Lecture 2

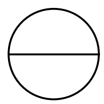
Grecian Scale of the Cosmos

Outline of Lecture 2

- Size of Earth, Moon, Sun
- Distance of Moon and Sun
- Heliocentric versus Geocentric View
  - Aristarchus
  - Archimedes
  - Aristotle

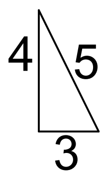
Geometry:
Science of Measuring the Land

- Egyptians:



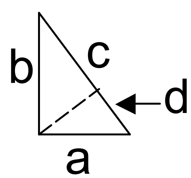
Circumference/Diameter = $\pi \approx 25/8 = 3.125$

(Actually, $\pi = 3.14159\ldots$ is not a rational number.)



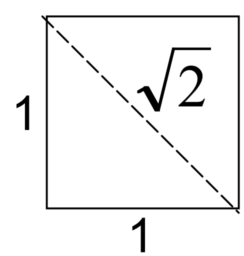
$$3^2 + 4^2 = 5^2, \text{ i.e., } 9 + 16 = 25$$

- Pythagoras (590-497 BC): $a^2 + b^2 = c^2$



$$\frac{a}{d} = \frac{c}{a} \Rightarrow a^2 = cd. \quad \frac{c}{b} = \frac{b}{c-d} \Rightarrow c^2 - cd = b^2 \Rightarrow c^2 = a^2 + b^2.$$

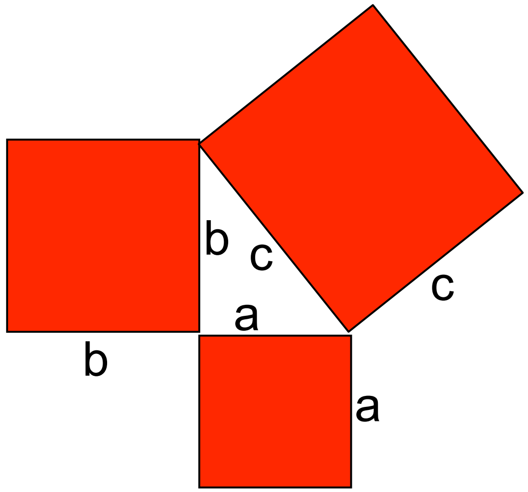
- Pythagoras: “All things are numbers.”
- Rise of the irrational:



$$\sqrt{2} \neq \frac{p}{q} \quad \text{where } p, q = \text{integers}$$

Eclipse of Algebra by Geometry Sets Science Back a Thousand Years

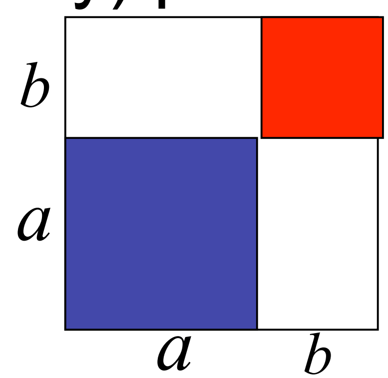
- Euclid's proof of Pythagoras's theorem:



The square on the hypotenuse equals the sum of the squares on the two sides of a right triangle.

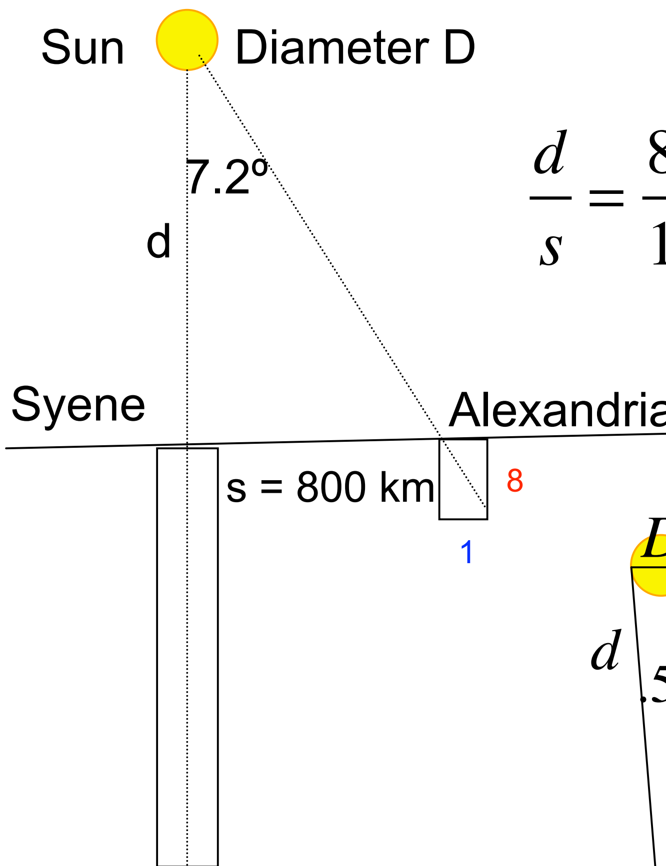
- Al Kowarizimi (ca. ninth century) proof of

