Lecture 14

Power of the Sun

Outline of Lecture 14

- For *hydrostatic equilibrium* to prevail, the pressure and temperature in the interior of the Sun are so high that the constituent atoms are completely ionized into a *plasma state*, which behaves as an *ideal gas*.
- High temperatures at the center of the Sun produce lots of X-rays, which slowly diffuse to the surface via a *random walk* of the photons, degrading to visible light in the process, with the gradual leakage giving the observed *luminosity of the Sun*.
- In steady state, the Sun adjusts its radius so that *nuclear fusion reactions* occur in its core, converting 4 H into 1 He, at a rate that exactly makes up for the loss of energy to the surface.

Symbolism of the Rising Sun

- Before the great sheets of ice receded 15,000 years ago, humans survived by hunting and fishing, and *animals* such as bison and deer were revered and *worshiped* as divine spirits.
- Ever since agriculture and animal husbandry replaced hunting and fishing, poets and writers have instinctively and correctly depicted the *Sun* as the *source of all life* on Earth.
- Homer in the *Iliad* describes the *Sun* as "*tireless*."
- Margaret Mitchell at the end of her Civil War novel *Gone with the Wind* has her heroine declare "*After all, tomorrow is another day*."
- During the 20th century, people widely identified power and optimism with the image of the *rising Sun*.

Japan = Land of the Rising Sun. The Sun rises in the East. China is the country to which Japan is East. China therefore refers to Japan as 日本, "jih pun" = "Nippon" in Japanese = "Japan" in English, meaning "the Sun's home."

- But what is the Sun really like in its *interior*? What is the Sun like below its visible surface, its *photosphere*.

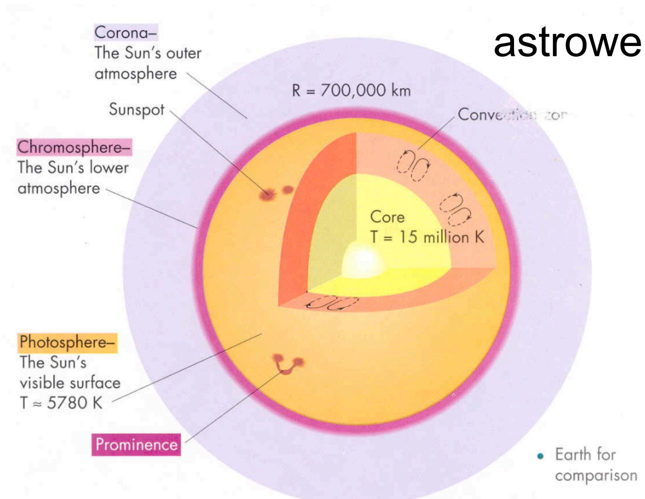
Review of Properties of Sun

- Yellow color of Sun and approximation of Sun as ideal thermal radiator (blackbody) $\Rightarrow$ effective temperature of Sun's surface: $T_e = 5800\ \text{K}.$
- Distance of Earth from Sun, $r = 1\ \text{AU} = 1.5\ \times 10^{11}\ \text{m},$ and angular size of Sun, $\theta = 0.5^\circ, \Rightarrow$ $R_\odot = 7.0 \times 10^8\ \text{m}.$
- Radiant energy emitted per second by Sun: $L_\odot = 4.0 \times 10^{26}\ \text{watts}.$
- Orbital period (1 yr) of Earth at 1 AU $\Rightarrow$ $M_\odot = 2.0 \times 10^{30}\ \text{kg}.$

www.atpm.com

Mean and Central Density of Sun

- Mean mass density:
  $$\bar{\rho} = \frac{\text{mass}}{\text{volume}} = \frac{M_\odot}{4\pi R_\odot^{\,3}/3} = 1400 \text{ kg m}^{-3},$$
  somewhat more dense than liquid water.
- Central density of Sun is actually about 100 times more dense than mean value. Taking into account the chemical composition of the central regions of the Sun and its ionization state (more later), the corresponding *number density* of particles is
  $$n_c = 1.0 \times 10^{32} \text{ m}^{-3}.$$
- Does this mean that the interior of the Sun is a liquid, or even a dense solid?
- No! The interior of the Sun is actually a *plasma*, which is a special kind of gas.
- To see this, consider the pressure in the interior of the Sun.
- The Sun is not shrinking or expanding, which implies that its interior pressure must exert an outward force that just balances the inward pulling self-gravity – a state called *hydrostatic equilibrium.*



