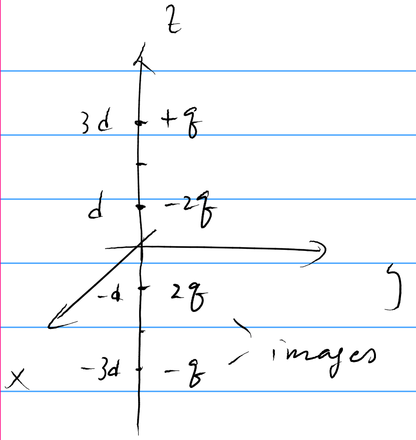
3.6



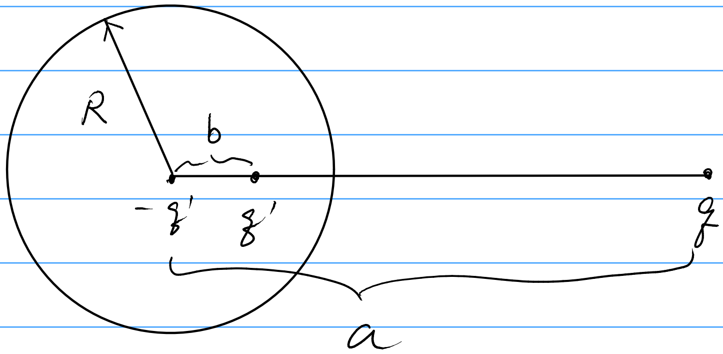
$$F = \frac{q}{4\pi\varepsilon_0}\left(\frac{-2q}{(2d)^2} + \frac{2q}{(4d)^2} + \frac{-q}{(6d)^2}\right)\hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\left(-\frac{1}{2} + \frac{1}{8} - \frac{1}{36}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\cdot\frac{29}{72}$$

3.8. ① $V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_i}{R} \Rightarrow q_i = 4\pi\varepsilon_0 R V_0$ image charge at center.

②



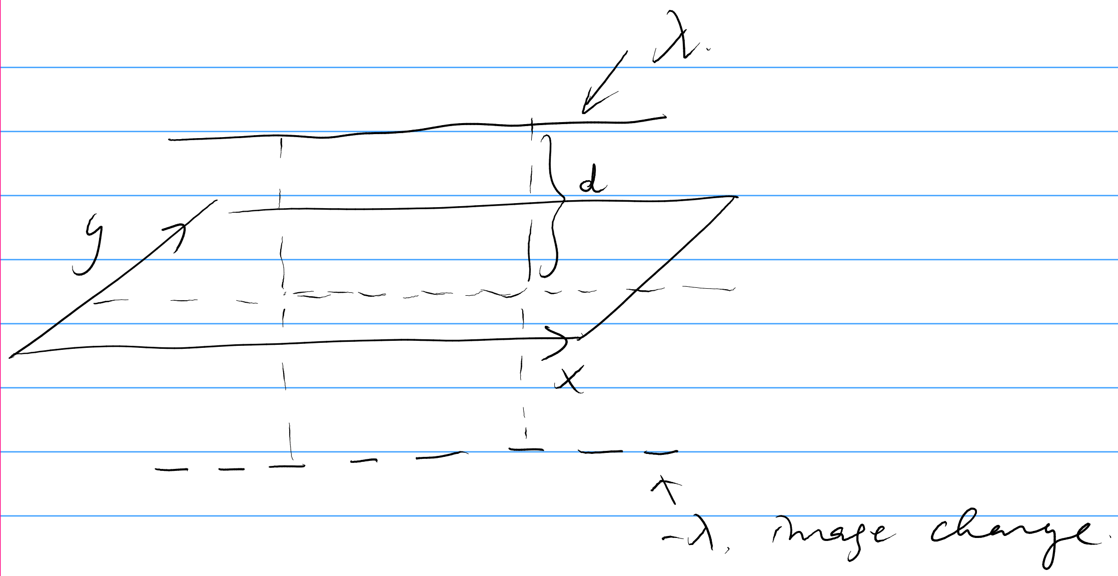
$$q' = -\frac{R}{a}q, \quad b = \frac{R^2}{a} \Rightarrow a-b = a - \frac{R^2}{a} = \frac{a^2-R^2}{a}$$

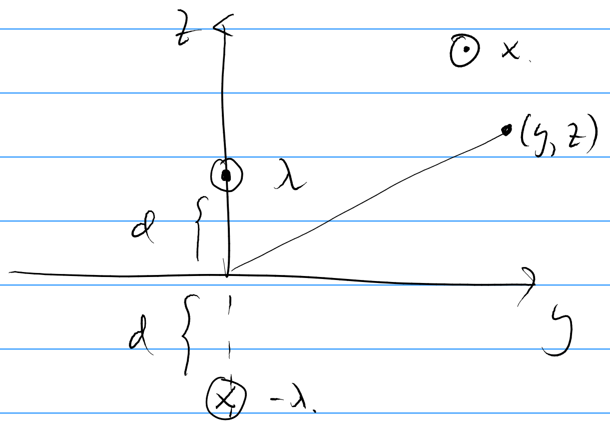
$$\Rightarrow F = \frac{q}{4\pi\varepsilon_0}\cdot\left[\frac{q'}{(a-b)^2} - \frac{q'}{(a)^2}\right]$$

