Lecture 22

Relativistic Cosmology

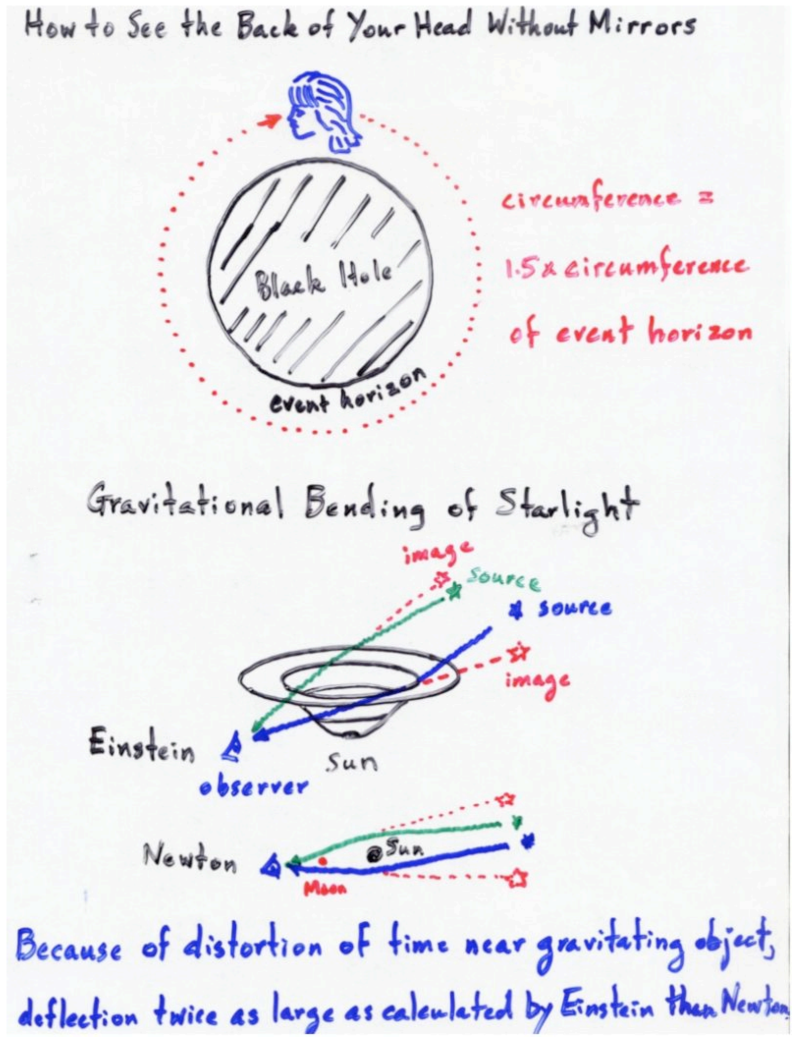
Outline of Lecture 22

- Einstein’s static cosmological model with a nonzero cosmological constant Λ:
  - Time-dependent generalizations by Friedmann and Lemaitre
  - Repudiation of Λ after Hubble discovers expansion of universe
- Relativistic cosmological models and their Newtonian analogs (when Λ= 0):
  - Bound ↔ Closed; Critical ↔ Flat; Unbound ↔ Open
  - Great lesson from relativistic cosmology: The passage of time and the amount of space in the universe is not given a priori, but is manufactured by the action of gravitation.
- Modern reintroduction of cosmological constant:
  - Reinterpretation as positive energy density and negative pressure of the quantum vacuum
  - Required by measurement of recent acceleration of the universe
  - Evidence that we live in a flat (Euclidean) universe composed currently of 4% ordinary matter, 22% dark matter (matter that is not part of periodic table), and 74% dark energy (vacuum that is not nothing).

Gravitation and Cosmology

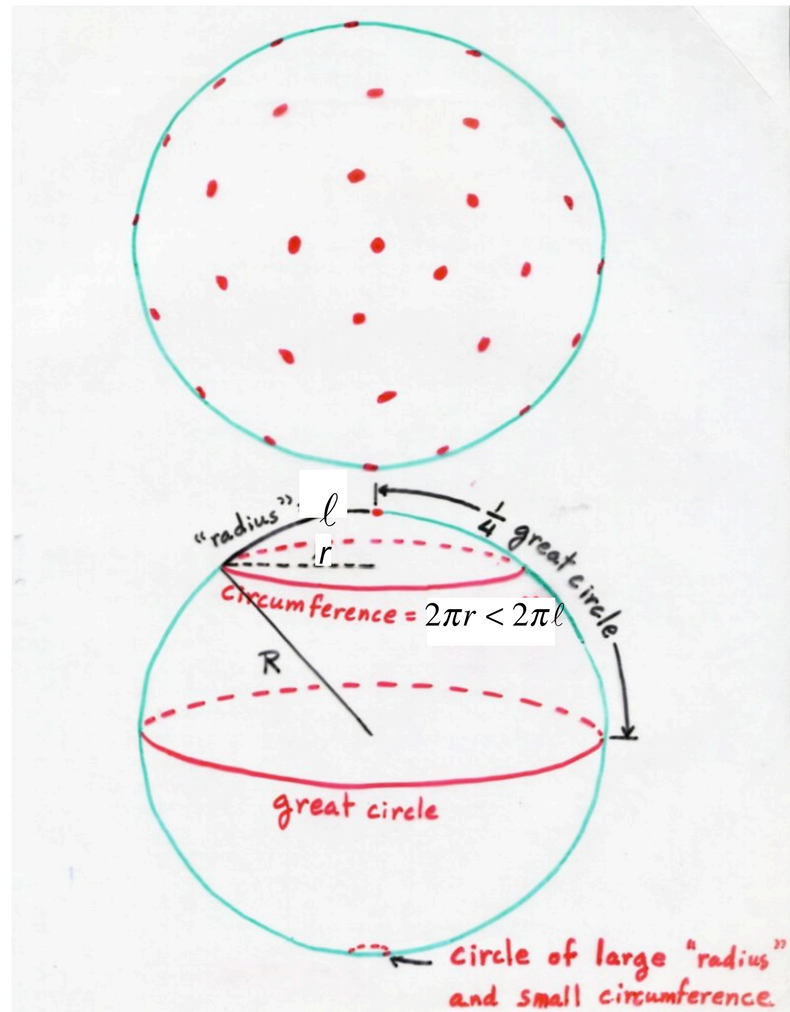
- In 1916, Einstein replaced Newton's conception of gravitation as a force with general relativity, which views gravitation as the dynamics of spacetime.
- In 1917 he applied his theory to the universe as a whole. (Recall that this is before the Great Debate when most scientists thought the "fixed stars" constituted the whole universe.) He made two assumptions: the universe is homogeneous on average and static; and it is closed on itself, a curved volume of space with no boundary (a scientific return back to Aristotle's conception of the cosmos).
- However, Einstein found that his equations have no such solutions unless an extra term is inserted that acts as a repulsion to offset the gravitational attraction of matter for itself. Thus were born both modern cosmology and the notion of a "cosmological constant" Λ.
- Cannot visualize uniform space curvature in 3-D because we are not 4-D creatures. Can make analogy with uniform space curvature in 2-D, which we can visualize because we are 3-D creatures. Consider then the world of ants.