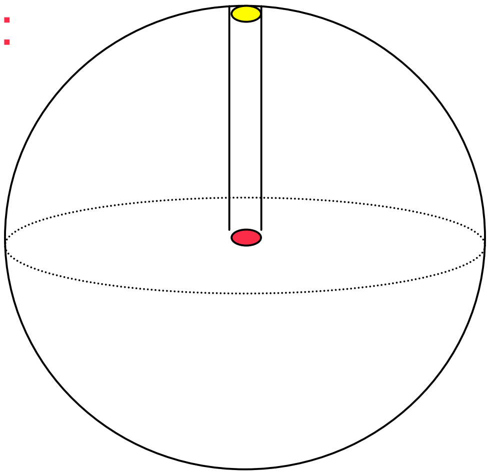
astroweb,.csru.edu

Although Sun has an extended, tenuous, outer atmosphere (corona, chromosphere), most of its mass is contained interior to its easily visible surface, the photosphere. When astronomers speak of the Sun's interior, they mean the layers deeper than the photosphere, consisting of a convective envelope, a radiative interior, and an energy-generating core.

Hydrostatic Equilibrium

- Earth:
  - Pressure at surface of Earth = weight of column of air per unit area above you:
    $P = \mu g$ where μ = mass per unit area.
  - With $P = 1.01 \times 10^5$ kg m^{-1} s^{-2} and
    $g = 9.80$ m s^{-2}, we deduce
    $\mu = 1.03 \times 10^4$ kg m^{-2}
    of air lies above us at sea level.
  - Considerations hold at any height. For example, volume density of water is 10^3 kg m^{-3}. Therefore going 10 m below the sea's surface adds a column
    $$\Delta\mu = 1 \times 10^4 \text{ kg m}^{-2},$$
    roughly doubling the pressure to which the body is subjected (about weight of textbook on thumb nail). Scuba divers rarely go beyond a depth of 30 m, which quadruples *P*.

- Sun:



- Pressure at center of Sun = weight of column of material per unit area above center

$$\text{Mean pressure} \sim \left(\frac{GM_\odot}{R_\odot^{\,2}}\right)\left(\frac{M_\odot}{\pi R_\odot^{\,2}}\right) = 4 \times 10^{14} \text{ kg m}^{-1} \text{ s}^{-2}.$$

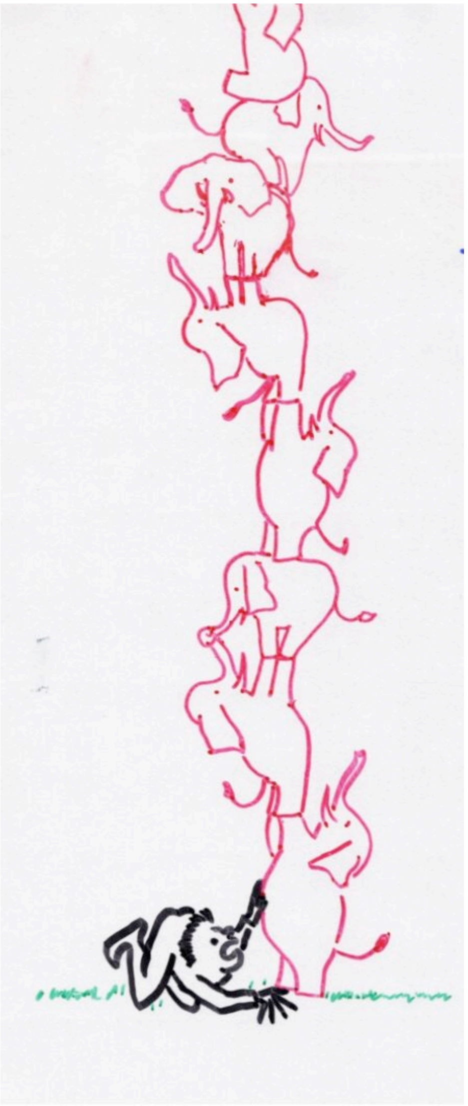
- Central pressure 50 times larger $\Longrightarrow$