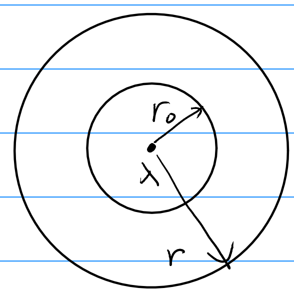
$$= \frac{qq'}{4\pi\varepsilon_0}\underbrace{\left[\frac{a^2}{(a^2-R^2)^2} - \frac{1}{a^2}\right]}_{=\frac{R^2(2a^2-R^2)}{(a^2-R^2)^2a^2}}$$

$$= -\frac{q^2}{4\pi\varepsilon_0}\cdot\left(\frac{R}{a}\right)^3\cdot\frac{2a^2-R^2}{(a^2-R^2)^2}$$

3.9







line charge: $E(r) = \dfrac{\lambda}{2\pi r \varepsilon_0}$

$$\Rightarrow V(r) = V(r_0) - \frac{\lambda}{2\pi\varepsilon_0} \log \frac{r}{r_0}$$

Assume z-axis passing through λ, and set V@origin to be 0, then

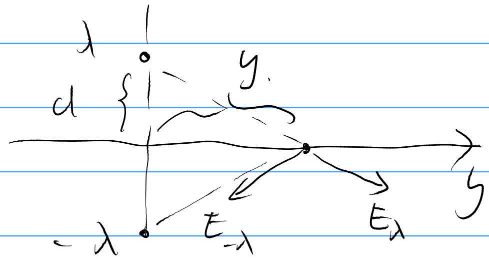
⇒ @ (y, z),

$$V_\lambda = \overset{=0}{V_{origin}} - \frac{\lambda}{2\pi\varepsilon_0} \log \frac{\sqrt{y^2+(z-d)^2}}{d}$$

$$V_{-\lambda} = \overset{=0}{V_{origin}} + \frac{\lambda}{2\pi\varepsilon_0} \log \frac{\sqrt{y^2+(z+d)^2}}{d}$$

$$\Rightarrow V_{tot} = V_\lambda + V_{-\lambda} = \frac{\lambda}{4\pi\varepsilon_0} \frac{y^2+(z+d)^2}{y^2+(z-d)^2}$$

(b)



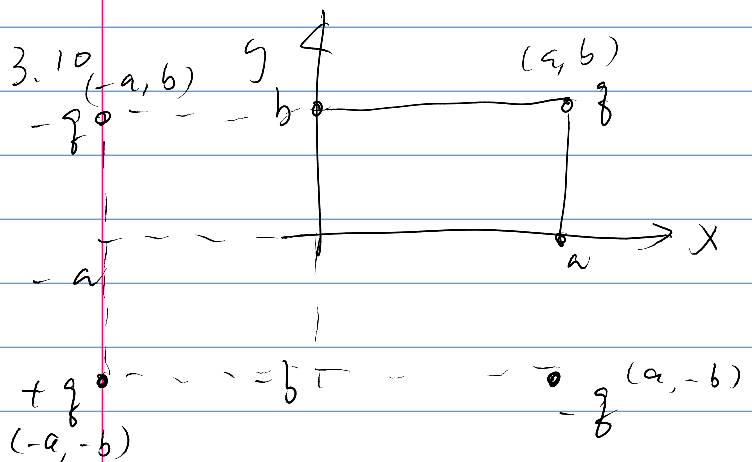
$$\vec{E}(y) = -\hat{z}\,\frac{\lambda}{2\pi\sqrt{d^2+y^2}\,\varepsilon_0}\cdot\frac{d}{\sqrt{d^2+y^2}}\cdot 2$$

both $\downarrow$ component

$$= -\hat{z}\cdot\frac{\lambda d}{2\pi\varepsilon_0(d^2+y^2)}$$

$$\Rightarrow \sigma(y) = \varepsilon_0 E = -\frac{\lambda d}{2\pi(d^2+y^2)}$$

3.10



① image configuration as left. easy to check that $E \perp$ boundary surfaces.

② $$V(x,y,z) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{(x-a)^2+(y-b)^2+z^2}} + \frac{1}{\sqrt{(x+a)^2+(y+b)^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+(y+b)^2+z^2}} - \frac{1}{\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]$$

③ Force

$$\vec{F} = \frac{q}{4\pi\varepsilon_0}\cdot\left(\frac{-q}{4a^2}\hat{x} + \frac{-q}{4b^2}\hat{y} + \frac{q}{4a^2+4b^2}\cdot\frac{a\hat{x}+b\hat{y}}{\sqrt{a^2+b^2}}\right)$$

④ work = PE

$$= \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{a} + \frac{-q^2}{b} + \frac{q^2}{\sqrt{a^2+b^2}}\right]$$

$$= \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{\sqrt{a^2+b^2}} - \frac{1}{a} - \frac{1}{b}\right]$$

(5). It only works for angles $\frac{2\pi}{N}$, $N \in \mathbb{N}$ and will have $N-1$ img charges distributed symmetrically around origin.

For other angles, symmetry will require, instead, a continuous (i.e., circular) img + real charge distribution, not allowed outside the conductor.