Static Space Curvature in Other Contexts:

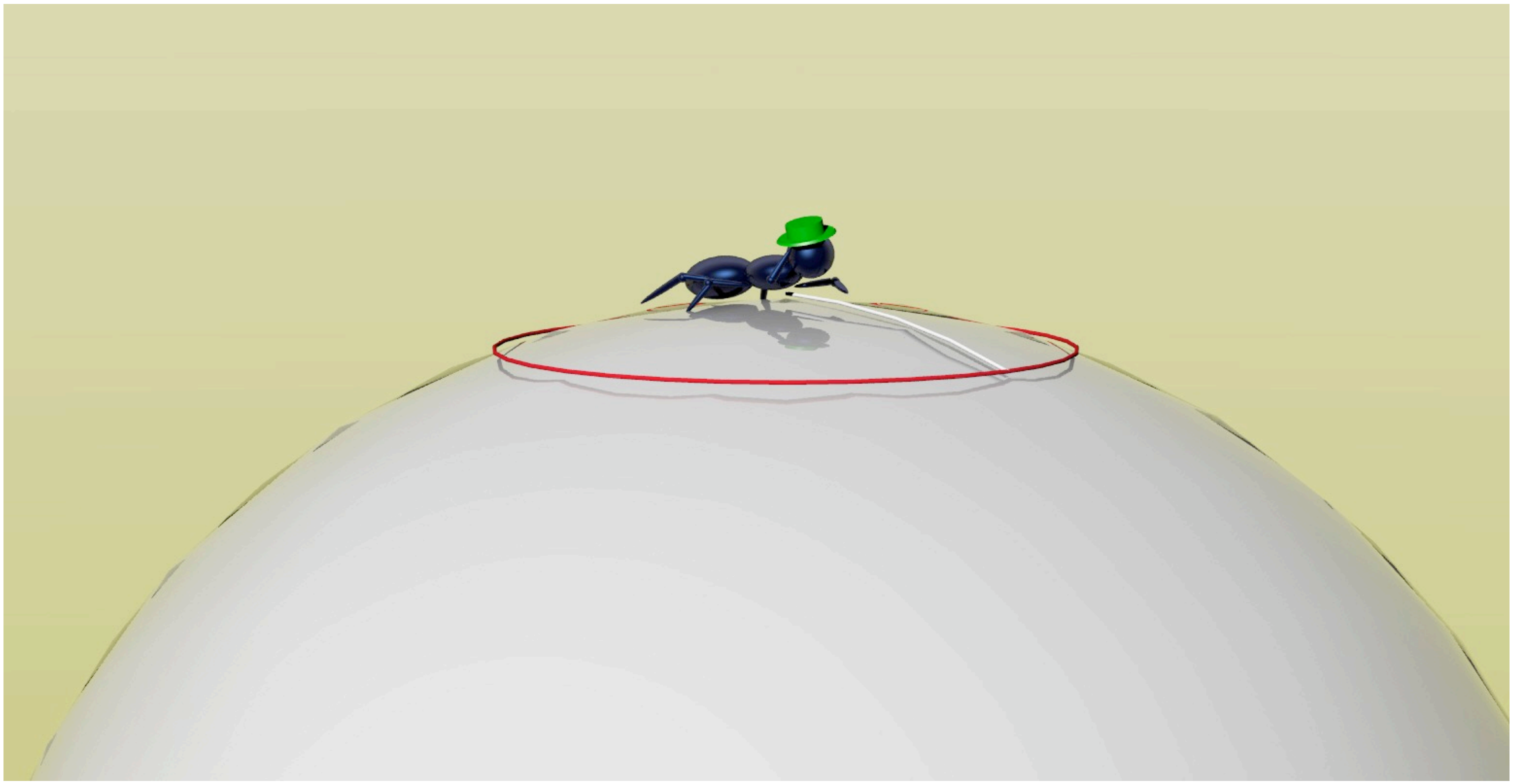


Ant Analogy of Closed Universe

- Ant cities (galaxies) distributed homogeneously on average on the curved surface of the ant world.
- This world has a finite surface area, yet no boundary. It also has no center (on the surface).
- Radius of curvature *R* for surface of sphere lies in third dimension, a direction that has no meaning for ants.
- For them, circles are drawn when one pulls a string of length ℓ ('radius of the circle") taut (against the surface of the world) and walks around with the other end tacked down.