$$P_c = 2 \times 10^{16} \text{ kg m}^{-1} \text{ s}^{-2}.$$

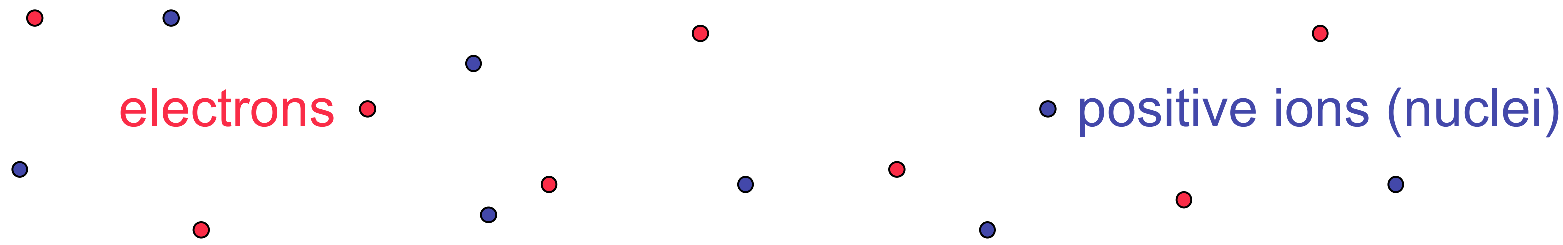
- How to picture such a large value?

How Large is $P_c = 2\times10^{16}\ \mathrm{kg\ m^{-1}\ s^{-2}}$?

- Air pressure of 1 atmosphere on Earth ~ weight of book pressing on thumbnail of area 1 cm^2:
  $$(1\ \mathrm{kg})(9.8\ \mathrm{m\ s^{-2}})/(10^{-4}\ \mathrm{m^2}) \sim 10^5\ \mathrm{kg\ m^{-1}\ s^{-2}}.$$
- P_c at center of Sun is larger yet by factor
  $$2\times10^{11}.$$
- Thus, the pressure at the center of the Sun is equivalent to the weight of more than 40 million circus elephants standing on one's thumb.
- Under such a pressure, not only would one's thumb be crushed, the atoms in one's thumb would also be crushed. The electrons would be freed from the nuclei of their atoms, with the positively charged nuclei (ions) and negatively charged electrons existing in a *plasma state*.



Solar Plasma



- A lot of space between individual particles, unlike the case if unionized atoms were pressed up against each other at mean density of Sun, where most of volume is occupied by the fuzzy electronic shells of the atoms. Arthur S. Eddington (1882-1944) was the first person to realize the far-ranging implications of this simplification.
- Electrons and nuclei therefore behave as freely moving particles, i.e., as an ideal gas satisfying

$$P = nkT \, ,$$

where P = pressure, n = number density, k = Boltzmann's constant, T = temperature on Kelvin scale.

- For the central values, $P = 2\times10^{16}\ \mathrm{kg\ m^{-1}\ s^{-2}}$, and $n = 1\times10^{32}\ \mathrm{m^{-3}}$, we get $T = 1.5\times10^{7}\ \mathrm{K}$ at the center of the Sun.

Thermal X-Rays

- For *T* = 1.5×10^7 K, Wien's displacement law implies $\lambda_{\max} = 0.19$ nm, i.e., the center of the Sun is full of X-rays.
- From common medical examinations, we know that X-rays of this wavelength can travel about a cm in matter like water before it is absorbed (mostly by losing energy to the electrons in matter).
- The average density of the Sun is somewhat denser than liquid water, so it is not surprising that even X-rays in the interior of the Sun cannot travel farther, on average, than about $\ell =$ 0.5 cm before they are scattered by the electrons of the Sun. (This is a result of theoretical calculation given as extra material in a later slide.)
- Photon paths inside the Sun are more like a *random walk* (e.g., Brownian motion) than free flight. For a random walk in 1-D:

$$\langle x_N \rangle = 0, \text{ but } \langle x_N{}^2 \rangle^{1/2} = N^{1/2}\ell.$$

- Proof (extra material):

  Going right is as likely as going left; therefore, $\langle x_N \rangle = 0.$

  But $\langle x_{n+1}{}^2 \rangle = \left\langle \frac{1}{2}(x_n+\ell)^2 + \frac{1}{2}(x_n-\ell)^2 \right\rangle = \langle x_n{}^2 + \ell^2 \rangle = \langle x_n{}^2 \rangle + \ell^2.$

  By induction, $\langle x_N{}^2 \rangle = N\ell^2;$ thus, $\langle x_N{}^2 \rangle^{1/2} = N^{1/2}\ell.$

Perrin: *Atoms*

3-D Random Walk of Photons Out of Sun

- Random walk in 3-D similar to 1-D:

$$\left\langle \left|\vec{r}_N\right|^2 \right\rangle = N\ell^2.$$

- Want $\left\langle \left|\vec{r}_N\right|^2 \right\rangle^{1/2} = R_\odot$. Thus, $N = \dfrac{R_\odot{}^2}{\ell^2}$.

