

Lecture 10

Heat Engines and Heat Radiation

This material is difficult, but essential for you to grasp if you are to have an informed opinion about the energy crisis and global warming.

Outline of Lecture 10

- Laws of Thermodynamics:
 - Zeroth law: Heat flows from hot to cold. The temperature becomes uniform when equilibrium is reached. (no further discussion needed)
 - First law: Heat is a form of energy. When this is taken into account, energy is conserved in a closed system.
 - Second law: There is a quality as well as a quantity to energy. Although work can always be totally transformed to heat, heat cannot be turned into work without some waste. The inefficiency in real systems inevitably leads to an increase in the entropy of closed systems.
 - Third law: There is an absolute zero to temperature. At absolute zero, all systems are perfectly ordered (usually, for homogeneous systems, perfect crystals). (no further disc)
- Continuous spectrum from thermal sources:
 - Stefan-Boltzmann law: $f = \sigma T^4$.
 - Wien displacement law: $\lambda_{\max} T = \text{const} = 2.9 \times 10^6 \text{ nm K}$,

where f = energy radiated per s per unit area of surface of temperature T , and λ_{\max} is wavelength of maximum emission.

- $1/r^2$ law of radiation:

$$f = \frac{L}{4\pi r^2},$$

where f = apparent brightness (energy per s per unit area on collector), L = luminosity = absolute brightness, and r = distance of collector from source.

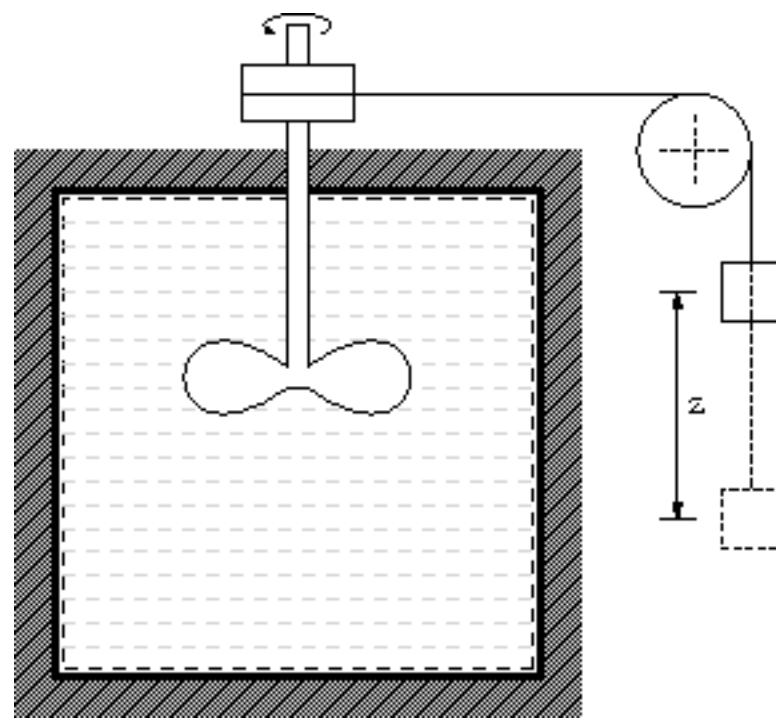
What is Heat?

- Caloric Theory: Heat is a kind of conserved fluid, which when added to things makes them hot (gives archaic concept of *heat capacity*.)
- Demise of caloric theory when Count Rumford observed that a limitless amount of heat could be generated in the boring of cannons.
- Evidently, mechanical work (here, provided by horses) can be transformed into heat, so heat is a form of energy (the random motions or jiggling of atoms and molecules in gases, liquids, and solids).