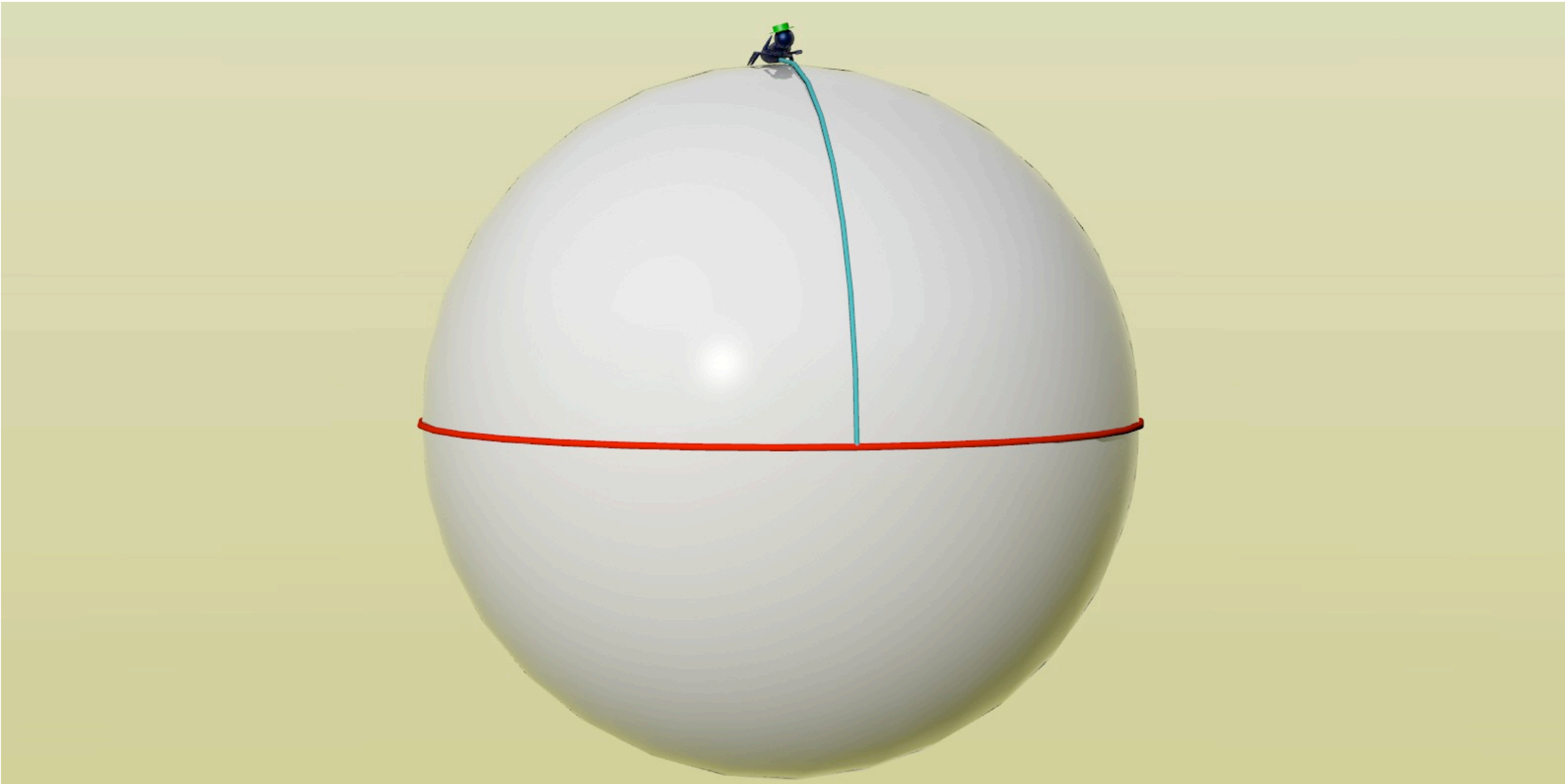


Circle of Small Radius in Ant World



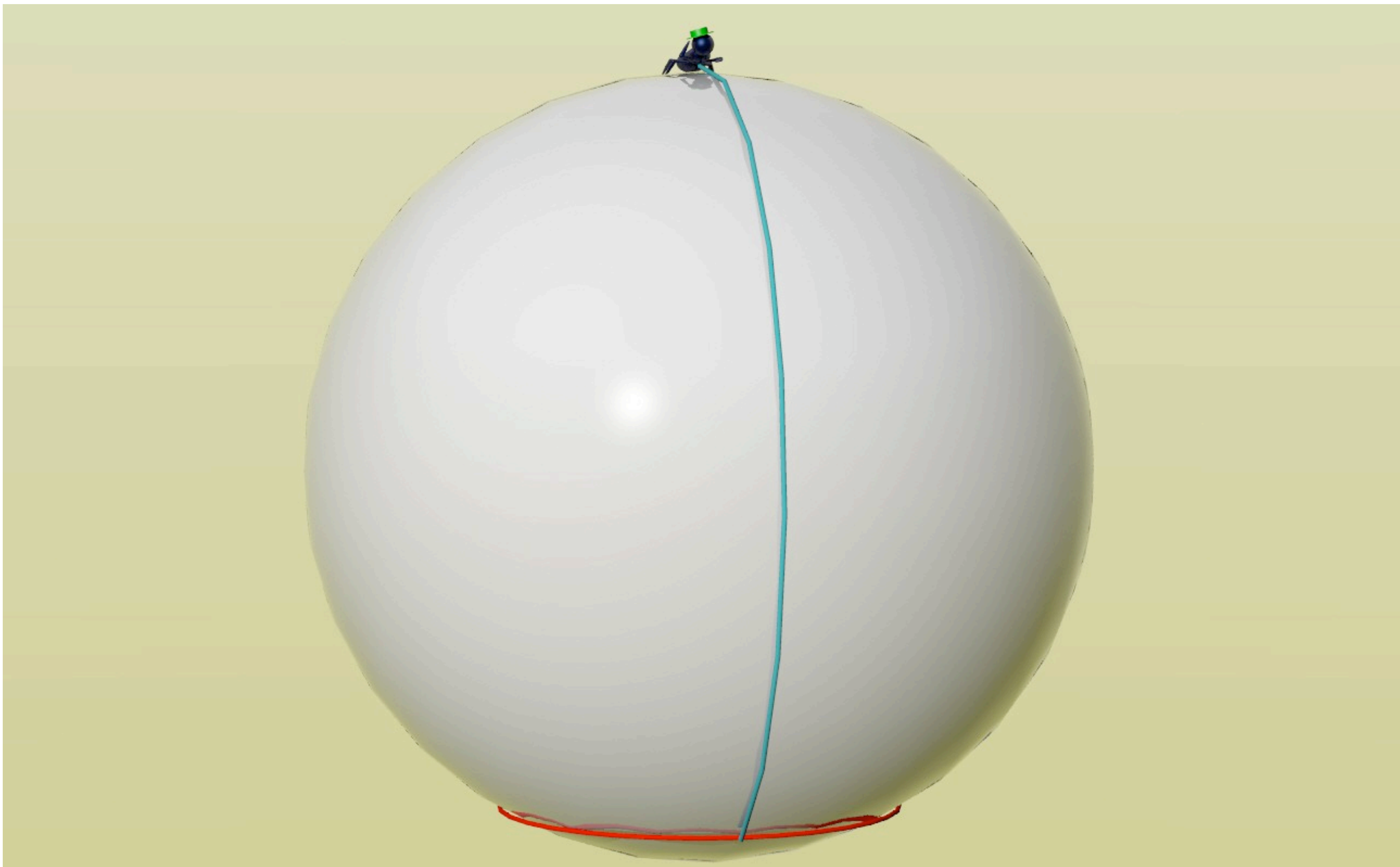
Circumference = $2\pi \cdot$ radius: $C = 2\pi\ell$ when $\ell << R$.

Circle of Moderate Radius in Ant World



Circumference = 4 · radius: $C = 4\ell$ when $\ell = \pi R / 2$.

Circle of Large Radius in Ant World



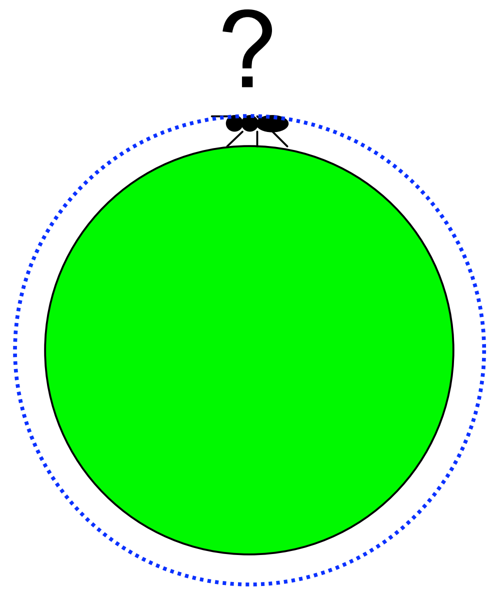
Circumference $\rightarrow$ $0 \cdot$ radius: $C \rightarrow 0 \cdot \ell = 0$ when $\ell \rightarrow \pi R$.

