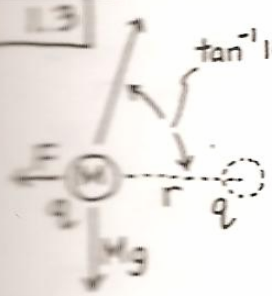
1.1 The ratio of the gravitational force Gm^2/r^2 to the electrical repulsion is Gm^2/e^2, which is

$$6.7 \times 10^{-8} \times (1.6 \times 10^{-24})^2/(4.8 \times 10^{-10})^2 = 7.4 \times 10^{-37}$$

For $r = 10^{-13}$ cm the electrical force e^2/r^2 is 2.3×10^6 dynes, equivalent to 23 newtons – about 5 pounds!

1.3



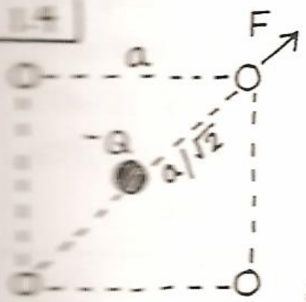
$$M = 0.3 \text{ kg} \quad g = 9.8 \text{ ms}^{-2} \quad r = 0.5 \text{ m}$$

$$F = 0.1\,Mg \qquad F = q^2/4\pi\epsilon_0 r^2$$

$$q = r(4\pi\epsilon_0 F)^{\frac{1}{2}}$$

$$q = 0.5(1.11 \times 10^{-10} \times 0.1 \times 0.3 \times 9.8)^{\frac{1}{2}} = 2.86 \times 10^{-6}\,\text{C}$$

1.4

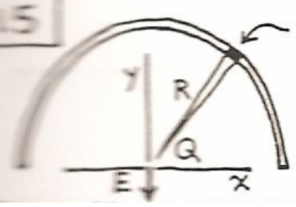


$$F = 2\left(\frac{\sqrt{2}}{2}\right)\frac{q^2}{a^2} + \frac{q^2}{(\sqrt{2}a)^2} - \frac{Qq}{(a/\sqrt{2})^2}$$

$$F = 0 \text{ for } Q = \left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)q = 0.957q$$

Equilibrium is unstable: inward displacement of a corner charge results in inward force on it.

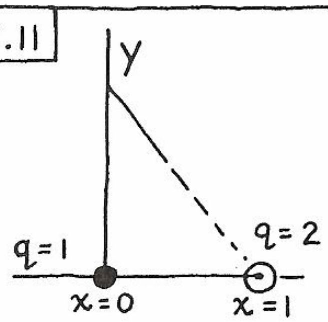
1.5



$$dQ = Q\frac{d\theta}{\pi} \qquad -dE_y = \frac{dQ}{R^2}\sin\theta$$

$$E = \frac{Q}{\pi R^2}\int_0^{\pi} \sin\theta\, d\theta = \frac{2Q}{\pi R^2}$$

$\boxed{1.11}$

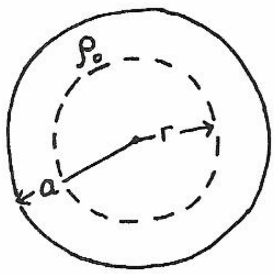


(a) The field E cannot be zero anywhere <u>between</u> two charges of opposite sign, or at any point closer to the greater than to the lesser charge. Hence the point we seek must lie on the negative x-axis. It is well to be clear about this before plunging into algebra. Let the point lie at $x = -s$. The field will vanish there if $1/s^2 = 2/(1+s)^2$, giving us the quadratic equation $s^2 - 2s - 1 = 0$, with roots $s = 1 \pm \sqrt{2}$. The positive root locates the point of vanishing field at $x = -2.414$. What is wrong with the other root? (b) At $(0, y)$ the field component E_y has the value $\frac{1}{y^2} - \frac{2y}{(1+y^2)^{3/2}}$. This vanishes

if $2y^3 = (1+y^2)^{3/2}$ which can be written

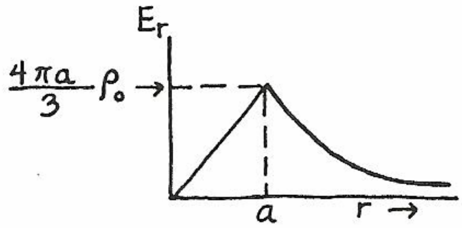
$2^{2/3} y^2 = 1 + y^2$, giving $y = 1/(2^{2/3} - 1)^{\frac{1}{2}} = 1.305$