$$(a+b)^2 = a^2 + b^2 + 2ab$$



Incorrect Distance to Sun

Anaxagoras (500-428 BC):



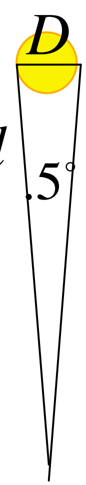
$$\frac{d}{s} = \frac{8}{1}$$

$d = 8s = 6400$ km,

closer to Greece than China!

Use small-angle formula;

see next slide.

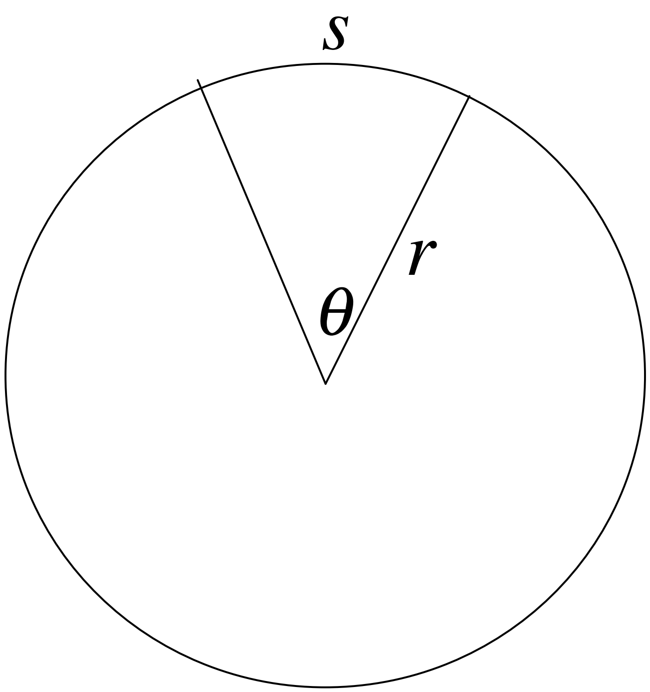


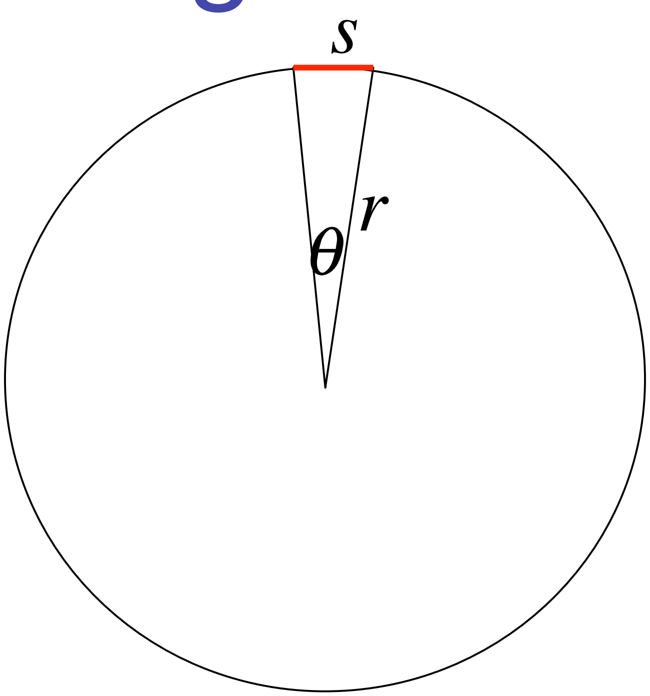
$$\frac{D}{d} = 0.5^{\circ} \times \frac{\pi \text{ rad}}{180^{\circ}} = 0.0087 \Rightarrow D = 56 \text{ km.}$$

Sun is smaller than Peloponnesus!

