C. Surko
100A

MT 1 - Solutions Fall '08

(1) (a) $\vec{\nabla} \times \vec{F}_1 = \text{"det"} \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ z^2 - ay & -ax & 2xz \end{pmatrix}$

$= \hat{x}(0) - \hat{y}(2z - 2z) + \hat{z}(-a + a) = \underline{\underline{0}}$

(b) $\nabla \cdot F_1 = \underline{2x}$

(c) Check $\vec{\nabla} \times \vec{F}_2 = \text{"det"} \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ z^2 - ay & -2ax & 2az \end{pmatrix}$

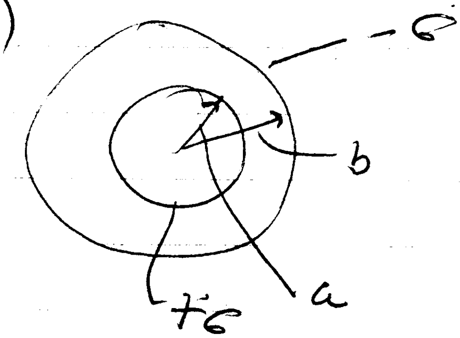
$= \hat{x}(0) - \hat{y}(-2z) + \hat{z}(-2a + a) = 2z\hat{y} - a\hat{z} \neq 0$

If we could write $F_2 = \nabla V$, then

$\vec{\nabla} \times \vec{F}_2 = \vec{\nabla} \times \vec{\nabla} V = 0$.

It isn't zero, so no, we cannot.

(2)



(a) $\cancel{4\pi r^2} E(r) = \dfrac{Q_{enc.}}{\epsilon_0} = \dfrac{4\pi\sigma(a^2-b^2)}{\epsilon_0} \quad r \geq b$

$= \dfrac{4\pi\sigma a^2}{\epsilon_0} \quad a \leq r \leq b$

$= 0 \quad r < a$

$E(r) = \dfrac{\sigma(a^2-b^2)}{\epsilon_0 r^2} \quad r \geq b$

$= \dfrac{\sigma a^2}{\epsilon_0 r^2} \quad a \leq r \leq b$

$= 0 \quad r \leq a.$

(b) $V(r) = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q_{encl.}}{r} + \cancel{\text{const.}} \to 0$

$r \geq b \quad V(r) = \dfrac{\sigma(a^2-b^2)}{\epsilon_0 r} < 0$

$b \geq r \geq a \quad V(r) = \dfrac{\sigma(a^2-b^2)}{\epsilon_0 b} + \dfrac{1}{4\pi\epsilon_0}\displaystyle\int_b^r \dfrac{4\pi\sigma a^2}{r^2}\,dr$