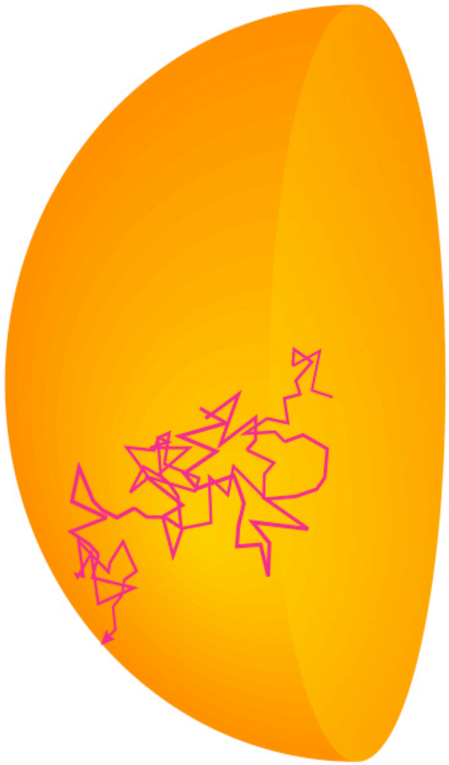
- The random-walk time required to take *N* steps to reach $R_\odot$, each step lasting a duration ℓ / c, is

$$t_{\text{random-walk}} = N\frac{\ell}{c} = \frac{R_\odot{}^2}{\ell c}.$$

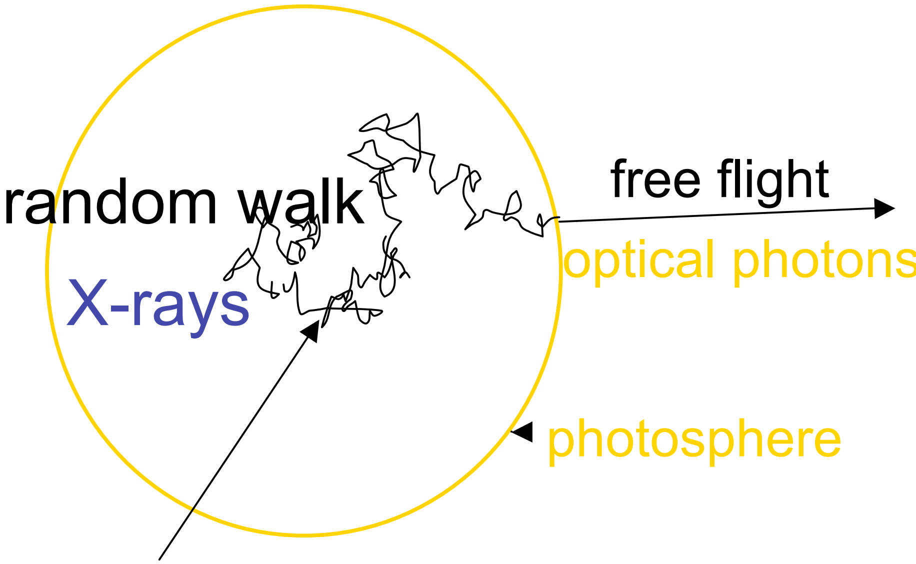
- Note random-walk time is longer by factor $R_\odot / \ell \sim 10^{11}$ (optical depth through Sun) than free-flight time of

$$R_\odot / c \sim 2 \text{ s}.$$

- Typical time for photon to random walk out of Sun is $R_\odot{}^2 / \ell c \sim 10^4$ yr.



Rate of Leakage of Photons from Interior of the Sun



Slow photon diffusion from interior to surface regulates energy leakage to the rate

$$L_\odot = 4.0 \times 10^{26} \text{ watts.}$$

Derivation (extra material):

•The total radiative energy content inside the Sun at any instant is *E* = energy per unit volume times volume:

$$E = \left(a\bar{T}^4\right)\left(\frac{4\pi R_\odot^{\,3}}{3}\right),$$

where $\bar{T} \sim 3\times 10^6$ K is the mean temperature of the Sun, and *a* = 4σ/*c* is the radiation constant for energy density.