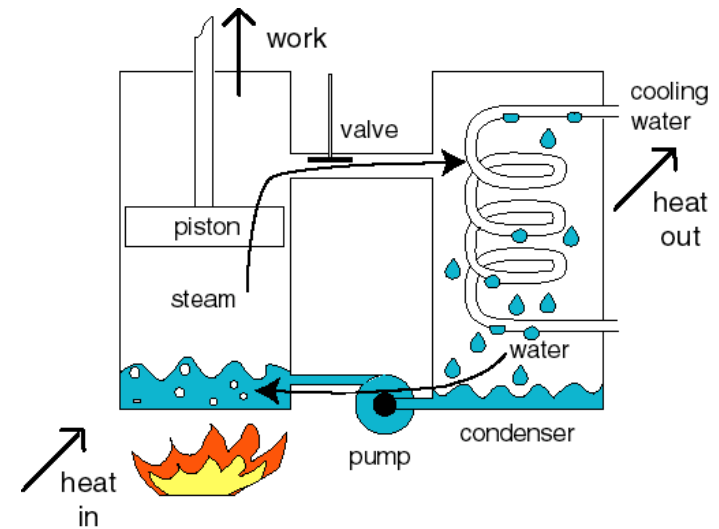
First Law of Thermodynamics

- Using paddle-wheels driven by falling weights to heat water, James Joule (1818-1879) determined that one calorie of heat (as then measured in the caloric theory) is equivalent to 4.15 joules of work ($\text{joule} = \text{kg m}^2 \text{s}^{-2}$).
- In 1847, Helmholtz formulates the First Law of Thermodynamics, which we will state in words as: **Heat is a form of energy. When this is taken into account, energy is conserved in closed systems.**
- The discovery of the First Law destroyed the hopes of all those who had hoped to construct perpetual motion machines of the first kind, whereby one could get useful work for nothing, thereby violating the principle of the conservation of energy.



Impossibility of Perpetual Motion Machines of the Second Kind

- Invention of steam engine was the breakthrough that allowed the Industrial Revolution at the turn of the 18th-19th century.
- The dream of engineers was then to build an engine that could produce work not by burning any fuel (say, coal), but by extracting the necessary energy for making steam from the heat contained in the same general environment (say, air) as one would dump the waste heat. Such hypothetical engines were called “perpetual motion machines of the second kind.”
- However, not only could engineers not build engines that gave output power at the expense of the heat of the surroundings, they could not even break even with the burning of any fuel. Real engines always produce less work than the energy burnt as fuel, with the remainder having to be exhausted as waste heat into the surroundings. The maximum efficiency of even the best steam engines was only about 20%. (Engine efficiency of gasoline internal combustion is about 35%, but auto efficiency it still at about 20%.)

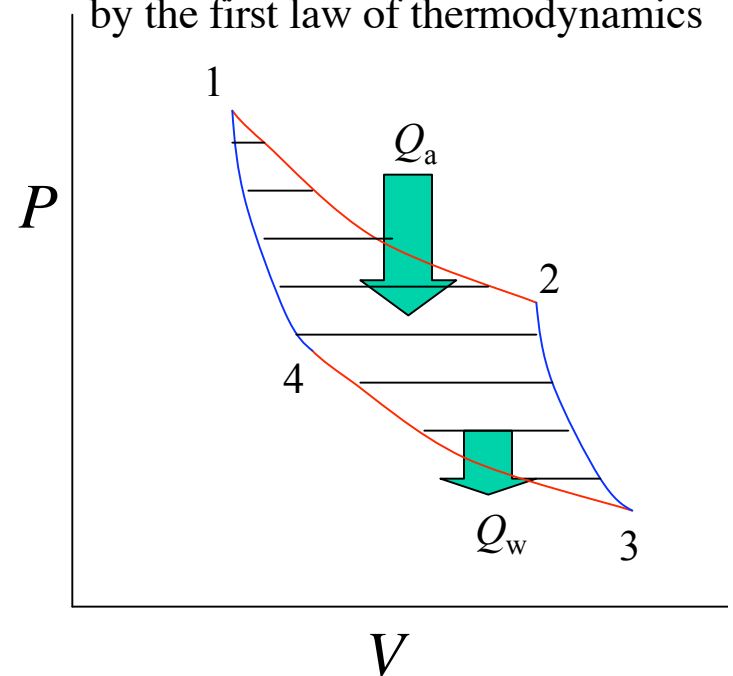


Efficiency of a Reversible Carnot Engine (extra material)

