

Useful formulas:

For a sphere

$$V = (4\pi/3)R^3$$

$$A = 4\pi R^2$$

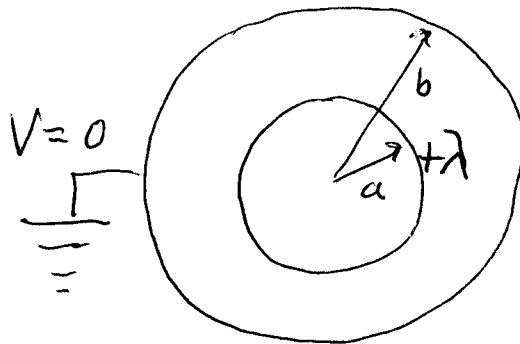
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}') \hat{r}}{r^2} d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{r} d\tau'$$

$$W = \frac{1}{2} \int \rho V d\tau$$

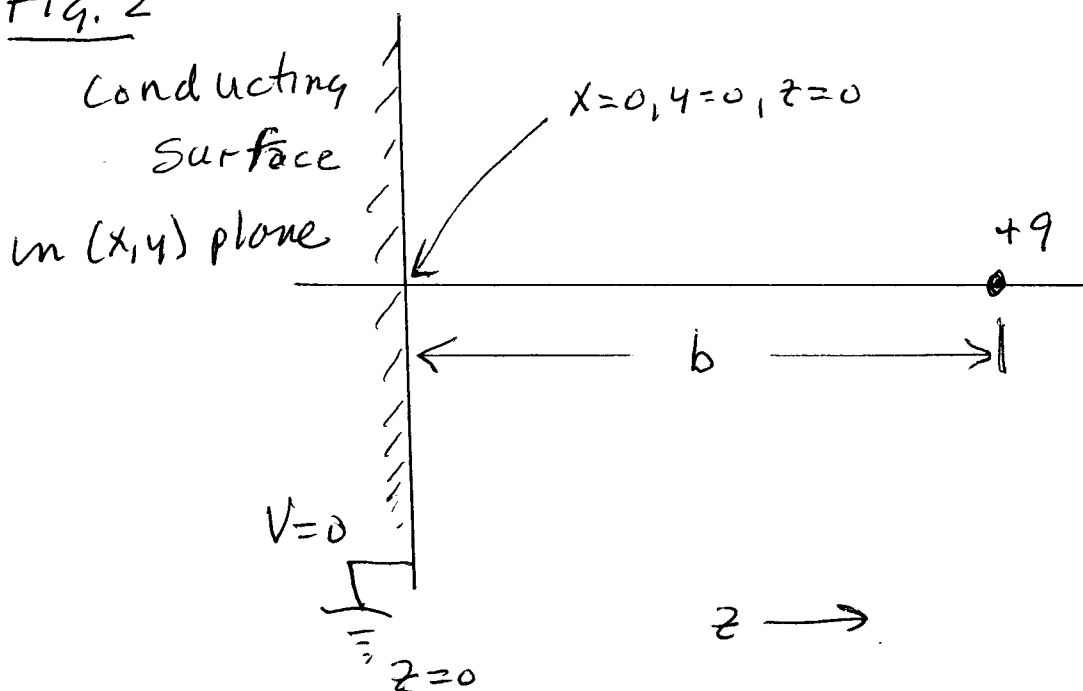
$$W = \frac{\epsilon_0}{2} \int E^2 d\tau$$

Fig. 1



cylindrical
 conductors
 length L out of plane
 $L \gg a, b$

Fig. 2



Please note: *Be sure to state clearly the reasoning behind your answers. Answers without explanation or supporting work will receive little or no credit.*

1. This problem relates to Fig. 1. Consider two concentric conducting cylinders of length L and radius a and b , as shown, where $L \gg a$ and $L \gg b$. The outer cylinder is grounded (i.e., at $V = 0$) and the inner one is charged with a charge per unit length $+\lambda$.
 - (a) Find the electric field $E(s)$ and the potential $V(s)$ in the region between the two conductors.
 - (b) Find the capacitance per unit length, C/L , for this configuration of coaxial cylinders.
 - (c) Find the electrostatic energy per unit length of conductor, W/L , required to establish this configuration. Use any method to do this that you'd like, but show your work clearly.

2. Consider a spherical conductor of radius b in free space with a charge $+q$.
 - (a) Find the work required to charge the sphere.
 - (b) Find the energy stored in the electrostatic field and compare your answer with that in (a).
 - (c) What is the capacitance of the sphere (i.e., assuming that the "second conductor" is at infinity)?

3. This problem relates to Fig. 2. A charge $+q$ is at position $x = 0, y = 0, z = b$, a distance $z = b$ from a grounded conducting surface oriented in the (x, y) plane and located at $z = 0$.
 - (a) What is the force of the conductor on the charge (magnitude and direction)?
 - (b) What is the work required to establish this configuration (magnitude and sign), starting with the charge $+q$ at $z = \infty$? Explain clearly the sign of your answer in terms of the change in the electrostatic field energy (i.e., why is the electrostatic field energy greater or less?).
 - (b) Find the electric field, $E(z)$, normal to the surface at $x = y = 0$, and use it to calculate the surface charge density (sign and magnitude) at that point on the surface.