$\boxed{1.15}$



On the sphere of radius r, the field is the same as if all charge inside that sphere, which amounts to $\frac{4\pi r^3 \rho_0}{3}$, were at the center:

$$E_r = \left(\tfrac{4}{3}\pi r^3 \rho_0\right)/r^2 = \frac{4\pi \rho_0 r}{3} \quad (r \leqslant a)$$

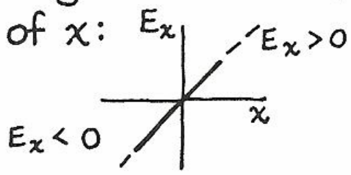


Outside the whole charge distribution the field is

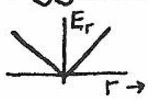
$$E_r = \frac{4}{3}\,\frac{\pi a^3 \rho_0}{r^2} \quad (r \geqslant a)$$

E_r is <u>continuous</u> at $r = a$

To discuss the continuity of the electric field in the neighborhood of the origin, look at E_x as a function of x:



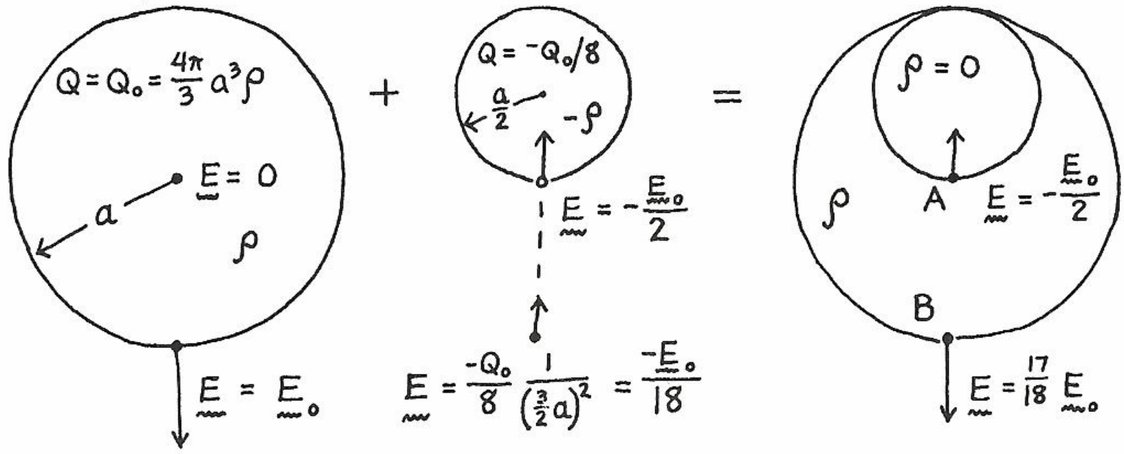
clearly there is no discontinuity; there is no singularity whatever. The singularity suggested by plotting E_r like this:



is spurious.

There is, properly, no negative range of r.

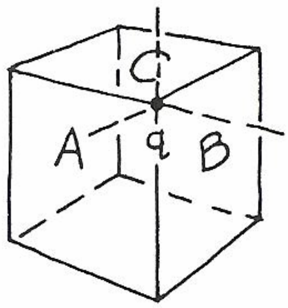
$\boxed{1.16}$ Use superposition:



A more remarkable fact, not hard to prove, is that within the cavity on the right the field E is perfectly uniform in magnitude and direction. It is $-E_0/2$ at <u>every</u> point in the cavity.