- Sadi Carnot (1796-1832) thought about this problem in terms of an idealized four-stroke piston (Carnot cycle): $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1 \rightarrow \dots$
- The two red strokes occur isothermally at temperatures T_1 and T_3 with the input and output of heat Q_a and Q_w , respectively; the two blue strokes occur adiabatically (from the Greek, meaning “without the addition of heat”). Carried out completely between two temperatures T_1 and T_3 , with $T_1 > T_3$, the cycle is completely *reversible* if the processes occur slowly enough so that the system has time to remain in thermodynamic equilibrium at each step. Carnot was then able to argue that the thermodynamic efficiency of such an engine is independent of the working material enclosed within the piston (i.e., it could be steam but it need not be); the efficiency is only a function of the two operating temperatures T_1 and T_3 :

$$\varepsilon \equiv \frac{\Delta W}{Q_a} = \frac{T_1 - T_3}{T_1}.$$

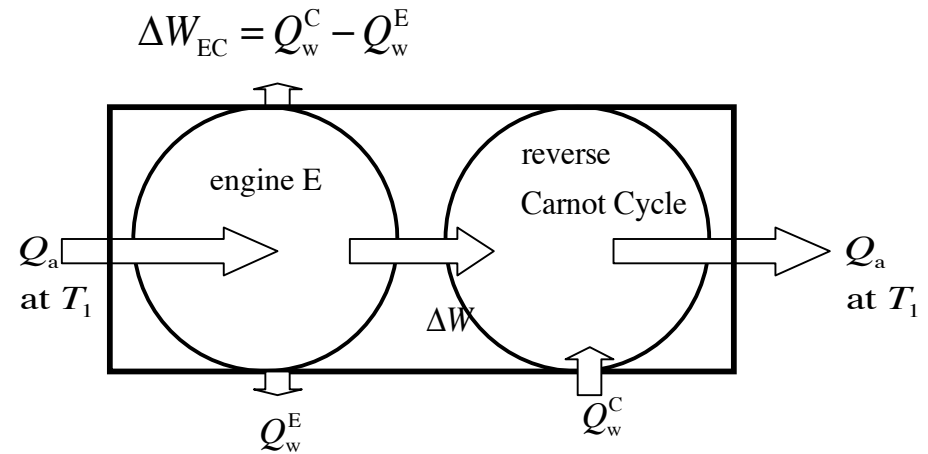
$\Delta W = \text{shaded area} = Q_a - Q_w$
by the first law of thermodynamics



For $T_3 = 373 \text{ K}$ and $T_1 = 473 \text{ K}$,
 $\varepsilon = 0.21$

Second Law of Thermodynamics

- Extra material: Efficiency of Carnot cycle C is maximum for any engine operating between two temperatures T_1 and T_3 .
- If there were a more efficient engine E, then by operating engine C in reverse and engine E in forward direction, the combination would be a perpetual motion machine of second kind.
- Since no one has ever succeeded in building a machine of this kind (taking heat from the surroundings to do work), efficiency of any reversible engine must be the same and maximum possible.



- Heat, like work, is a form of energy; however, energy has a **quality** as well as a quantity. Although work can be transformed entirely into heat (e.g., boring of a cannon); it is impossible to transform heat entirely into work (otherwise perpetual-motion machine of second kind would be possible).

Relationship of Macroscopic Irreversibility with Probability

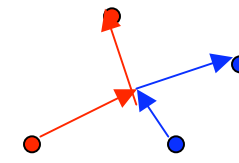
- Clausius in 1871 identifies amount of heat added in reversible process as

$$\Delta Q = T \Delta S,$$

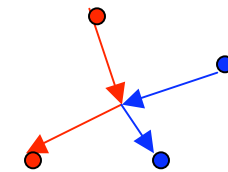
where S is the **entropy** of the system (a state variable like P , V , or T , but not Q).

- Gibbs (1839-1903) and Boltzmann (1844-1906) relate entropy S to the probability of achieving a given macrostate from a great number of possible microstates.
- Alternative statement of Second Law: **Natural processes occur in a direction so that the entropy of the universe increases, or at best, stays constant with time. Macroscopic events are usually not time-reversible.**
- In time, disorder tends to replace order.

- Two colliding molecules versus a dropped eraser colliding with ground.