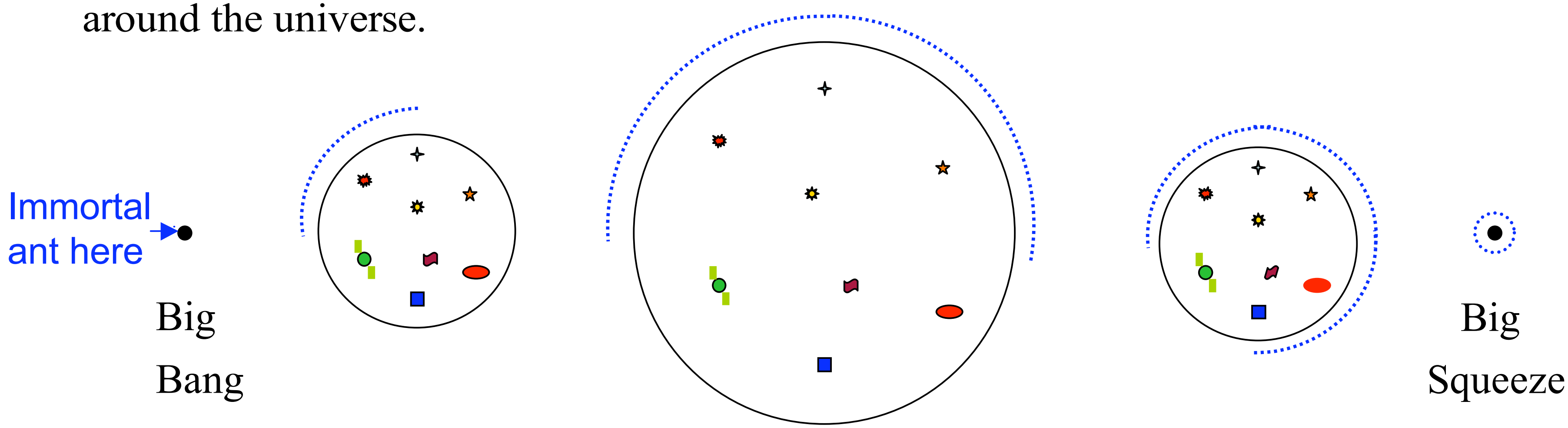
Closed Ant World

Since ant light travels on geodesics (great circles), can an ant see its own rear by looking straight ahead? This would be much simpler than actual surveying.

Complication 1: Ant does not live long enough for light to go all the way around the universe.

Complication 2: Even if the ant is immortal, because the ant world began with a big bang, the ant world may not live old enough for the light to go all the way around the universe.

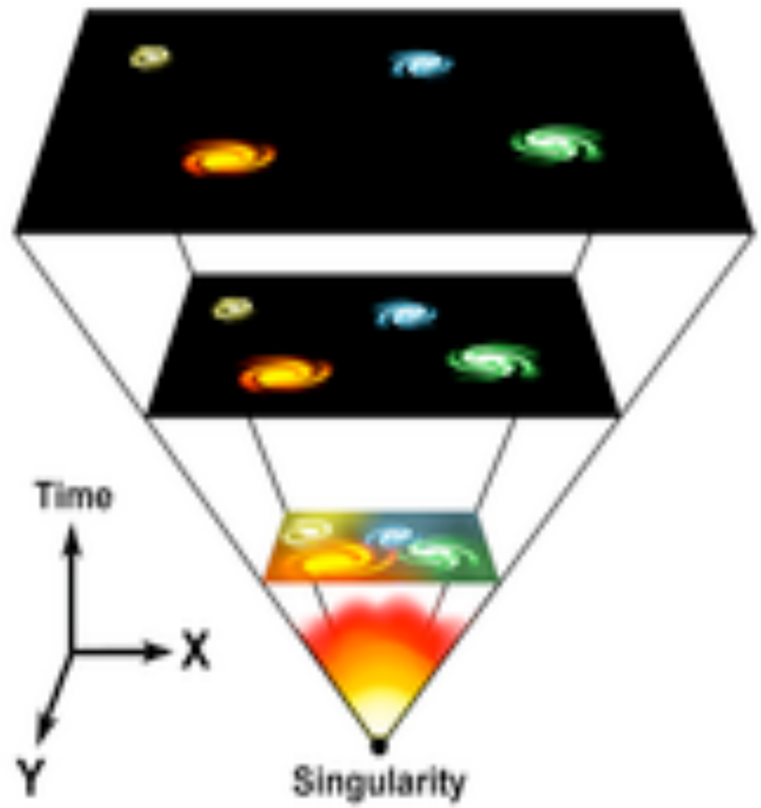




Expansion and recontraction of closed ant world

Time-Dependent Cosmologies

- In early 1920's, Aleksandr Friedmann (1888-1925) & Georges Lemaitre (1894-1966) show that Einstein's equations have solutions, with or without a cosmological constant Λ, that are time-dependent.
- With the assumption of the cosmological principle (that the universe is homogeneous and isotropic), the universe generally has to start with expansion from a big bang at $t = 0$ when universe had a vanishingly small scale factor and 3-D volume.
- In 1929 Hubble found that the universe *is expanding*, a feature that Friedmann and Lemaitre had shown were necessary consequences of Einstein's equations if Λ were zero.
- Realizing that he might have predicted the expansion of the universe himself had he not supposed a nonzero Λ, Einstein repudiated its introduction as "the biggest blunder of his life."



Wikipedia

Newtonian & Relativistic Cosmologies (extra material)

- Newton plus Birkhoff's Rule for small mass *m*:

$$E = \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} = \text{const} \quad \text{with} \quad M = \rho \frac{4\pi}{3} r^3 = \text{const.}$$

$$\frac{1}{r^2}\left[\left(\frac{dr}{dt}\right)^2 - \frac{2E}{m}\right] = \frac{8\pi G}{3}\rho \quad \text{with} \quad E <,=,> 0 \quad \text{as bound, critical, unbound.}$$

- Relativistic (Friedmann-Lemaitre) equations for scale factor *a*:

time curvature space curvature

$$\frac{1}{a^2}\left[\left(\frac{da}{dt}\right)^2 + kc^2\right] = \frac{8\pi G}{3}\rho \qquad \text{with} \quad k = +1, 0, -1 \text{ as closed, flat, open.}$$

$$\text{First law of thermodynamics with zero entropy change}: \frac{d}{dt}\left(\rho c^2 a^3\right) = -P\frac{d}{dt}\left(a^3\right)$$