•The time for any given photon to random walk to the surface equals

$$t_{\text{random-walk}} = \frac{R_\odot^{\,2}}{\ell c}.$$

•The luminosity of the Sun therefore can be estimated as

$$L_\odot = \frac{E}{t_{\text{random-walk}}} = \frac{16\pi}{3}\sigma\bar{T}^4 R_\odot \ell.$$

•Numerically, above formula yields

$$L_\odot \sim 3\times 10^{26} \text{ watts.}$$

•A more accurate theoretical calculation (the so-called “standard solar model”) yields the empirically measured value from the surface of the Sun: $L_\odot = 4.0\times 10^{26}$ watts.

Opacity Inside the Sun (extra material)

- Crucial realization by Sir Arthur Eddington that the matter of the Sun, although of a mean density comparable to that of liquid water, is
  - actually a perfect gas and
  - nearly completely ionized.
- In their interactions with electromagnetic radiation, the free electrons in the Sun behave as if they had a cross sectional area equal to

$$A = \frac{8\pi}{3} r_e^{\,2} = 6.6 \times 10^{-29}\ \text{m}^2.$$

where r_e is the "classical radius" of the electron:

$$r_e = \frac{e^2}{m_e c^2} = 2.8 \times 10^{-15}\ \text{m}.$$

- As a consequence, the mean free length $\ell = 1/nA$ that photons in the deep interior of the Sun can travel before they are deflected into a different direction is, as assumed in the previous slides, on average only about 0.5 cm.
- In the outer regions of the Sun, where atoms are incompletely ionized, the cross sectional area for interaction, per particle, is larger than the value *A* given above; however, the density is also less. As a consequence, by the time photons reach the surface of the Sun, the mean free length that photons travel before they are scattered or absorbed has increased to become a significant fraction of the solar radius. This means that photons make a transition from "walking" to "flying" at the surface of the Sun defined by its *photosphere*.

Implication of Loss of Energy for Stellar Evolution

- Eddington's calculations of
  - hydrostatic balance and
  - transfer of radiation

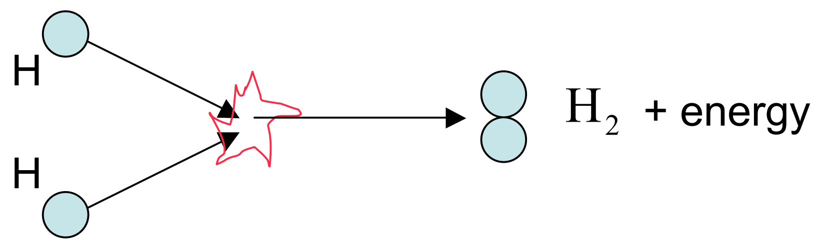
  allowed him to determine the interior structure of the Sun and other stars.
- However, without a clear knowledge of the mechanisms by which the lost energy was replenished, Eddington could not compute the rate that stars age ("evolve"), nor could he specify the consequences of this aging.
- Eddington suspected that the solution had something to do with

  $$E = mc^2,$$

  but physics had not developed sufficiently for him to develop this idea.
- The theory of stellar evolution had to wait for the maturation of nuclear physics (in which UCSD physicist Maria Mayer played a crucial role). The proper development of the theory of nuclear reactions inside stars took place in the second half of the twentieth century and represents perhaps the greatest triumph of modern astrophysics (in which UCSD astronomers Geoffrey and Margaret Burbidge played leading parts). It gave rise to an understanding of the lives of stars, and also to the origin of the chemical elements.

Source of Energy Output of Sun

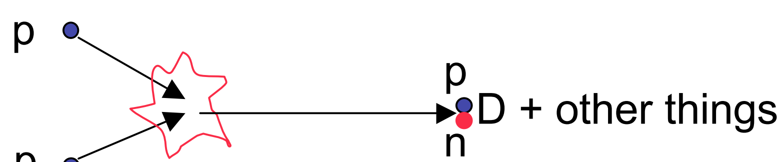
- Every second, the Sun radiates more energy than lies under the sands of all Arabia.
- When will the Sun (and the other stars) run out of energy? (Not “tireless.”)
- If the Sun shone by burning fossil fuel, and it were entirely made of coal or oil, it could last only about 10 thousand years. This is far too short – shorter than how long humanoids have been on Earth (7 million years), much shorter than how long life has existed on Earth (3.5 billion years).
- More powerful energy sources:
  - Chemical reaction with greatest release of energy per kg: H+H = H_2.