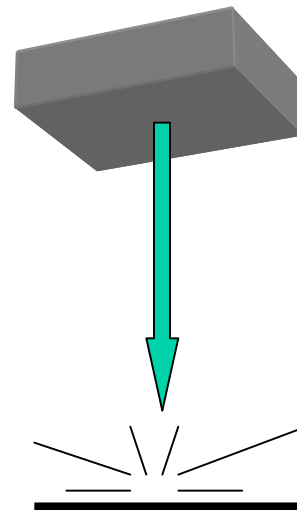


actual event

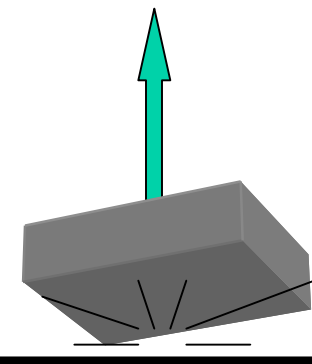


time-reversed event

equally likely to happen

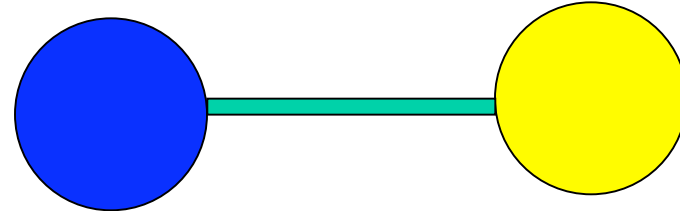


actual event



reverse process never happens

Heat Radiation from “Blackbodies”



If two opaque sources have the same temperature T , but differed in the amount of radiation that they emit in a band of wavelengths λ , then connect the two sources with a filter light pipe that passes only light of that range of wavelengths. Light energy would flow in net from the brighter to the fainter source. The flow could in principle do work, allowing us to construct a perpetual motion machine of the second kind since we can tap the heat content of two sources at the same temperature T . The second law of thermodynamics, which forbids this ability, therefore implies that the spectral intensity of all opaque bodies of given T have a universal functional form: $I_\lambda = B_\lambda(T)$.

- Kirchoff (1824-1887): Intensity spectrum emitted by opaque body in thermodynamic equilibrium at temperature T depends only on T :

$$I_\lambda = B_\lambda(T).$$

- What is functional form of $B_\lambda(T)$?
- Partial answers:
 - **Stefan-Boltzmann law:** Total energy emitted per unit area per unit time from the surface of an opaque body at absolute temperature T :
$$f = \sigma T^4,$$
where $\sigma =$ Stefan-Boltzmann constant = 5.67×10^{-8} watt m^{-2} K^{-4} and watt = joule s^{-1} is the SI unit of power.
 - **Wien's displacement law:** Where does most of energy come out? Answer is given by
$$\lambda_{\text{max}} T = C = 2.9 \times 10^6 \text{ nm K}.$$

Example:

Incandescent Light Bulb

- Tungsten filament of surface area
$$A = 2 \text{ cm} \times 1 \text{ mm} = 2 \times 10^{-5} \text{ m}^2$$
at a temperature $T = 3000 \text{ K}$. From $\lambda_{\text{max}} T = C$, we get $\lambda_{\text{max}} = 970 \text{ nm}$, which is in the near infrared.
- From $L = fA = \sigma T^4 A$, we get $L = 92 \text{ watts}$, close to the rating of a 100-watt bulb.
- Because of relatively low T , an incandescent bulb gives off mostly heat radiation and not visible light. Thus, fluorescent bulbs are more efficient generators of light for a given consumption of electric power. Even more efficient are solid-state light-emitting diodes (LEDs) that radiate light in a narrow band of optical wavelengths.



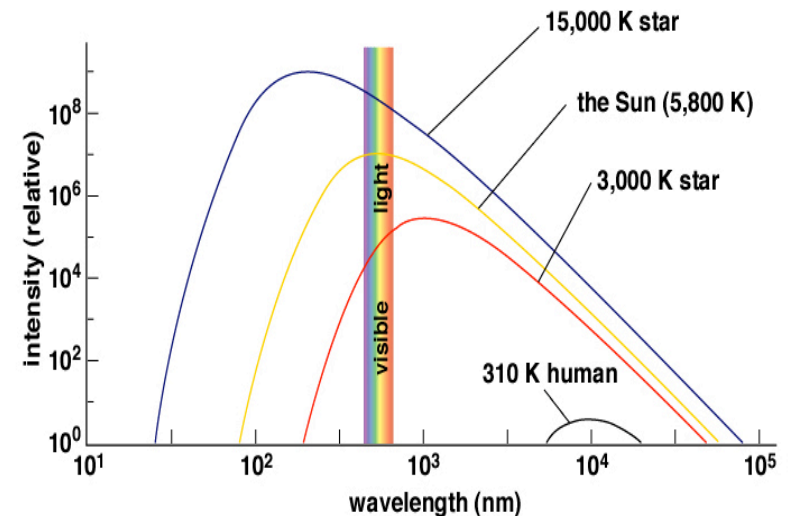
Max Planck (1858-1947)