Anaxagoras concludes Sun is just a big flaming stone, consistent with his theory that hot meteorite fell from Sun.

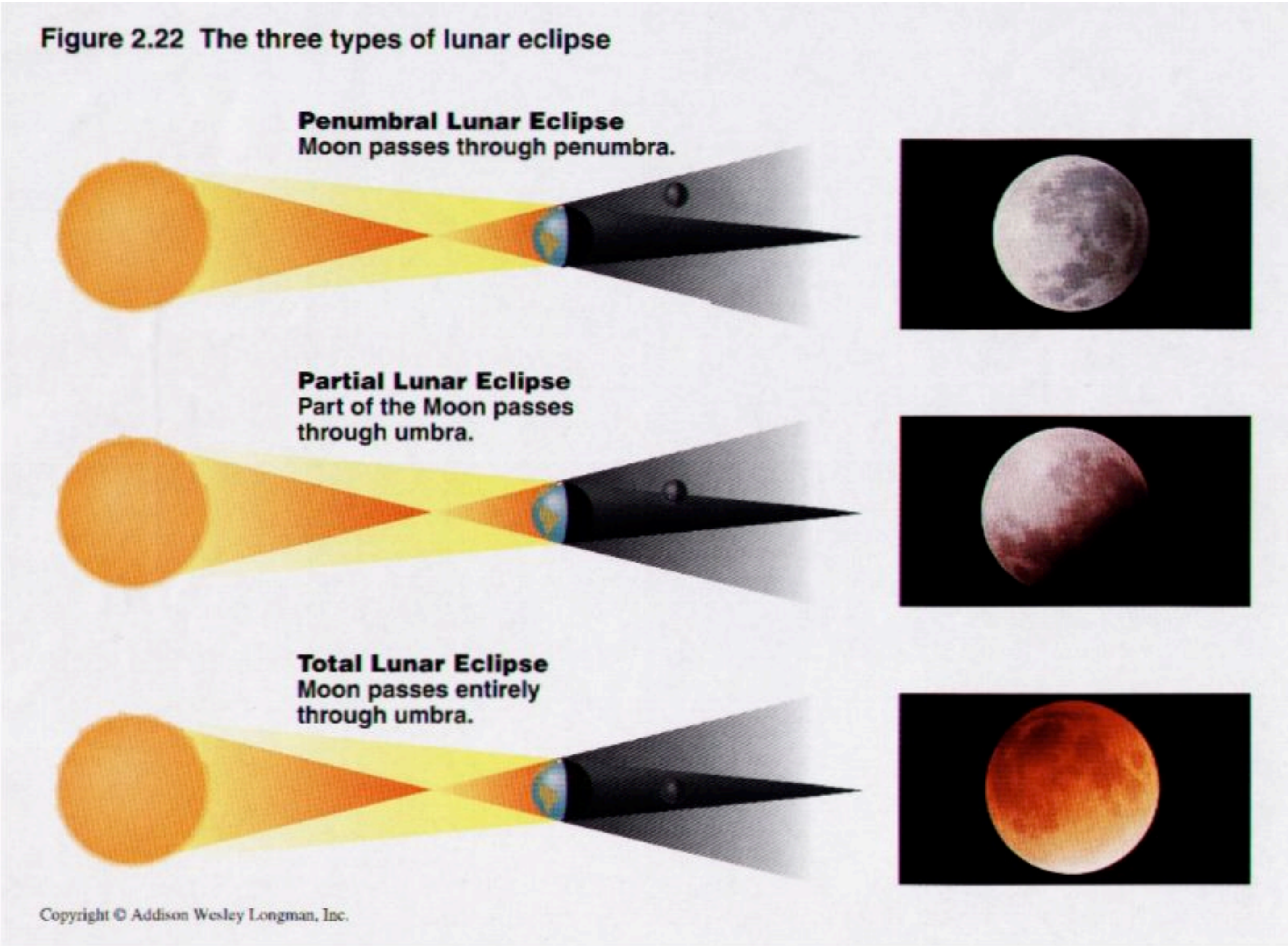
Radians & Small-Angle Formula





- In general, the angle θ in radians subtended by an arc segment *s* on a circle of radius *r* is given by the formula $\theta = s\,/\,r$. It follows that there are 2π radians in a full circle of degree angle 360º, or 1 rad = 180º/$\pi \approx 57.3^{\circ}$.
- The small-angle formula amounts to the approximation that the arc segment approaches a straight line (red) when $\theta << 1\ \mathrm{rad} = 57.3^{\circ}$.