- For *E* = 0 or *k* = 0, $H^2 = 8\pi G\rho/3$.
- Special cases:
  - Cold matter: *P* = 0 → ρ*a* ^{3}= const → $\rho = C/a^3$.
  - For universe dominated by cold matter in which *k* = 0,

$$a \propto t^{2/3} \quad \text{since } a = 0 \text{ when } t = 0 \Rightarrow H \equiv \frac{\dot{a}}{a} = \frac{2}{3t} \Rightarrow t = \frac{2}{3}H^{-1}.$$

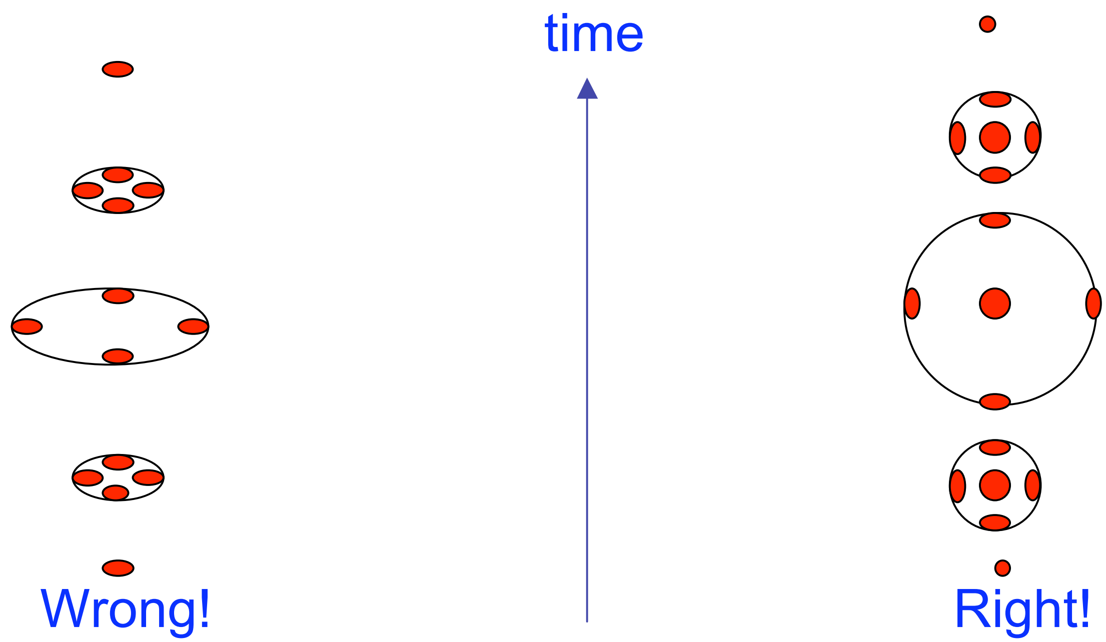
Cosmological Models

Density Compared To Critical Value	Newtonian Models	Relativistic Models
$\rho > \frac{3H^2}{8\pi G}$	Bound	Closed
$\rho = \frac{3H^2}{8\pi G}$	Critical	Flat (Euclidean)
$\rho < \frac{3H^2}{8\pi G}$	Unbound	Open

- In Newtonian models, all three cases, bound, critical, unbound, have infinite amount of space and infinite number of galaxies.
- In relativistic models, amount of space is infinite only for flat & open models. In closed model, volume of space and number of galaxies are finite.
- How can space be finite without coming to an edge where cosmological principle does not apply?

3-D space on the large scale can be curved (in an artifical "fourth dimension" that we do not experience), fooling us in the same way that ants are fooled by a large, round, rather than flat, Earth. Physical 3-D space need not satisfy Euclidean geometry on the global scale of the universe. Experiments are needed to tell whether the actual universe is flat.