- Historical determination of σ and C was empirical. i.e., from experimental measurements.
- Planck demonstrated how these constants could be derived from a **quantum hypothesis** involving h , a new constant of nature:

$$\sigma = \frac{2\pi^5 k^4}{15h^3 c^2}, \quad C = 0.2014 \frac{hc}{k},$$

where k = Boltzmann's constant is not as fundamental because it always enters in the energy combination kT , and T has been given an arbitrary scale in terms of absolute K.

- The three most fundamental constants of nature are therefore:
 - Gravitational constant $G = 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$,
 - Speed of light $c = 3.00 \times 10^8 \text{ m s}^{-1}$,
 - Planck's constant $h = 6.62 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$.
- Sometimes use $\hbar \equiv h / 2\pi$.



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Planck's law (extra material)

$$B_{\lambda}(T) = \frac{2hc^2}{\lambda^5} \left[\frac{1}{e^{hc/\lambda kT} - 1} \right]$$

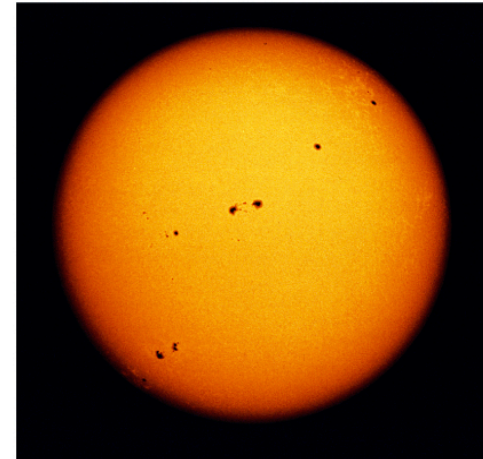
Application to the Sun

- For Sun, peak emission occurs in yellow light $\rightarrow \lambda_{\max} = 500 \text{ nm}$
 $\rightarrow T = 5800 \text{ K}$ at surface of Sun.
- Diameter of Sun subtends angle of 0.5° at a distance of 1 AU \rightarrow
Radius of Sun:

$$R_{\odot} = 7.0 \times 10^8 \text{ m.}$$

- Total energy output per unit time
= Luminosity of Sun:

$$L_{\odot} = f_{\odot} A_{\odot} = (\sigma T_{\odot}^4)(4\pi R_{\odot}^2) = 4.0 \times 10^{26} \text{ watts.}$$

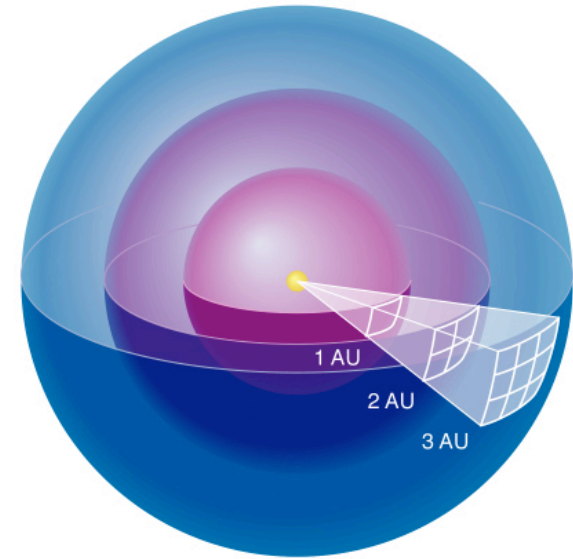


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Sun has luminosity of 4 trillion trillion 100-watt bulbs!

Check on Distance of Sun

- Approximate check: A 100 watt bulb at a distance of 10 cm feels about as warm as 4 trillion trillion 100 watt bulbs (Sun) at a distance of 1 AU. This implies that the Sun must be 2 trillion times = 2×10^{12} farther away than 10 cm
 $\Rightarrow 1 \text{ AU} \approx 2 \times 10^{13} \text{ cm} = 2 \times 10^{11} \text{ m}$.
- Actual value $1 \text{ AU} = 1.50 \times 10^{11} \text{ m}$.