Pythagoras: Earth is not Flat; It is a Sphere.



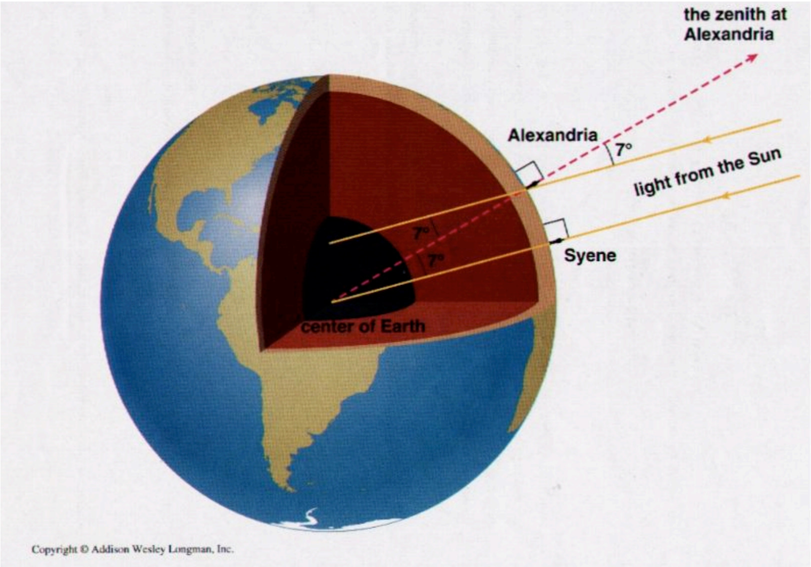
Shadow of Earth on Moon is curved. Curvature of shadow is 3 to 4 times larger than the Moon.

Size of Earth

- Erastothenes (276-194 BC): Circumference of Earth = $2\pi R$

$$\frac{2\pi R}{s} = \frac{360^{\circ}}{7.2^{\circ}} = 50 \Rightarrow 2\pi R = 50s \approx 40{,}000 \text{ km.}$$

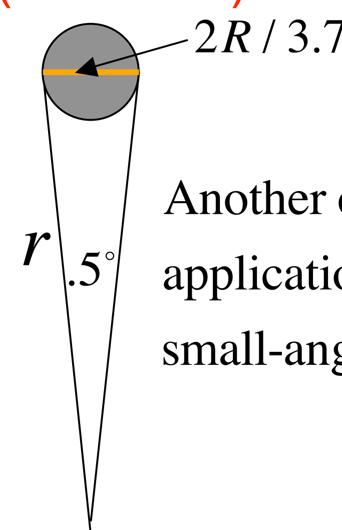
In a later age of Napoleon, French scientists would define the meter so that the polar circumference of the Earth exactly equals 40,000 km. If we use the Egyptian approximation $\pi \approx 25/8$, we get $R = 8s = 6{,}400$ km. What Anaxagoras got as distance to the Sun, Erastothenes now calculates correctly as the radius of Earth!



- Consistent with assumption of parallel rays for the light from the Sun, the Sun might be at a very great distance from Earth, and it is therefore of an unknown size.

Size and Distance of Moon

- Aristarchus (310-230 BC):
  - From Earth's shadow on Moon during lunar eclipse, we deduce that Earth is 3 to 4 times bigger than the Moon (3.7 times).
  - If distance of Moon is *r*, then



$$\frac{2R/3.7}{r} = 0.5^\circ \times \frac{\pi \text{ rad}}{180^\circ} = 0.0087 \Rightarrow$$