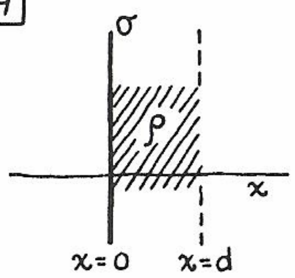
1.17 (a) The total flux is $4\pi q$ (Gauss's law). The flux through every face of the cube must be the same, because of symmetry. Hence, over any one of the six faces: $\int \underset{\sim}{E} \cdot d\underset{\sim}{a} = 4\pi q/6 = 2\pi q/3$

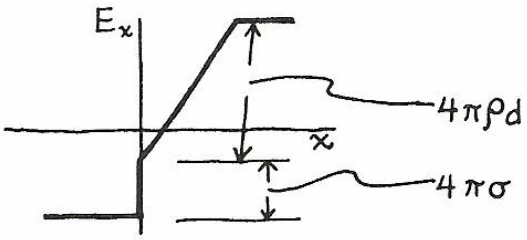
(b)



Because the field of q is parallel to the surface on each of the three faces A, B and C, the flux through these faces is zero. The flux through the other three faces must therefore add up to $4\pi q/8$, because our cube is one of eight such surrounding q. The three faces being symmetrically located with respect to q, the flux through each must be $\frac{1}{3} \cdot \frac{4\pi q}{8} = \frac{\pi q}{6}$.

1.19

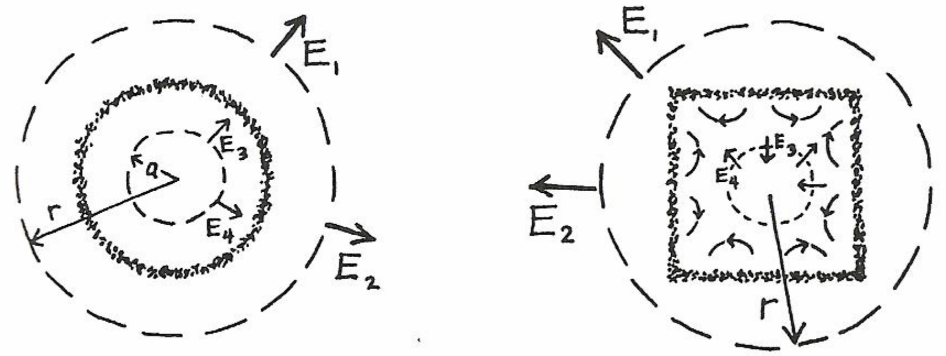




The total charge per cm² is $\sigma + \rho d$. For $x < 0$, $E_x = -2\pi(\sigma + \rho d)$ and for $x > d$ $E_x = 2\pi(\sigma + \rho d)$. More generally, at any point x, $E_x = -2\pi$ (charge per cm² to right of x) $+ 2\pi$ (charge per cm² to left of x) or $E_x = -2\pi(d-x)\rho + 2\pi(\sigma + x\rho)$ for $0 < x < d$.

$E_x = 2\pi\sigma - 2\pi\rho d + 4\pi\rho x$

1.20



The cylindrical tube of charge on the left has perfect axial symmetry. Hence $\mathbf{E}_1$ and $\mathbf{E}_2$ must be radial and equal in magnitude. Applying Gauss's Law, $2\pi r E_1 = 4\pi\lambda$, where λ is the charge per cm length of tube, and $E_1 = 2\lambda/r$, just as if the charge were concentrated on the axis. Inside the tube symmetry also demands that $E_3 = E_4$. But Gauss's Law requires that the surface integral over the cylinder of radius a be zero. Hence $E_3 = E_4 = 0$.

For the square tube of charge the integral over the cylinder of radius r must equal $4\pi \times$ charge enclosed, but nothing requires that $E_1 = E_2$. The integral of $\mathbf{E}$ over the small inner cylinder vanishes, but it can do so with $\mathbf{E}_3 \neq \mathbf{E}_4$ if, as is the case, they point in opposite directions. By comparing this tube with a square charged <u>conducting</u> tube, within which the field is indeed zero, you can deduce that E_4 must point inward (if the charge is positive).