For steady light source of luminosity L , conservation of energy

$$\Rightarrow f \cdot \text{area} = L \Rightarrow$$

$$f = \frac{L}{4\pi r^2}$$

Kepler is first person to realize that brightness of light follows a $1/r^2$ law.

Application to Earth

- By the time the solar luminosity L_{\odot} has spread to a distance $r = 1$ AU, the apparent brightness has dropped to

$$f = \frac{L_{\odot}}{4\pi r^2} = 1.4 \times 10^3 \text{ watt m}^{-2}.$$

- This energy flux falls onto the Earth with a cross-sectional area πR_{\oplus}^2 where R_{\oplus} is the radius of the Earth; thus, the intercepted power is

$$P = f\pi R_{\oplus}^2.$$

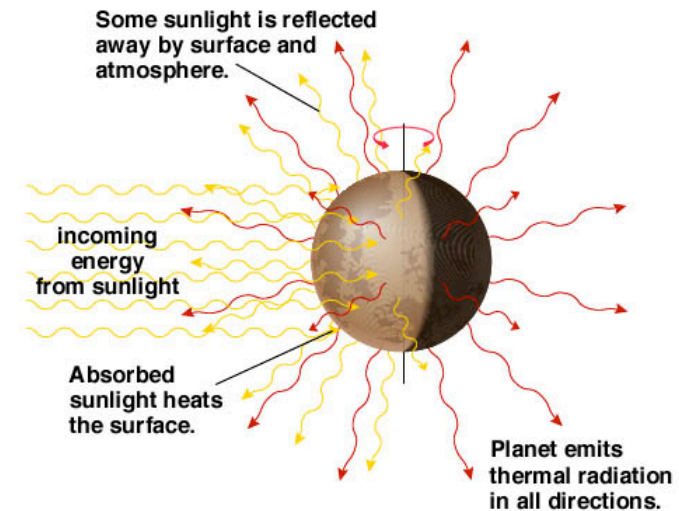
- Averaged over day-and-night and over seasons, 20% of the intercepted power P is reflected back to space; 80% is absorbed and reradiated as blackbody radiation at the surface temperature T_{\oplus} of the Earth over a surface area of $4\pi R_{\oplus}^2$,

$$0.80P = (\sigma T_{\oplus}^4)(4\pi R_{\oplus}^2).$$

- We may solve now for the average surface temperature of the Earth and obtain

$$T_{\oplus} = \left(\frac{0.80f}{4\sigma} \right)^{1/4} = 250 \text{ K}.$$

- The discrepancy between the above and the empirical average $\bar{T} = 290$ K can be attributed to the *greenhouse effect*.



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The Earth has an atmosphere which partially absorbs infrared radiation emitted by the solid surface. This warms the surface just as a blanket partially traps your body heat at night and warms you. The greenhouse effect has increased in recent decades because of human burning of fossil fuels, contributing to the problem of *global warming*.

Renewable Energy Sources

- Since continents cover only 1/4 the surface of the Earth, the usable solar power for human use is potentially

$$\frac{1}{4} \times 0.8P = 3.6 \times 10^{16} \text{ watts.}$$

- In another fifty years, the Earth is likely to have about 10^{10} humans. Each person's share of sunshine is therefore 3.6×10^6 watts.
- If we were to raise plants with this sunshine, the efficiency that plants have for converting sunshine into biofuel is about 10^{-3} , i.e., each person's fair share of biofuel would be

$$3.6 \times 10^3 \text{ watts} = 3.6 \text{ kw.}$$

- This is about 1/3 the power consumption of the average US citizen. Even if we were to devote all the land area of the Earth to biofuel production, it is impossible for everyone to have the living standards of America.



Wind power? No. Tidal power? No. Solar cells? Only if have technological breakthrough in cost. Conservation? Yes, but still need power at somewhat reduced level. Nuclear power? Yes, but nuclear power plants not long-term solution. Fusion? Yes, but fifty years away. Need viable 50-yr strategy.