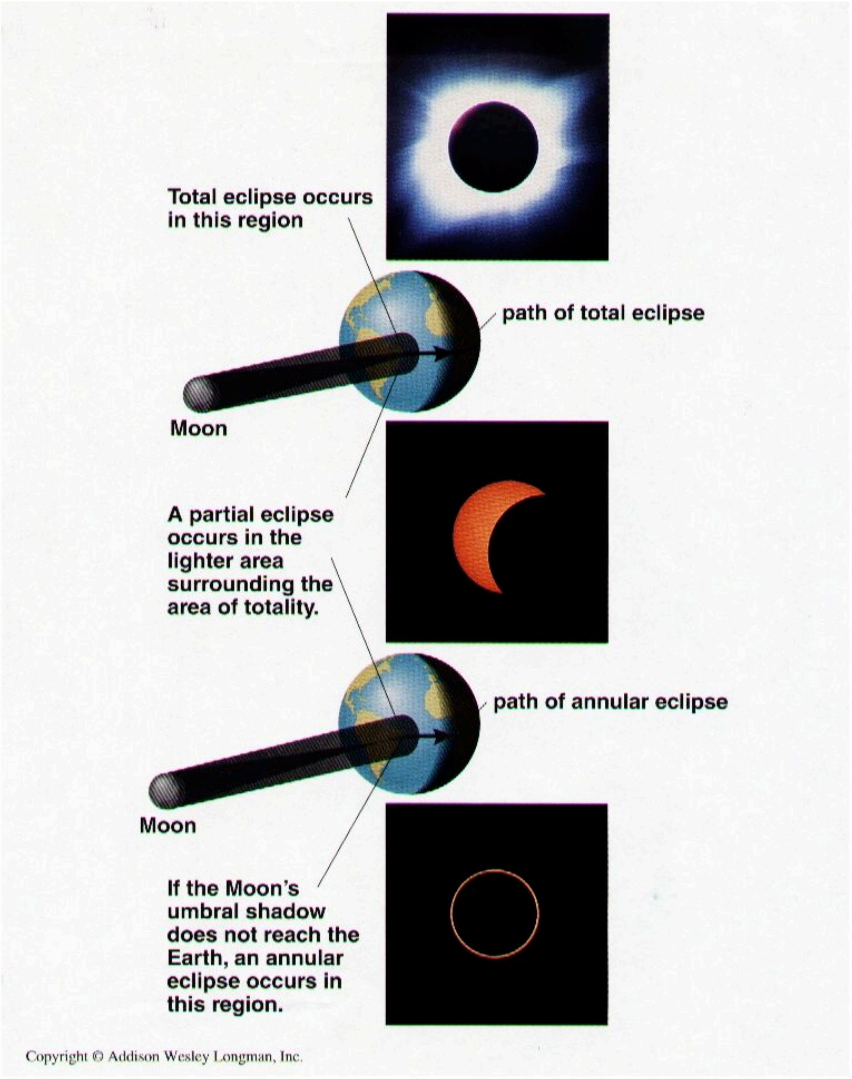
$r = 62\,R$. The Moon lies at a distance of about 60 Earth radii, a value important for Newton's future deduction of the law of universal gravitation.

Another example of application of small-angle formula.

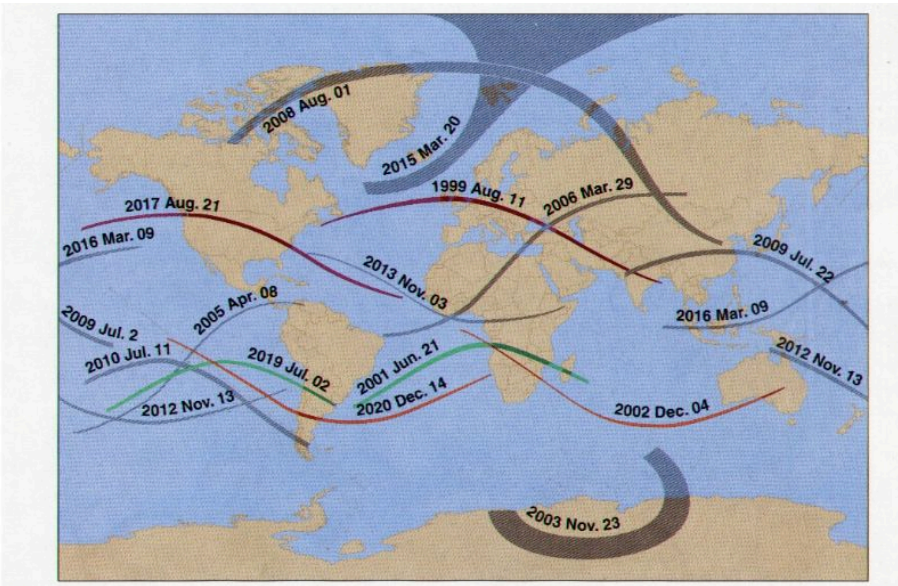
- If we use Erastothene's later determination, *R* = 6400 km, we get *r* = 400,000 km, close to the modern value.