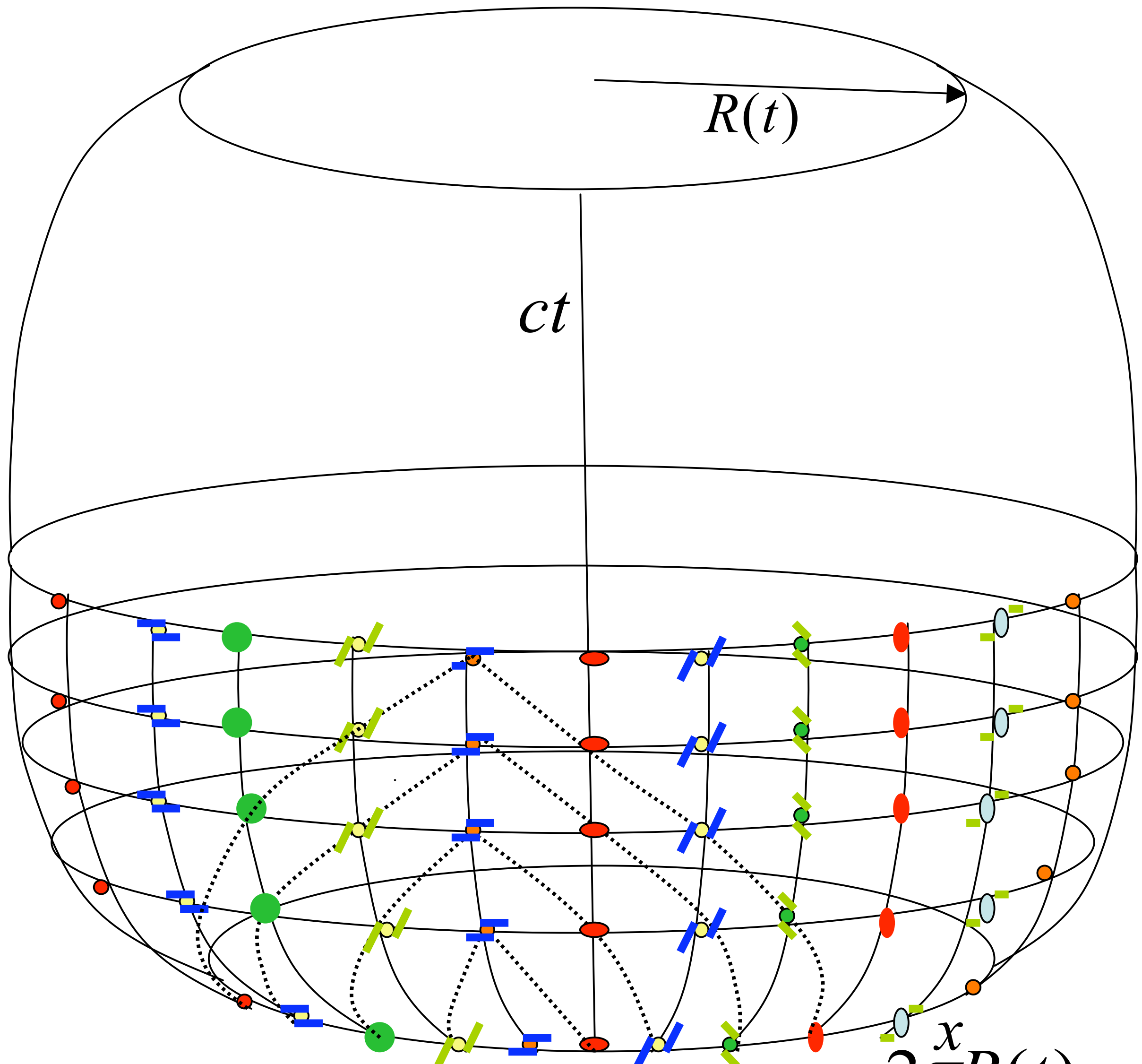
Common Ant World Misconceptions



- Expansion of ant world (closed universe) takes place not along observable two (three) spatial directions, but by world (universe) being carried in time to a new 2-D surface (3-D volume) in an unobservable third (fourth) "spatial" dimension.
- There is no preexisting 2-D area (3-D volume) into which the ant world (universe) expands. The evolution of space available is governed by Einstein's theory of gravitation (general relativity).
- In a crucial sense, therefore, the fabric of spacetime is manufactured by the gravitation of all the energy & stress existing in the universe. In particular, time began with the big bang; there was no "before the big bang."

Curved Spacetime in a Closed Universe

Galaxies on average remain stationary in space as they are carried forward (upward) in time.

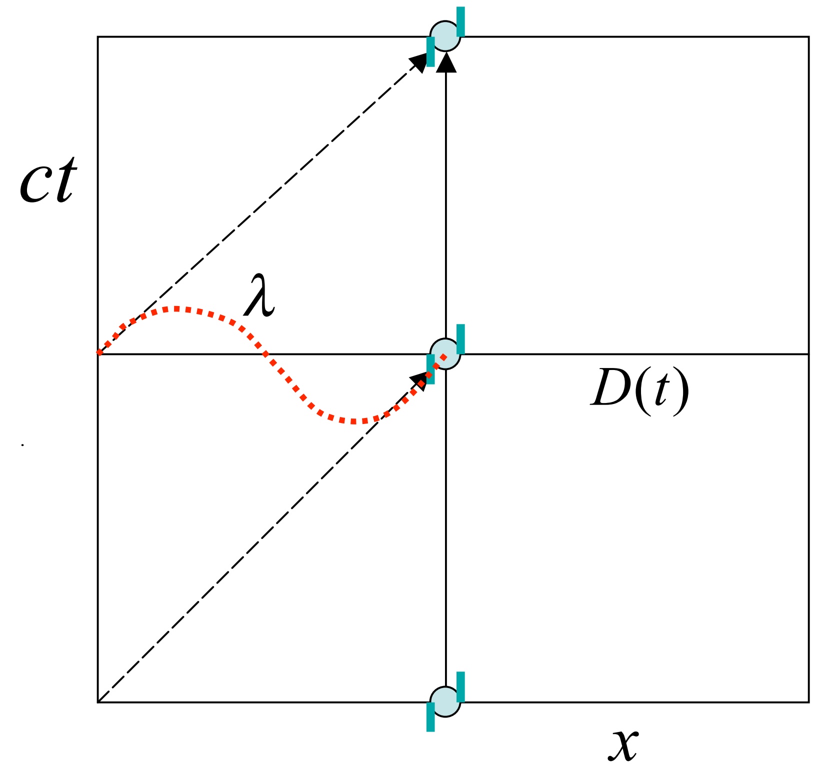


$$N \text{ galaxies in } x \text{ direction} \Rightarrow D(t) = \frac{2\pi R(t)}{N}$$

Galaxies are separating not because they have *velocities* with respect to each other, but because more space is being added between them. There is no limit to how fast space can be added (or subtracted) in time. This removes the objection that galaxies at large separations seem to be moving faster than the speed of light with respect to each other. There is no actual *motion*!

