PHYSICS 4C — QUIZ 4 — FALL QUARTER 2007

PROF. HIRSCH — November 16, 2007

Formulas:

$\vec{F}_2 = \frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$ Coulomb's law ; $\vec{E} = \vec{F}/q_0$ electric field ; $\vec{E}(x,y,z) = \int \frac{\rho(x',y',z')(\hat{r} - \hat{r}')}{|\vec{r}-\vec{r}'|^2} dx'dy'dz'$

$\oint \vec{E}\cdot d\vec{a} = 4\pi q_{enc} = 4\pi \int \rho dv$ Gauss' law 1 charge at the origin: $\vec{E}(\vec{r}) = \frac{q}{r^2}\hat{r}$

Linear, surface, volume charge density : $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

Electric field of : charge : $E = \frac{q}{r^2}$; line of charge : $E = \frac{2\lambda}{r}$; sheet of charge : $E = 2\pi\sigma$

Potential of single charge q : $\phi(\vec{r}) = \frac{q}{r}$; charge distribution : $\phi(\vec{r}) = \int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} dx'dy'dz'$

$\phi(x,y,z) - \phi(x_0,y_0,z_0) = -\int_{(x_0,y_0,z_0)}^{(x,y,z)} \vec{E}\cdot d\vec{s}$; $\vec{E} = -\nabla\phi$; $\nabla^2\phi = -4\pi\rho$; $\text{div}\vec{E} = 4\pi\rho$

$\text{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; $u = \frac{E^2}{8\pi}$ electric energy density

energy of 3 charges: $U = \frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}$; energy of q in potential ϕ: $U = q\,\phi(x,y,z)$

Electric field right next to a conducting surface: $E=4\pi\sigma$

Capacitors: Q=CV ; Parallel plates: $C = \frac{A}{4\pi s}$ A=area, s=dist. betw. plates; $U = \frac{Q^2}{2C}$ energy

$I = \int \vec{J}\cdot d\vec{a}$, $I = \frac{dq}{dt}$, $\vec{J} = nq\vec{u}$; $div\vec{J} = -\frac{\partial\rho}{\partial t}$; Power: $P = I^2 R$; $P = \varepsilon I$

V=IR , $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$; $R = \rho\frac{L}{A}$; $\sigma = \frac{ne^2\tau}{m_e}$; $Q(t) = C\varepsilon(1 - e^{-t/RC})$

Ampere's law: $\oint_C \vec{B}\cdot d\vec{s} = \frac{4\pi}{c} I_{enc} = \frac{4\pi}{c}\int_S \vec{J}\cdot d\vec{a}$; Biot-Savart law: $d\vec{B} = \frac{I d\vec{\ell}\times\hat{r}}{cr^2}$

$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J}$; $\vec{\nabla}\cdot\vec{B} = 0$; $\vec{\nabla}\times\vec{E} = 0$ (electrostatics) ; $\vec{\nabla}\times\vec{A} = \vec{B}$; $\vec{\nabla}\cdot\vec{A} = 0$

Lorentz force : $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c}\times\vec{B})$; force on wire : $d\vec{F} = \frac{I}{c} d\vec{\ell}\times\vec{B}$; cyclotron : $\omega = \frac{qB}{mc}$

Field of: long wire: $B = \frac{2I}{cr}\hat{\varphi}$; ring: $\vec{B} = \frac{2\pi b^2 I}{c(b^2+z^2)^{3/2}}\hat{z}$; solenoid: $\vec{B} = \frac{4\pi I n}{c}\hat{z}$

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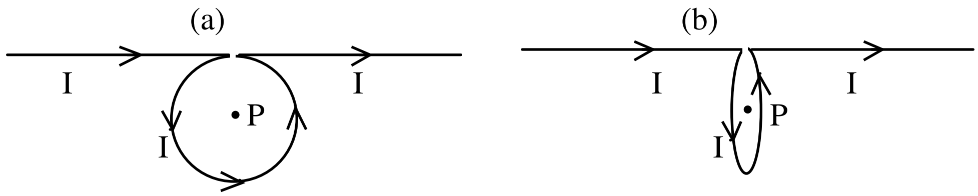
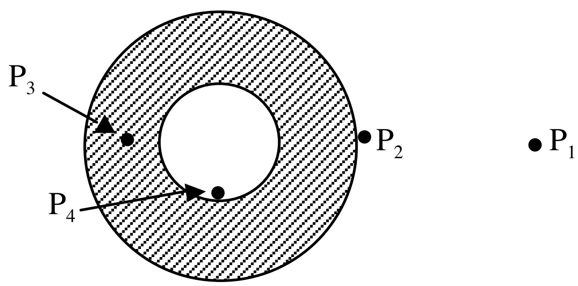
3 problems, 10 points each:

Problem 1 (10 pts)



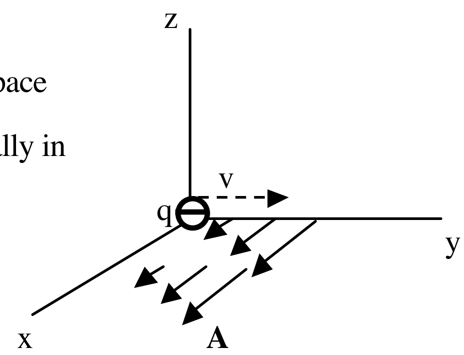
(a) A long wire carrying current I is bent into the shape shown in the figure to the left ((a)). The radius of the circle is R. Find the magnitude and direction of the magnetic field at point P at the center of the circle. Express the magnitude in terms of I and R.
(b) Imagine the loop part of this wire is now rotated (without breaking the wire) 90 degrees around a vertical axis going through point P, so that it now points perpendicular to the paper (figure to the right, (b)). Find the magnitude of the magnetic field at point P at the center of the loop in this case.

Problem 2 (10 pts)



The hollow metallic cylinder shown in the figure has inner radius R and outer radius 2R. It carries current I in direction perpendicular to the paper pointing <u>into</u> the paper uniformly distributed across its cross section. The magnitude of the magnetic field at point P_1 which is at distance 4R from the center is 10 gauss.
(a) What is the magnitude (in gauss) and direction of the magnetic field at point P_2 right on the outer surface of the cylinder?
(b) What is the magnitude and direction of the magnetic field at point P_3 at distance 1.5R from the center, indicated by the arrow in the figure?
(c)What is the magnitude and direction of the magnetic field at point P_4 on the inner surface of the cylinder (at distance R from the center)?
Justify all your answers.

Problem 3 (10 pts)



The magnetic vector potential in a region of space is given by : A_x=Cy, A_y=0, A_z=0
Some A-field vectors are indicated schematically in the figure. C is a positive constant.

(a) Find the magnetic field components (B_x,B_y,B_z). Draw arrows indicating the magnitude and direction of B in another figure like the one shown.

(b) A particle of negative charge q (q<0) is at the origin (x,y,z)=(0,0,0) and has velocity in the y direction: **v**=(0,v_0,0) indicated by the dashed arrow in the figure. Draw its trajectory and state in which plane it is moving (xy plane or yz plane). Justify your answer.

(c) Find at what distance from the origin the trajectory of the particle will first hit one of the coordinate axis, expressed in terms of C and v_0. Which axis will it hit? At positive or negative value of the coordinate?