Solar Eclipses

- Ancient Chinese thought a solar eclipse occurs when a dragon swallows the Sun.
- Actually, sometimes the Moon can come between us and the Sun, putting the Sun into eclipse. This shows that the Sun is farther from the Earth than is the Moon.
- When a total solar eclipse occurs, the Moon has an angular size that just covers the Sun (when it doesn't, the eclipse is called *annular*). This shows that the Sun also has an angular diameter of about 0.5º.
- Since the Sun is farther, for it to have the same angular size, the Sun must be physically larger than the Moon. How much larger?



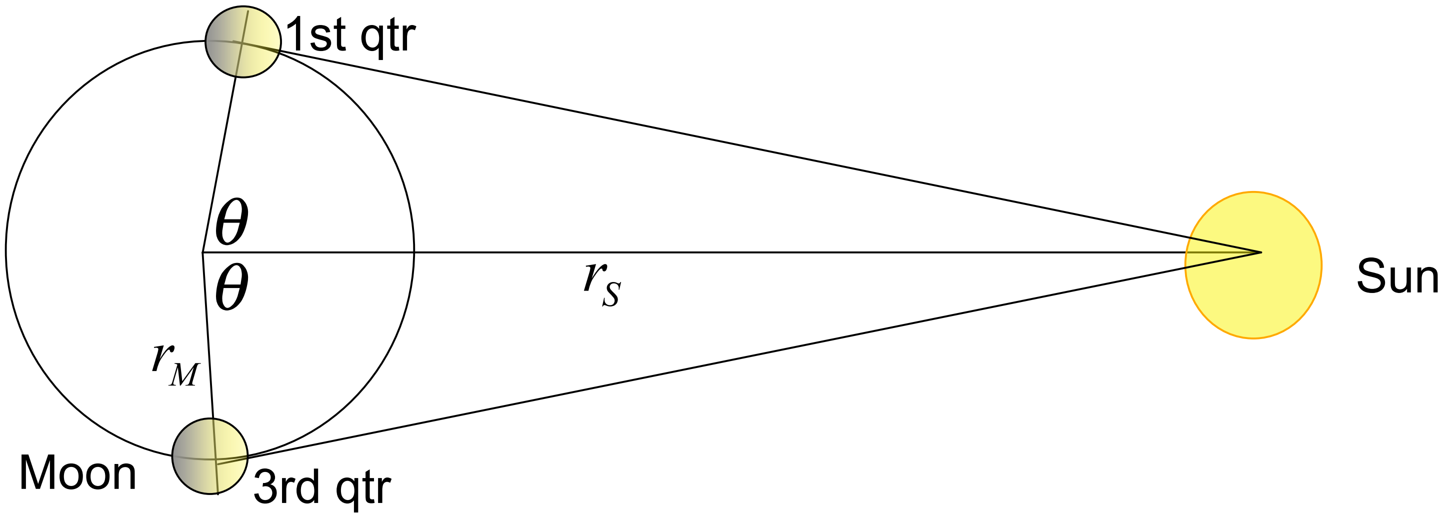
Total Solar Eclipses



- Total solar eclipses are spectacular sights, so if you have an opportunity to view one, you should take it.
- Because the Sun, Moon, and Earth repeat their positions to one another only once every 19 years (recall discussion of lunar calendar), the chances of catching one near any given location on Earth occurs only once every couple of decades or so.

Incorrect Size of Sun and Its Distance

- Lunar period (as seen from Earth to go through phases) = 29 d.



- According to Aristarchus, 15 d to go from 1st qtr to 3rd qtr, 14 d from 3rd qtr to 1st qtr.

