PHYSICS 4C — QUIZ 3 — FALL QUARTER 2007
PROF. HIRSCH — November 2nd, 2007

Formulas:

$\vec{F}_2 = \frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$ Coulomb's law ; $\vec{E} = \vec{F}/q_0$ electric field ; $\vec{E}(x,y,z) = \int \frac{\rho(x',y',z')(\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} dx'dy'dz'$

$\oint \vec{E}\cdot d\vec{a} = 4\pi q_{enc} = 4\pi\int \rho dv$ Gauss' law — 1 charge at the origin: $\vec{E}(\vec{r}) = \frac{q}{r^2}\hat{r}$

Linear, surface, volume charge density : $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

Electric field of : charge : $E = \frac{q}{r^2}$; line of charge : $E = \frac{2\lambda}{r}$; sheet of charge : $E = 2\pi\sigma$

Potential of single charge q : $\phi(\vec{r}) = \frac{q}{r}$; charge distribution : $\phi(\vec{r}) = \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dx'dy'dz'$

$\phi(x,y,z) - \phi(x_0,y_0,z_0) = -\int_{(x_0,y_0,z_0)}^{(x,y,z)} \vec{E}\cdot d\vec{s}$; $\vec{E} = -\nabla\phi$; $\nabla^2\phi = -4\pi\rho$; $\text{div}\vec{E} = 4\pi\rho$

$\text{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; $u = \frac{E^2}{8\pi}$ electric energy density

energy of 3 charges: $U = \frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}$; energy of q in potential ϕ : $U = q\,\phi(x,y,z)$

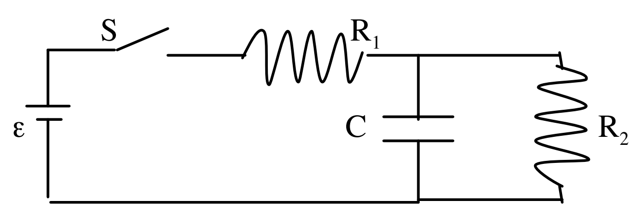
Electric field right next to a conducting surface: E=4πσ

Capacitors: Q=CV ; Parallel plates: $C = \frac{A}{4\pi s}$ A=area, s=dist. betw. plates; $U = \frac{Q^2}{2C}$ energy

$I = \int \vec{J}\cdot d\vec{a}$, $I = \frac{dq}{dt}$, $\vec{J} = nq\vec{u}$; $div\vec{J} = -\frac{\partial\rho}{\partial t}$; Power: $P = I^2 R$; $P = \varepsilon I$

V=IR , $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$; $R = \rho\frac{L}{A}$; $\sigma = \frac{ne^2\tau}{m_e}$; $Q(t) = C\varepsilon(1 - e^{-t/RC})$

Problem 1 (10 pts)



In the circuit above, ε=120V, R_1=20Ω, R_2=40Ω, C=1F. Initially the switch S is open and the capacitor is uncharged. At time t=0, S is closed.

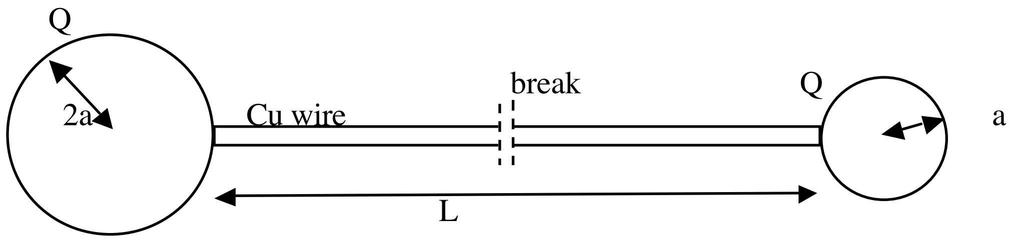
(a) What is the current flowing through R_1 right after S is closed, in A? Justify.

(b) Find the charge in C a long time after S is closed, in C (Coulombs).

(c) A long time after S is closed, it is opened again. Find the current through R_1 and through R_2 right after S is opened again, in A.

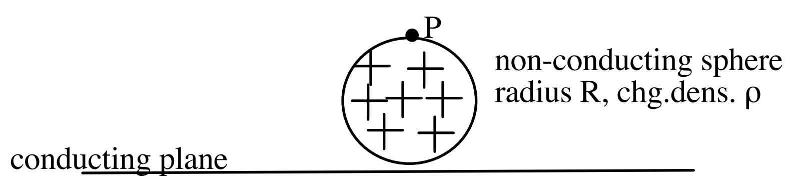
(d) Estimate roughly how long after S is opened again will C have lost its charge, in seconds.

Problem 2 (10 pts + 5 pts extra credit). Use cgs units for this problem.



Consider 2 conducting spheres of radius a and 2a, with a=1cm, each sphere with charge Q=1 esu, at distance L=1m from each other. Since L>>a you can assume that the potential of each sphere is independent of the potential of the other sphere.
A thin wire of Cu of cross-sectional area $A=1mm^2$ and resistivity $\rho=1.7x10^{-18}s$ runs between the spheres and has a break at the center. You can neglect the capacitance of the wire.
At time t=0 the two ends of the wire across the break are brought together so electrical contact is established.
(a) Find the resistance R of this Cu wire over its entire length, in sec/cm.
(b) Find the current I that flows through the wire immediately after t=0, in esu/s.
(c) After a long time passed, the charge on the big sphere is found to be (4/3)Q. What will be the charge on the small sphere? Explain why the charges take those values.
(d (extra credit) If I(t) is the current through the Cu wire at time t, find the value of the integral $\int_0^\infty dt I(t)^2 R$ in terms of Q and a, and expressed in ergs.
Hint: The capacitance of a sphere of radius R is R. The cgs unit of potential is statvolt.

Problem 3 (10 pts)



A non-conducting sphere of radius R and uniform volume charge density ρ rests on an infinite conducting plane. No charge transfer between the sphere and the plane occurs because the sphere is non-conducting.
(a) What is the total charge on the sphere, Q, in terms of ρ and R?
(b) Find the magnitude and direction of the electric field at point P shown in the figure, which is farthest away from the plane on the surface of the sphere.
(c) Find the magnitude and direction of the electric field at the center of the sphere.
(d) Find the surface charge density σ of the conducting plane at the point where the sphere touches the plane.
Hint: think about the similarities and differences between this situation and that of a point charge Q located at height R above the conducting plane.