MT-1

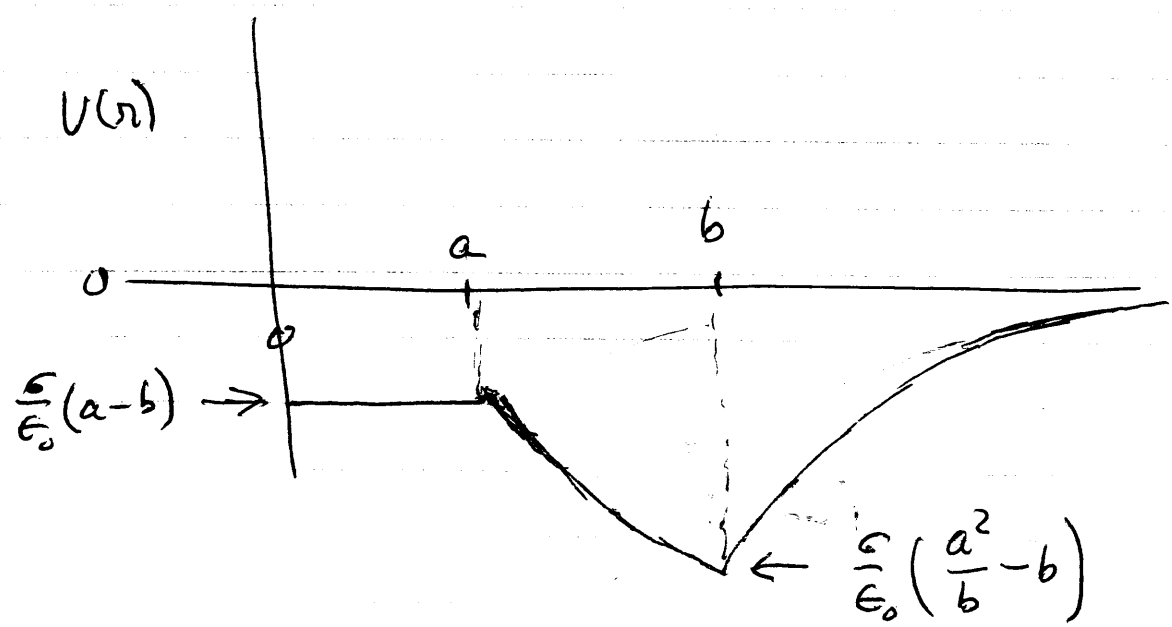
(2) cont'd

$$V(r) = \frac{\sigma(a^2-b^2)}{\epsilon_0 b} + \frac{\sigma a^2}{\epsilon_0}\left(\frac{1}{r} - \frac{1}{b}\right)$$

$$= \underline{\frac{\sigma}{\epsilon_0}\left(\frac{a^2}{r} - b\right)} \leftarrow < 0 \qquad (a \le r \le b)$$

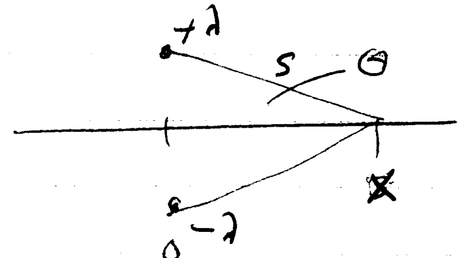
$r \le a$ $\quad V(r) = \text{Const} = \underline{\frac{\sigma}{\epsilon_0}(a-b)} < 0$

(c)



(3) One line charge w/ $+\lambda$

(a) $2\pi s L E = \frac{\lambda L}{\epsilon_0}$ $\quad E = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$

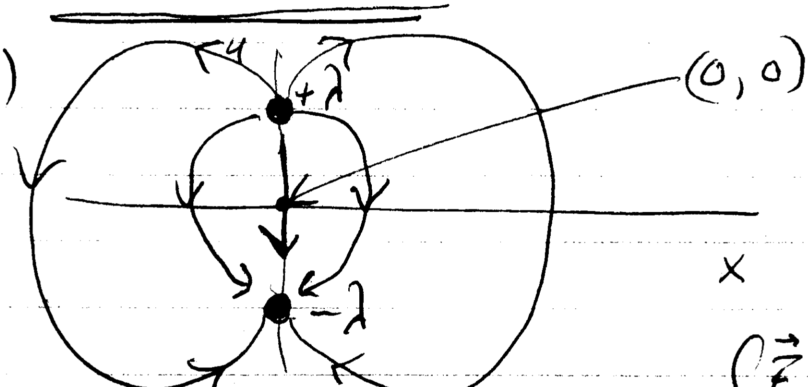


$\sin\theta = \frac{a}{\sqrt{x^2+a^2}}$

$$\vec{E} = \frac{2\lambda\sin\theta}{2\pi\epsilon_0 s}\hat{y}$$

$$= \frac{\lambda a}{\pi\epsilon_0(x^2+a^2)}\hat{y}$$

(b)



$= -\int \vec{E}\cdot d\vec{s}$

(c) $V(\vec{r}) = \frac{-\lambda}{2\pi\epsilon_0}\ln s_+ + \frac{\lambda}{2\pi\epsilon_0}\ln s_- + \text{const}$

$s_+ = \sqrt{(y-a)^2+x^2} \qquad s_- = \sqrt{(y+a)^2+x^2}$

$$V(\vec{r}) = \frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{x^2+(y+a)^2}{x^2+(y-a)^2}\right] + \cancel{\text{const}} \to 0$$

since $V(0) = 0$ without it.

On the x axis

$V(x,0) \propto \ln(1) = \underline{0}$