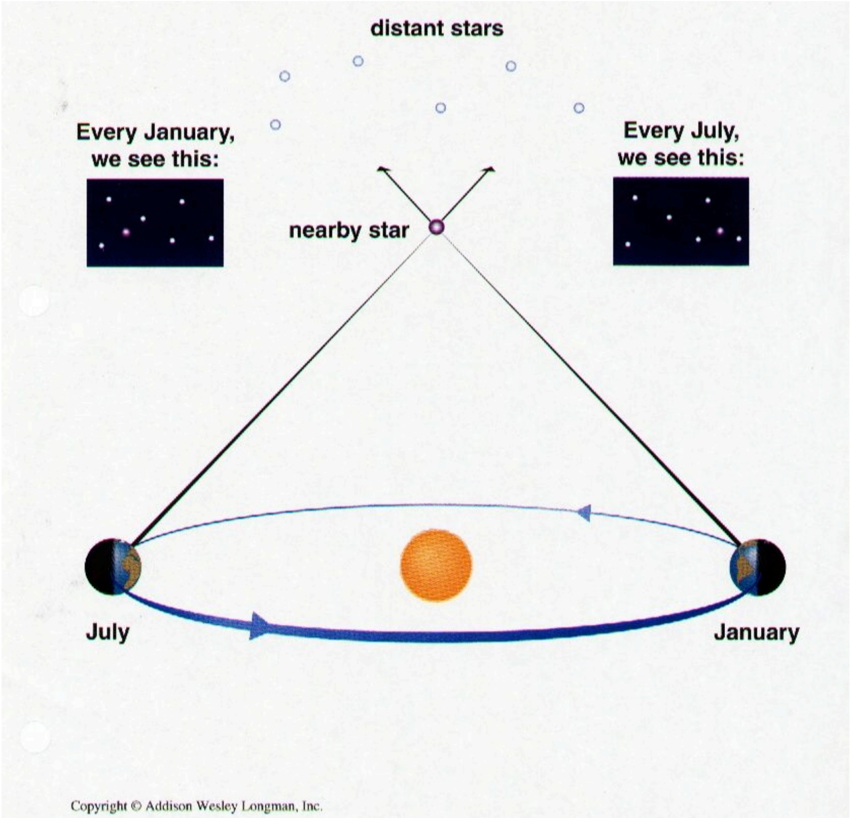
$$\frac{14\text{ d}}{29\text{ d}} = \frac{2\theta}{360^\circ} \Rightarrow \theta = 87^\circ$$

$$\frac{r_S}{r_M} = \frac{1}{\cos 87^\circ} = 19 \Rightarrow r_S = 19 r_M \Rightarrow D_S = 19 D_M = 5 D_E.$$

- Actual value, $D_S = 110 D_E$ because θ closer to 90°, variable, and hard to measure. Sun much bigger than Earth! Shouldn't Earth circle Sun?

Heliocentric vs. Geocentric View

- Aristotle (384-322 BC): If Earth circles Sun, we should see stellar parallax. We do not. Thus, Earth must be stationary, and Sun circles Earth.
- Aristarchus (310-230 BC): Sun is apparently much larger than Earth. Shouldn't Earth go around the Sun? We don't see stellar parallax because effect is not measurable (by Greeks) if stars are so far away as essentially to be at infinity.
- Archimedes (287-212 BC): The idea of an infinite universe is ridiculous. No matter how large is the universe of stars, Archimedes had a method to express it. He could even calculate the number of grains of sand it can hold (*Sand Reckoner*) via powers of 10. (In other words, Archimedes invented the notion of *logarithms*.)



Archimedes (287-212 BC)

- 223/71 < π < 220/70 (i.e., 3.1408 < π < 3.1428)
- Area of circle = πR^2
- Surface area of sphere = $4\pi R^2$
- Volume of sphere = $4\pi R^3 / 3$
- Principle of buoyancy (Eureka! = Calif. Motto)
- Law of the lever
- Great engines of war
- Death at hands of Romans
- Greatest mathematician of antiquity

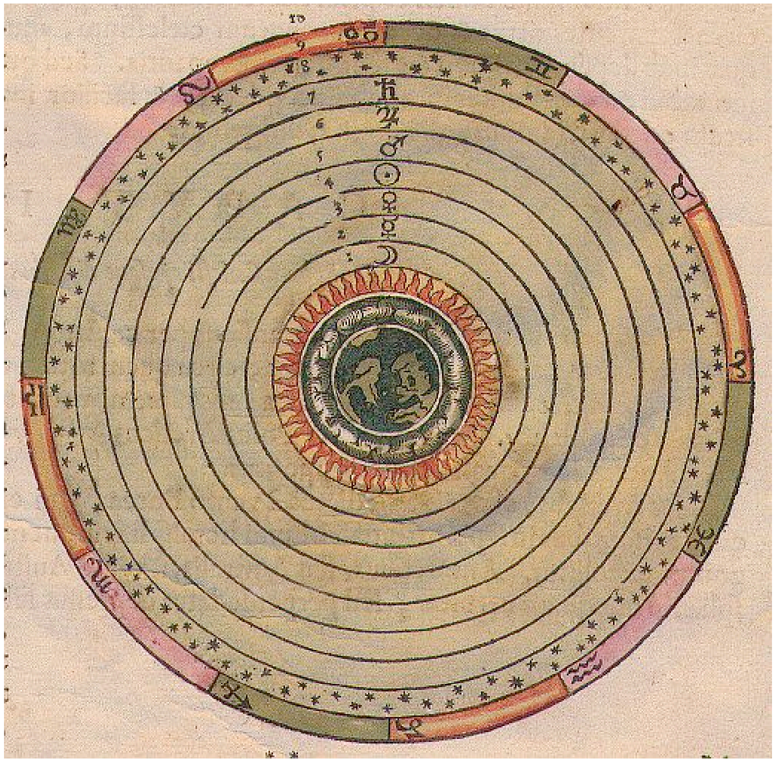
Uffizi Museum, Florence, Italy

Hipparchus (190-120 BC)