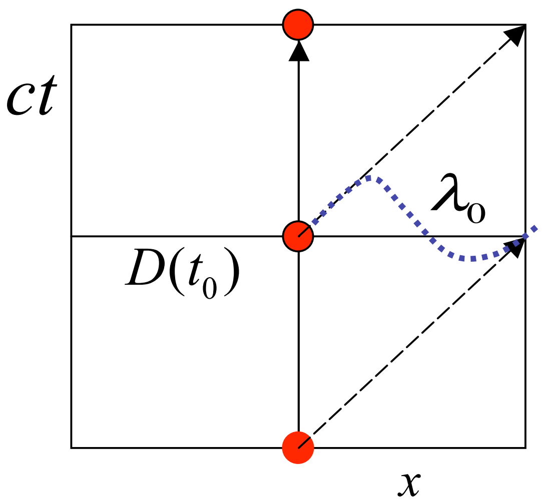
Cosmological Redshift

$$\frac{\lambda}{\lambda_0} = \frac{D(t)}{D(t_0)} \equiv 1 + z$$



Reception at
later epoch t

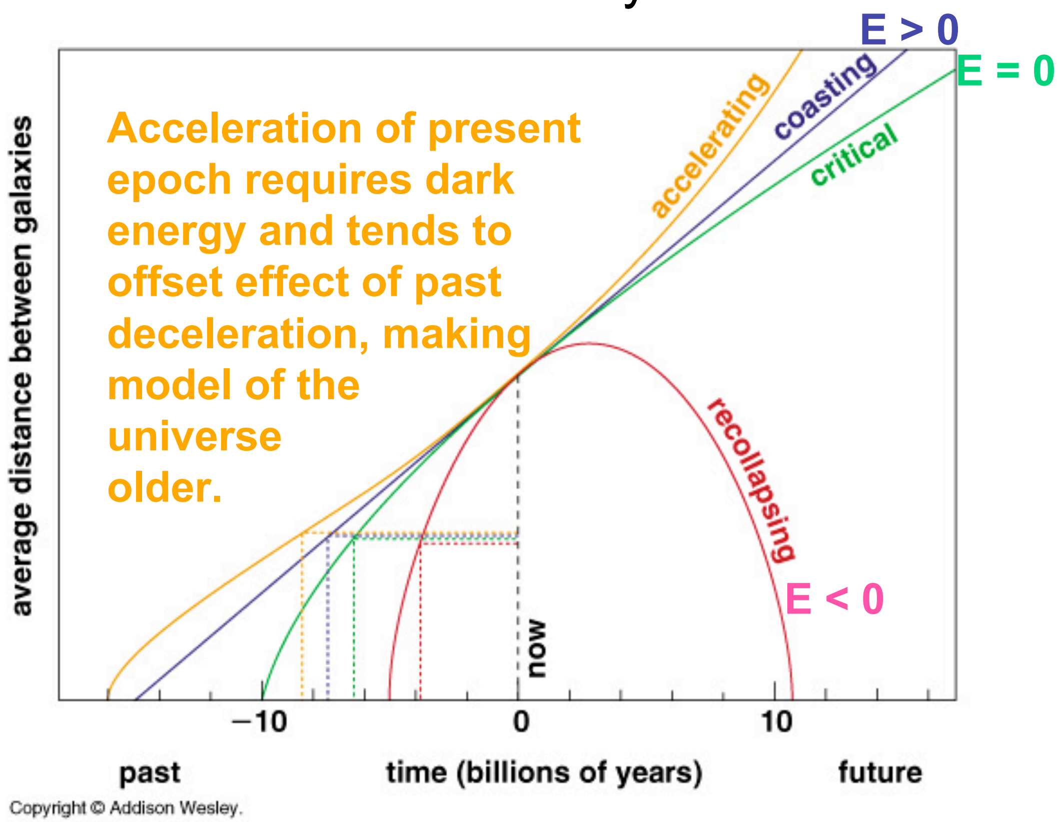
Transmission at
earlier epoch t_0



$D(t_0) =$ average spacing between
galaxies at time t_0

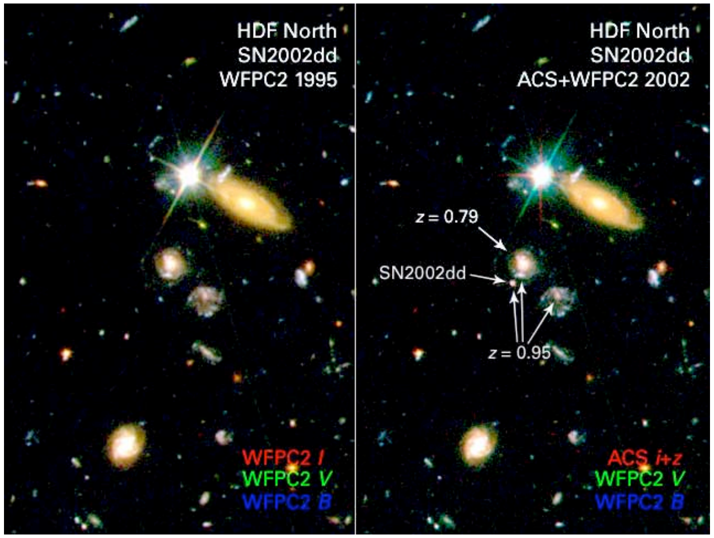
Problem with Flat Universe

- The age of a conventional flat universe, which is supported by observations of fluctuations in the cosmic microwave background (Lecture 23), equals 9 billion years, which is in conflict with the age, 13 billion years, of the oldest stars (see Lecture 21).
- An escape is to suppose that the vacuum has "dark energy" additional to that contained in ordinary matter and dark matter.



Accelerating Universes and Dark Energy

- Evidence for such acceleration has been claimed in the data of two teams concerning the brightness of distant supernovae.
  - Data favors universe in which dark energy is about 74% of the critical density.
  - Dark matter is matter that does not interact electromagnetically with light. Dark energy is what exists uniformly in space even in the absence of matter and radiation.
  - Combined with the CMB evidence that the universe is flat (Lecture 23), this means ordinary matter plus dark matter constitutes only about 26% of the critical density, in rough agreement with other estimates (galaxy rotation curves & primordial nucleosynthesis; Lecture 23).
- This combination is just right to fix cosmological age problem!

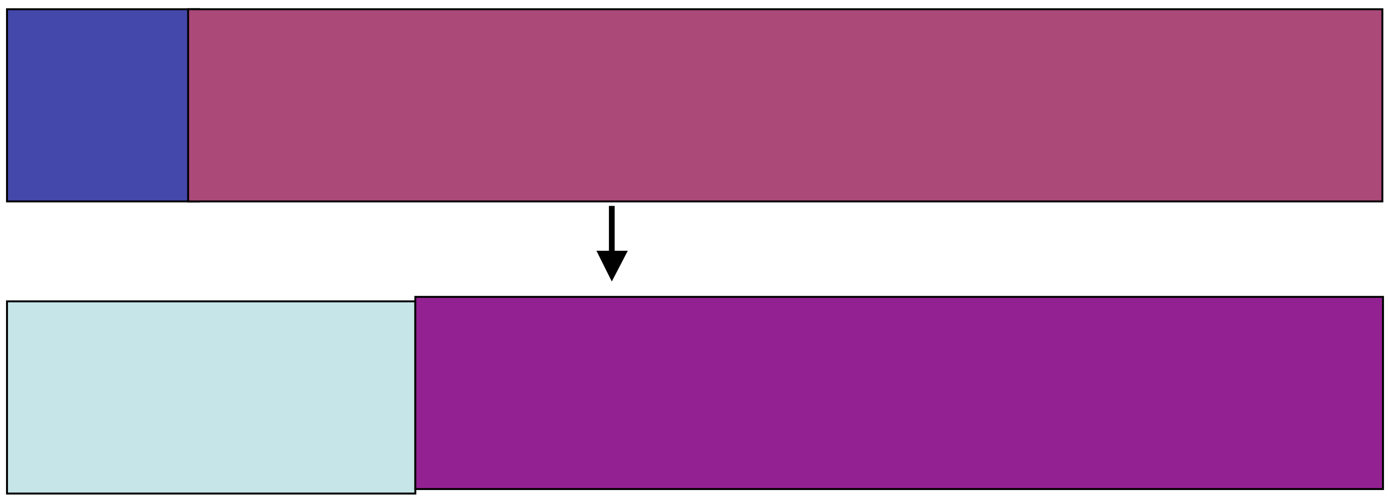


Apparent brightness of distant SN Ia as a function of redshift z depends on cosmological model.

NASA/ACS

Why Expanding Gases Cool

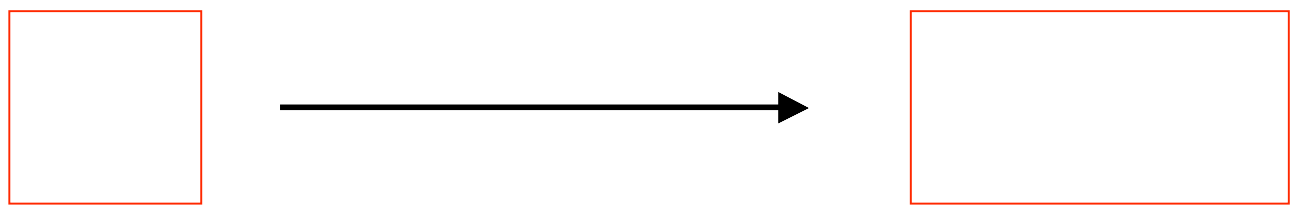
- Expansion of ordinary gas into external medium:



- Blue gas loses energy (and cools down) because it does work pushing against the pressure of the external purple gas.

