maximum core mass, which corresponds to r_c is given by

$$\frac{M_{ic}}{M_{tot}} \simeq 0.37\left(\frac{\mu_{out}}{\mu_{in}}\right)$$

μ = mean molecular weight

If: $\mu_{out} = 0.6$, $\mu_{in} = 4/3 = 1.33$