    For Sun, this reaction could last about 25 thousand years, still far too short.

  - Nuclear reaction:

$$p + p = {}^{2}H \equiv D.$$

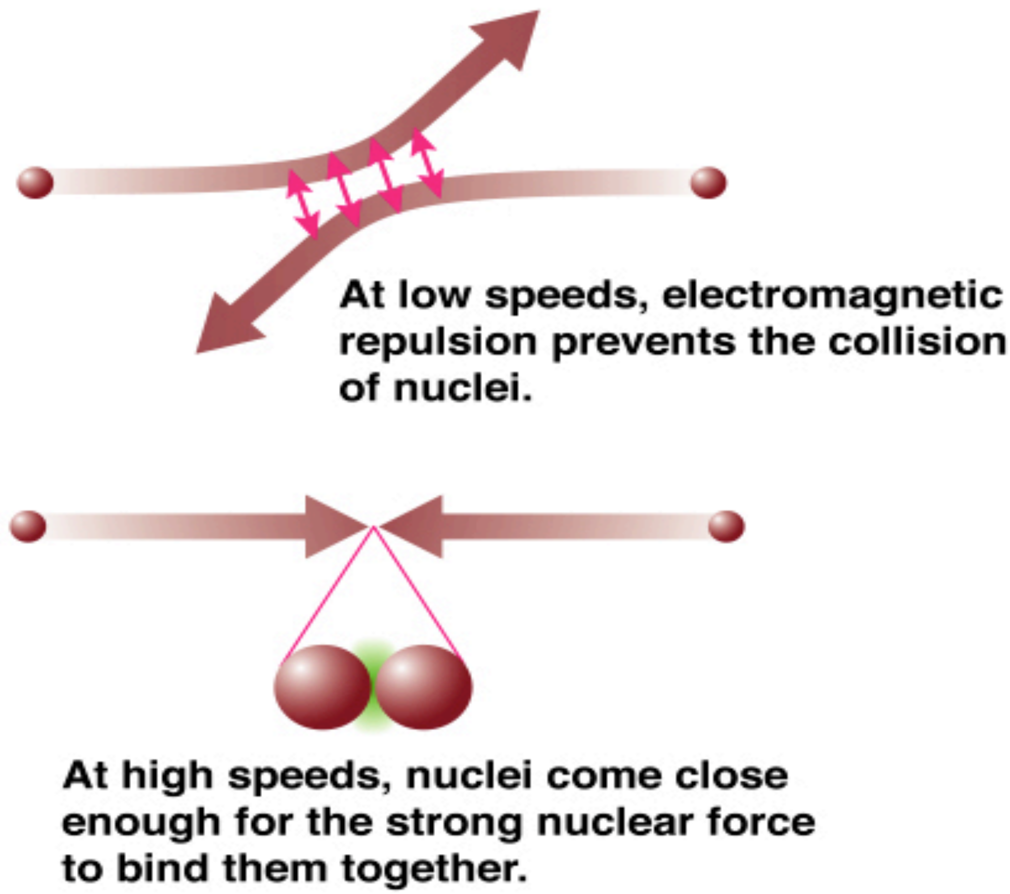


    For Sun, this reaction (and others like it) could sustain Sun for more than 10 billion years. Long enough. (Notice that nuclear reactions are about a million times more powerful than chemical reactions. This is their huge advantage.)

Nuclear Energy

- Net reaction: 4H→He (fusion reaction).
- Mass deficit: $m = 4m_{\mathrm{H}} - m_{\mathrm{He}} = 0.03m_H$.
- Energy release: $E = mc^2 = 0.007(4m_H c^2)$.
- Implication: If Sun were 100% H, fusion reaction of H into He ("hydrogen burning") would provide store of energy sufficient to last 100 billion years at present rate of solar usage: $L_{\odot}$.
- Since the Sun is actually only 70% H and since only inner 14% of mass (core) is hot enough to "burn" H during the so-called main-sequence stage, the main-sequence lifetime of the Sun = 10 billion years.
- The Sun, with an age of 4.5 billion years, is half way through its main-sequence life. What it will turn into after another 5 billion years or so is one of the more interesting stories of astronomy.

