PHYSICS 4C QUIZ 5 FALL QUARTER 2007
PROF. HIRSCH November 30, 2007

Formulas:

$\vec{F}_2 = \frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$ Coulomb's law ; $\vec{E} = \vec{F}/q_0$ electric field ; $\vec{E}(x,y,z) = \int \frac{\rho(x',y',z')(\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} dx'dy'dz'$

$\oint \vec{E} \cdot d\vec{a} = 4\pi q_{enc} = 4\pi \int \rho dv$ Gauss' law 1 charge at the origin: $\vec{E}(\vec{r}) = \frac{q}{r^2}\hat{r}$

Linear, surface, volume charge density : $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

Electric field of : charge : $E = \frac{q}{r^2}$; line of charge : $E = \frac{2\lambda}{r}$; sheet of charge : $E = 2\pi\sigma$

Potential of single charge q : $\phi(\vec{r}) = \frac{q}{r}$; charge distribution : $\phi(\vec{r}) = \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dx'dy'dz'$

$\phi(x,y,z) - \phi(x_0,y_0,z_0) = -\int_{(x_0,y_0,z_0)}^{(x,y,z)} \vec{E} \cdot d\vec{s}$; $\vec{E} = -\nabla\phi$; $\nabla^2\phi = -4\pi\rho$; $\mathrm{div}\vec{E} = 4\pi\rho$

$\mathrm{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; $u = \frac{E^2}{8\pi}$ electric energy density

energy of 3 charges: $U = \frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}$; energy of q in potential ϕ: $U = q\,\phi(x,y,z)$

Electric field right next to a conducting surface: $E=4\pi\sigma$

Capacitors: Q=CV ; Parallel plates: $C = \frac{A}{4\pi s}$ A=area, s=dist. betw. plates; $U = \frac{Q^2}{2C}$ energy

$I = \int \vec{J} \cdot d\vec{a}$, $I = \frac{dq}{dt}$, $\vec{J} = nq\vec{u}$; $div\vec{J} = -\frac{\partial\rho}{\partial t}$; Power: $P = I^2 R$; $P = \varepsilon I$

V=IR , $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$; $R = \rho\frac{L}{A}$; $\sigma = \frac{ne^2\tau}{m_e}$; $Q(t) = C\varepsilon(1 - e^{-t/RC})$

Ampere's law: $\oint_C \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{enc} = \frac{4\pi}{c}\int_S \vec{J} \cdot d\vec{a}$; Biot-Savart law: $d\vec{B} = \frac{Id\vec{\ell} \times \hat{r}}{cr^2}$

$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$; $\vec{\nabla} \cdot \vec{B} = 0$; $\vec{\nabla} \times \vec{E} = 0$ (electrostatics) ; $\vec{\nabla} \times \vec{A} = \vec{B}$; $\vec{\nabla} \cdot \vec{A} = 0$

Lorentz force : $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$; force on wire : $d\vec{F} = \frac{I}{c} d\vec{\ell} \times \vec{B}$; cyclotron : $\omega = \frac{qB}{mc}$

Field of: long wire: $B = \frac{2I}{cr}\hat{\varphi}$; ring: $\vec{B} = \frac{2\pi b^2 I}{c(b^2 + z^2)^{3/2}}\hat{z}$; solenoid: $\vec{B} = \frac{4\pi In}{c}\hat{z}$

Faraday law: $\varepsilon = \oint_c \vec{E} \cdot d\vec{s} = -\frac{1}{c}\frac{\partial}{\partial t}\Phi_B = -\frac{1}{c}\frac{\partial}{\partial t}\int \vec{B} \cdot d\vec{a}$; $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$

Inductance: $\varepsilon_{21} = -M_{21}\frac{\partial I_1}{\partial t}$; $M_{21} = \frac{\Phi_{21}}{I_1}$; $M_{21} = M_{12} = M$; $\varepsilon = -L\frac{\partial I}{\partial t}$; $L = \frac{\Phi}{I}$

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L-R circuit: $I = \frac{\varepsilon_0}{R}(1 - e^{-(R/L)t})$; Energy: $U = \frac{1}{2}LI^2$; density $u = \frac{B^2}{8\pi}$

RLC circuit: $V(t) = e^{-(R/2L)t}(A\cos\omega t + B\sin\omega t)$; $\omega = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$