Why Vacuum with Positive Energy Has Negative Pressure

- Expansion of box of vacuum:



- If the vacuum has positive energy density, the expanded box has twice the volume and therefore twice the energy of the original box. Where did the extra energy come from? The box must have done negative work against the pressure of the surrounding vacuum; i.e., the surrounding vacuum must have negative pressure!
  – Because pressure also acts as a source of gravitation, it acts like repulsive gravity (same effect as "cosmological constant")!
  – The net gravitation associated with dark energy, or a positive cosmological constant, then leads to an acceleration of the universe, not to a deceleration.
  – Einstein's "greatest blunder" may be his greatest triumph!

Equation of State (extra material)

- First law of thermodynamics, $\frac{d}{dt}\left(\rho c^2 a^3\right) = -P\frac{d}{dt}\left(a^3\right)$.
- Equation of state: $P = w\rho c^2$.
- With w = const, solution is $\rho \propto a^{-3(1+w)}$.
- Special cases:
  - Cold matter (ordinary or dark): $w = 0 \Rightarrow \rho = C / a^3$.
  - Radiation (or massless neutrinos): $w = 1/3 \Rightarrow \rho = D / a^4$.
  - Vacuum energy (cosmological const): $w = -1 \Rightarrow \rho = \mathrm{const} \equiv \rho_{\mathrm{v}}$.
  - Quintessence (ad hoc theoretical construct): $-1/3 > w > -1$.
- WMAP: $w < -0.78$ for dark energy, consistent with $w = -1$.
- Physical interpretation of $\rho_{\mathrm{v}} = const$, $P_{\mathrm{v}} = -\rho_{\mathrm{v}} c^2$.

Implications of Positive Cosmological Constant ↔ Vacuum Energy (extra material)

- Friedmann & Lemaitre equations for scale factor *a*:

$$\frac{1}{a^2}\left[\left(\frac{da}{dt}\right)^2 + kc^2\right] = \frac{8\pi G}{3}\rho \text{ with } k = +1,\ 0,\ -1 \text{ as closed, flat, open.}$$

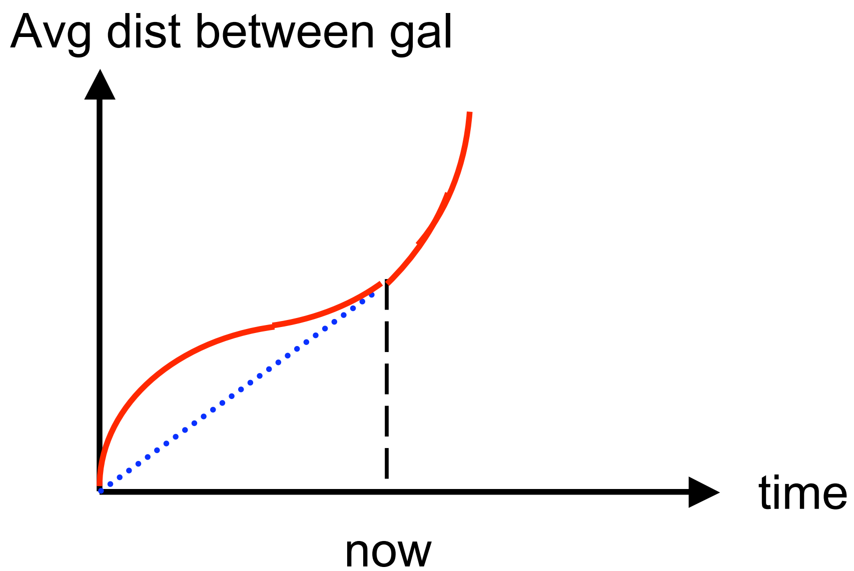
- EOS of mixture of matter, radiation, and vacuum:

$$\rho = \rho_{\mathrm{m}} + \rho_{\mathrm{r}} + \rho_{\mathrm{v}} = \frac{C}{a^3} + \frac{D}{a^4} + \rho_{\mathrm{v}}.$$

- For $k = 0$ and large *a*,

$$\frac{1}{a}\frac{da}{dt} = \pm\sqrt{\frac{8\pi G\rho}{3}} \Rightarrow a \propto \exp\left(+\sqrt{\frac{8\pi G\rho_{\mathrm{v}}}{3}}\,t\right) \text{ for large } t.$$

Flat Universe with Cosmological Constant



Vacuum density dominates at present, but matter density (ordinary and dark) dominates in past (after brief era of radiation dominance).

- Universe is presently accelerating (roughly exponentially).
- Universe was decelerating in the past (roughly as a 2/3 power law).
- In backwards extrapolation from present, effect of current acceleration just cancels effect of past deceleration! By pure coincidence, age of universe $\approx 1/H_0$ if the critical density consists (today) of about 26% ordinary plus dark matter and 74% dark energy (positive “vacuum energy” or “cosmological constant”).
- Best value for age of universe: 13.7 billion years.

Conflict Between Cosmology and the "Theory of Everything"? (extra material)

- Dark energy as vacuum energy, or a nonvanishing cosmological constant, may violate some of the most promising new developments in fundamental physics concerning string theory and supersymmetry.
- Since string theory is by consensus the best candidate for the "theory of everything," it is extremely disturbing that it is in seeming conflict with observational cosmology, which is literally the "observation of everything."
- Something is rotten in cosmology/particle physics, which represents a huge opportunity for progress.

- Magnitude of difficulty
  - Observed value:

    $$\rho_{\mathrm{v}} \approx 6 \times 10^{-27}\ \mathrm{kg\ m^{-3}}.$$
  - Natural value:

    $$\rho_{\mathrm{P}} \equiv \frac{c^5}{G^2 \hbar} \approx 5 \times 10^{96}\,\mathrm{kg\ m^{-3}}.$$
  - Ratio:

    $$\frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{P}}} \approx 10^{-123}.$$
- Where does such a small number, not exactly zero, come from?

Summary: Great Lessons from Modern Cosmology

- The passage of time and the amount of space in the universe is not given a priori, but is manufactured by the action of gravitation.
- Thus, not only does the instant of the big bang correspond to the creation of the entire mass-energy content of the universe, but also to the beginning of space and time.
- The geometry of space appears to be flat, i.e., given by Euclidean intuition and lying at the boundary between open and closed. This is surely an important clue to how the universe began. The best current idea is exponential inflation from some quantum-mechanically small state (Lecture 24).
- Current cosmological models require the critical density today to be composed of about 4% ordinary matter, 22% dark matter, and 74% dark energy. This combination is a very puzzling result, indicating that there are unsolved fundamental problems in cosmology/particle physics that may require a new generation of scientists to resolve.