Need for High Temperatures



Stability of Solar Thermonuclear Reactor

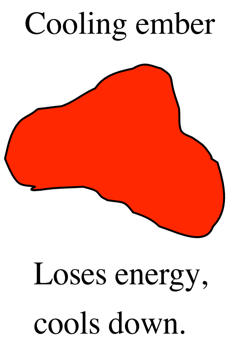
$L_c > L$ | $L_c < L$

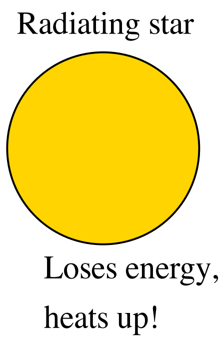
Sun expands; core cools; $L_c \to L$. | Sun contracts; core heats; $L_c \to L$.

At present (in equilibrium), the Sun has a size $R_\odot$ which generates thermonuclear power L_c in the core that just offsets the loss L from the surface: $L_c = L$.

State of War Exists Between Thermodynamics & Gravitation

- Note paradox when Sun runs out of nuclear fuel but still loses energy from surface:
  - Sun must contract, liberating gravitational potential energy to offset surface losses.
  - This makes Sun's interior hotter yet compared to surroundings, so it loses even more radiation, causing it to contract even more, etc.
- The inability of stars like to Sun to come into thermodynamic equilibrium with a cold universe is what gives the present-day universe interest and variety.
- However, it presents a big, long-term problem for stars; the subject of Lecture 15.





Zeroth Law of Thermodynamics:
Heat flows from hot to cold. At thermodynamic equilibrium the temperature throughout the system acquires a single uniform value.

Radiating ember satisfies zeroth law, but radiating, normal, star does not!

Summary
The Russell-Vogt Theorem

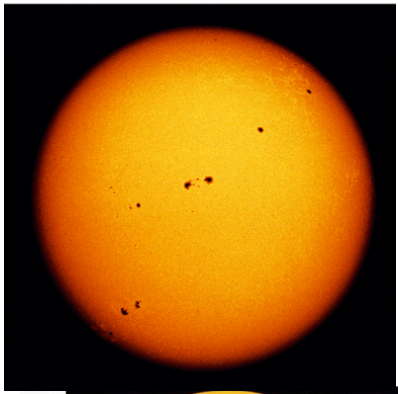
- We began with measurements of Sun's mass *M*, radius *R*, and effective temperature T_e, from which we could get its luminosity

$$L = 4\pi R^2 \sigma T_e^{\ 4}.$$

- For other stars, only T_e is directly measurable (from the color or "spectral type"). We can get the absolute brightness *L* from a measured apparent brightness *f* if we know the distance *r* of the star since

$$f = \frac{L}{4\pi r^2}.$$

- With knowledge of the Sun's current chemical composition and mass *M*, we showed that its luminosity *L* and radius *R* could be deduced theoretically (with which we can obtain the effective temperature T_e) from the principles of hydrostatic equilibrium, energy transfer, and nuclear energy generation.
- Knowing the Sun's current chemical composition is equivalent to knowing the Sun's initial chemical composition (similar to that of all other Population I stars) and its age *t*, since the theory of thermonuclear reactions allow us to compute the evolution of nuclear species with time.
- Thus, the mass *M* and the age *t* of a star (plus its initial chemical composition) are the ultimate determinants of its interior structure and external appearance. This statement is known as the *Russell-Vogt theorem*.



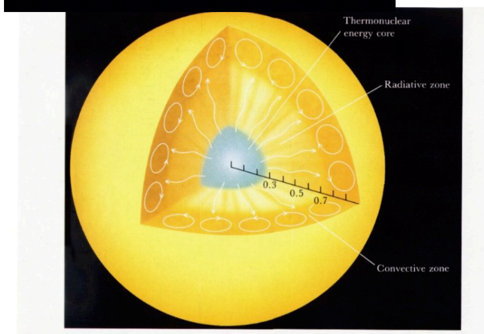


Figure 18-3
Kaufmann: UNIVERSE, Third Edition
© 1991, W. H. Freeman and Company

Observations allow us to deduce surface properties; theory allows us to deduce interior properties as well as, eventually, history and fate of the Sun.