Alternating current: $\varepsilon = \varepsilon_0 \cos\omega t$; $I = I_0 \cos(\omega t + \varphi)$; $\tan\varphi = \frac{1/(\omega C) - \omega L}{R}$

$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L - 1/(\omega C))^2}}$; Power: $< P >= \frac{1}{2}\varepsilon_0 I_0 \cos\varphi$

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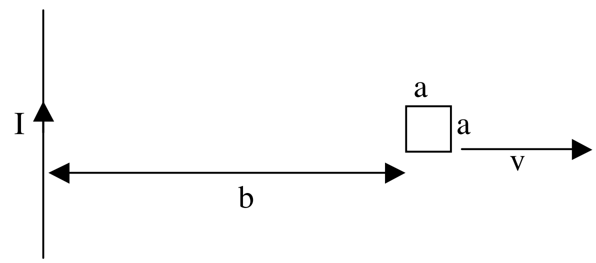
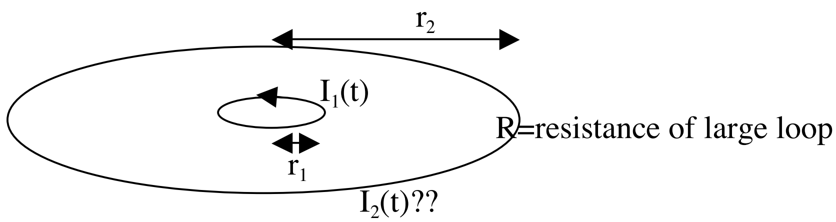
3 problems, 10 points each:

Problem 1 (10 pts)



The square loop of wire in the figure has side length a, and is at distance b>>a from an infinitely long straight wire carrying a constant current I. The magnetic field generated by the long wire at distance b from the long wire is 500 gauss.
(a) Find an approximate expression for the magnetic flux through the loop, assuming that b>>a. Give its numerical value in gauss cm^2 for a=10cm, b=5m.
(b) The loop is moving away from the long wire at speed v. In which direction is the induced current in the loop? Draw arrows in the figure and explain.
(c) Write an expression for the induced emf in the square loop. For $v/c=10^{-6}$, find the value of the induced emf, in statvolts.

Problem 2 (10 pts)



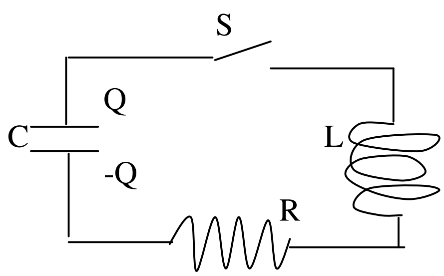
The two concentric circular loops of wire in the figure are in the same plane and have radius r_1 and r_2, with $r_2 >> r_1$. The large loop has total resistance R. There is a time-dependent current circulating in the small loop, given by: $I_1(t)=I_0e^{-t/\tau}$ flowing counterclockwise as seen from the top.

(a) Find an expression for the induced current circulating in the large loop, $I_2(t)$, in terms of I_0, τ, r_1, r_2 and R. Hint: use $M_{21}=M_{12}$. Explain all steps.

(b) Indicate in the figure in which direction $I_2(t)$ flows and explain why.

(c) Can the current $I_2(t)$ be larger than the current $I_1(t)$? Explain why or why not. Assume the self-inductance of the large loop can be ignored.

Problem 3 (10 pts)



In the circuit in the figure, C=1F, L=1H, $R=10^{-3}\Omega$. Initially the charge in the capacitor C is Q=2C and no current circulates. At time t=0 the switch S is closed.

(a) Estimate the maximum value of the current that will flow in this circuit, in Amps. Note that R is very small. Hint: one way to do this is use energy conservation.

(b) Estimate how long (in seconds) after S is closed will it take for the signs in the charges on the capacitor plates to switch (i.e. for the upper plate to have -Q, the lower plate to have +Q).

(c) Make a qualitative plot of the current versus time for this circuit. Estimate how long (in seconds) after S is closed will it take for the current in this circuit to have died down to close to zero and the charge in the capacitor plates to go down to nearly zero.

(d) Estimate the total energy dissipated in R after a long time (in Joules).