- Improved Aristarchus's distance to Moon
- Compiled extensive trignometric tables
- Discovered nova ("new star" → heavens can change)
- Catalogued stars according to magnitudes
  - 1st magnitude = brightest
  - 6th magnitude = faintest (can be seen by eye)
- Discovered precession of equinoxes
- Unable to find stellar parallax

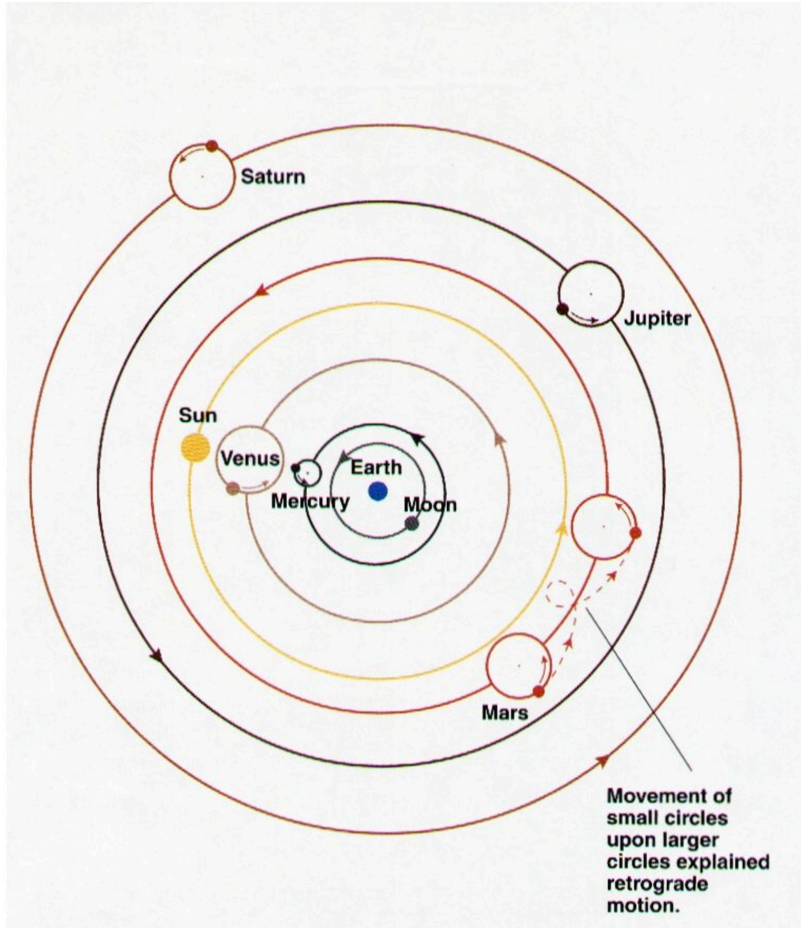
Schism Between Deductive and Empirical Parts of Science

- Plato (427-347 BC)
  - Ascendancy of role of deductive reasoning.
  - Subjective nature of sensory experience.
  - Work of the mind for philosophers; work of the hands for slaves.
- Aristotle (384-322 BC)
  - Great system of the world.
  - Filled Euclidean universe with Earth at center.
  - Motion imparted by prime mover but eventually stopped by friction.
  - Separation of Heaven and Earth.
  - Perfection of celestial sphere.
  - Gravitational attraction for terrestrial matter.



Ptolemaic Model

- Pure uniform circular orbit inadequate to explain to-and-fro motion of planets other than Sun and Moon.
- Introduction of device of epicycles on top of guiding circles (deferents).
- Eventually, also needed other fixes -- displaced centers (eccentrics), points not centered on Earth about which planets revoled at uniform circular speed (equants).
- In all, more than 80 epicycles introduced to fit the data ("save appearances" of planets where they do show up observationally).



Ptolemy (AD 85-165)