Example: Inadequacy of Wind Power

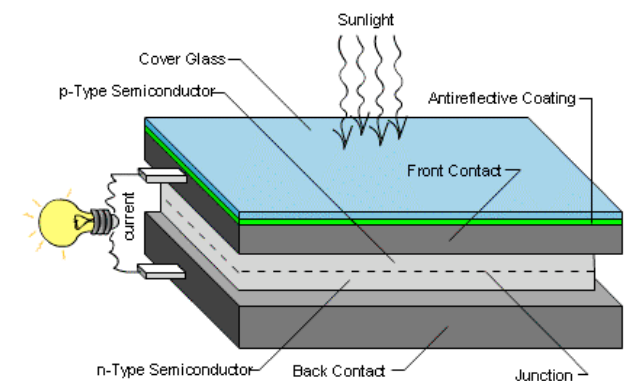
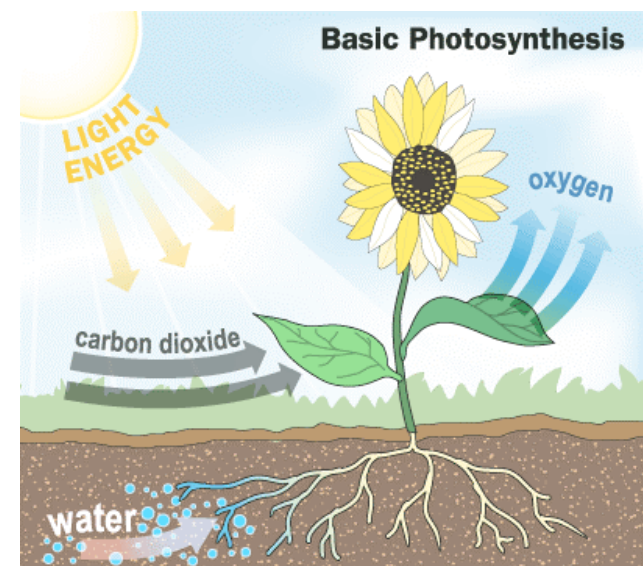
- Wind arises because of uneven solar heating.
- Temperature differential between surface and tropopause or between mid-latitudes and poles ~ 40 K. Maximum efficiency for generating air flow (wind) from temperature differential $= (T_1 - T_3)/T_1 = 40 \text{ K}/290 \text{ K} = 0.14$.
- Potentially usable wind power over continents $= 0.14 (3.6 \times 10^{16} \text{ watts}) = 5 \times 10^{15} \text{ watts}$.
- However, this power is distributed over 8 km of atmosphere (surface to tropopause), whereas typically we are building only windmills with arms of diameter ~ 8 m that tap the slowest moving part of the atmosphere (the part that's near the surface). Thus, the maximum efficiency is reduced by a another factor of at least $8 \text{ m}/8 \text{ km} = 10^{-3}$.
- Even if we carpet all the land mass of Earth with windmills, the average wind power available per person to 10^{10} humans is $10^{-3} (5 \times 10^{15} \text{ watts})/10^{10} = 0.5 \text{ kw}$, which is less than 5% of the per capita use in the US.



Closing Comments

- To a rough order of approximation, the average temperature on the surface of the Earth derives from its location at a distance of 1 AU from a yellow Sun with luminosity of $1L_{\odot}$.
- Since the Earth reradiates all the absorbed solar radiation it receives as infrared radiation at an equivalent blackbody temperature of 250 K, the Sun is not a net energy source for the Earth. What the Sun yields really is negative entropy. The Sun sends us high-quality visible photons; we send back into space low-quality infrared photons. It is the processes that accompany the transformation of the former into the latter which is the source of all life and almost all activity on Earth. (The heat trapped in the Earth's interior from a hotter past drives some geological activity, such as volcanoes and earthquakes, but this slow internal release of energy from the Earth's interior is smaller by about 5 orders of magnitude on average than what we receive per unit area of the Earth's surface from the Sun.)
- Many current discussions about renewable energy sources are based on wishful thinking. They ignore either the first or the second law of thermodynamics. The problems of the "energy" crisis and global warming do have solutions, but they require realistic planning for at least the 50-year horizon, and probably must include nuclear power plants as a temporizing intermediate step. Rational, realistic discussions of the limitations of various alternatives and possibilities are not occurring today.

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