GF 2

(1)

(a) $2\pi s L E = \frac{\lambda L}{\epsilon_0}$ $\quad E(\vec{s}) = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$

$$V(s) = -\int_b^s \vec{E}\cdot d\vec{s} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{s}\right)$$

(b)

$$\frac{C}{L} = \frac{Q}{LV(a)} = \frac{\lambda L}{LV(a)} = \frac{\lambda}{\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}$$

(c) Field method

$$\frac{W}{L} = \frac{\epsilon_0}{2L}\int |E|^2 d\tau = \frac{\epsilon_0 \lambda^2}{8\pi^2\epsilon_0^2 L}\int_a^b \frac{2\pi s L\, ds}{s^2}$$

$$= \frac{\lambda^2}{4\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$$

Potential method

$$\frac{W}{L} = \frac{1}{2L}\sum q_i V_i = \frac{1}{2L}\left(\lambda L V(a)\right)$$

$$= \frac{\lambda^2}{4\pi\epsilon_0}\ln\left(\frac{b}{a}\right) \quad \checkmark$$

(2)

(a) $W = \frac{1}{2}\int_0^q V\,dq' = \frac{q^2}{8\pi\epsilon_0 b}$

{ Alt method, bring up q incrementally }

$W = \int_0^q \frac{q'\,dq'}{4\pi\epsilon_0 b} = \frac{q^2}{8\pi\epsilon_0 b}$

(b) $4\pi r^2 E = \frac{q}{\epsilon_0} \qquad E = \frac{q}{4\pi\epsilon_0 r^2}$

$W = \frac{\epsilon_0}{2}\int_b^\infty |E|^2\,d\tau = \frac{\epsilon_0}{2}\frac{q^2}{16\pi^2\epsilon_0^2}\int_b^\infty \frac{4\pi r^2\,dr}{r^4}$

$= \frac{q^2}{8\pi\epsilon_0}\int_b^\infty \frac{dr}{r^2} = \underline{\frac{q^2}{8\pi\epsilon_0 b}}$ (equal to work in (a)

(c) $V = \frac{q}{4\pi\epsilon_0 b} \qquad C = \frac{q}{V} = \underline{4\pi\epsilon_0 b}$

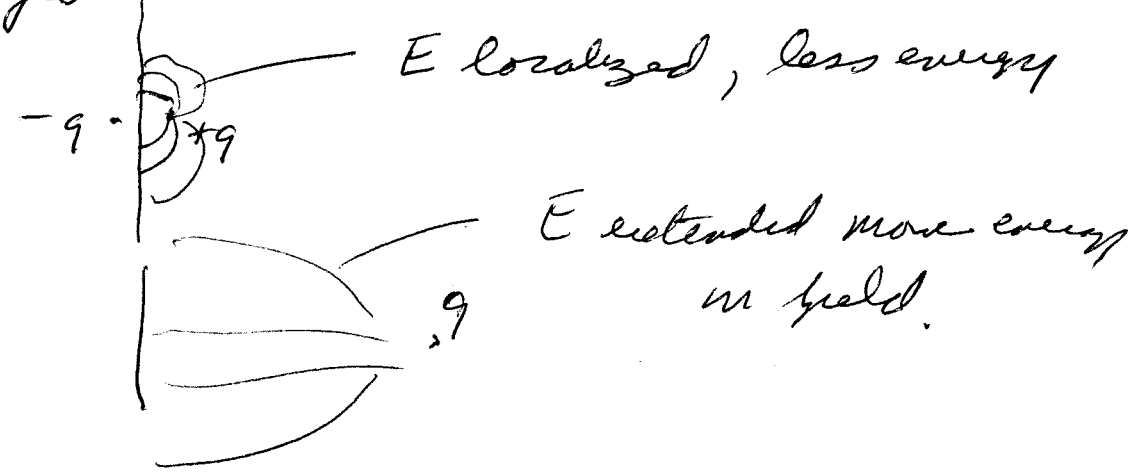
3. Use image $-q$ at $z = -b$

(a)
$$\vec{F} = \frac{-q^2}{4\pi\epsilon_0 (2b)^2}\hat{z} = \frac{-q^2}{16\pi\epsilon_0 b^2}\hat{z}$$

external force: $\int \vec{F}_{ext} \cdot d\vec{l} = -\int \vec{F} \cdot d\vec{l}$

(b)
$$W = -\int_{\infty}^{b} \frac{q^2 dr}{16\pi\epsilon_0 r^2} = \frac{-q^2}{16\pi\epsilon_0 b} < 0$$

$W < 0$ because less field energy when charges are close



(c) at $x = y = 0$
$$\vec{E} = -\frac{q\hat{z}}{4\pi\epsilon_0}\left(\frac{1}{(z-b)^2} + \frac{1}{(z+b)^2}\right)\hat{z}$$

3 (c) Cont'd.

$$\sigma = \epsilon_0 E_\perp \Big|_{z=0} = \frac{-2q}{4\pi b^2}$$