$\boxed{1.21}$ The fraction of the negative charge which lies within a sphere of radius r_1 will be given by

$$\int_0^{r_1} \rho dv \Big/ \int_0^{\infty} \rho dv. \quad \text{With } \rho = Ce^{-2r/a_0} \text{ and } dv = 4\pi r^2 dr$$

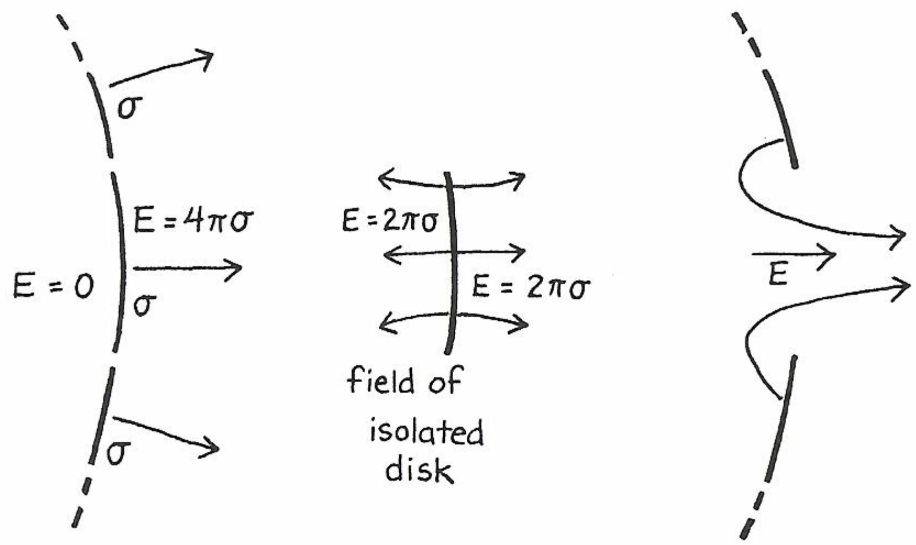
all we need is the integral

$$\int_0^{x_1} x^2 e^{-x} dx = -(x^2 + 2x + 2)e^{-x}\Big]_0^{x_1} = 2 - (x_1^2 + 2x_1 + 2)e^{-x_1}$$

For $x = \infty$ the integral is 2. For $r_1 = a_0$, $x_1 = 2$ and the integral is $2 - 10e^{-2}$. The fraction of electron charge inside $r = a_0$ is $(2 - 10e^{-2})/2 = 1 - \frac{5}{(2.178)^2} = 0.323$

The net positive charge inside $r = a_0$ is therefore $0.677 \times 4.8 \times 10^{-10}$ esu and the field E_r at that radius is $\dfrac{0.677 \times 4.8 \times 10^{-10}}{(0.53 \times 10^{-8})^2} = 1.15 \times 10^7$ dyne/esu

$\boxed{1.29}$



E at center of hole $= 2\pi\sigma$

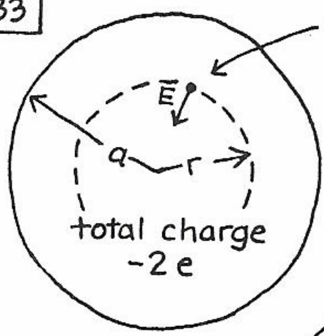
The field of the complete charged spherical shell is a superposition of the field of the charged disk and the field of the shell with a hole.

1.30

$$U = \frac{1}{8\pi}\int E^2 dv = \frac{1}{8\pi}\int_a^b \frac{Q^2}{r^4} 4\pi r^2 dr = \frac{Q^2}{2}\int_a^b \frac{dr}{r^2}$$

$$= \frac{Q^2}{2}\left(\frac{1}{a} - \frac{1}{b}\right)$$

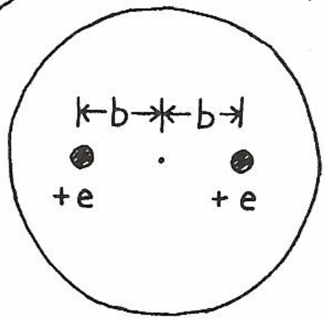
1.33



field at radius r :

$$E_r = \underbrace{-2e\left(\frac{r^3}{a^3}\right)}_{\text{charge inside radius } r}\frac{1}{r^2} = -\frac{2er}{a^3}$$

imbed protons in electron jelly:



total force on proton is zero if :

$$\frac{2e^2 b}{a^3} = \frac{e^2}{4b^2}, \quad \text{or}$$

$$b = a/2$$