5-4 Taking $\lambda = 0.1$ nm and using $p = \frac{h}{\lambda} = mv$, we get

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{Js}}{9.11\times10^{-31}\ \text{kg}}(0.1\times10^{-9}\ \text{m}) = 7.28\times10^{6}\ \text{m/s}.$$

As $v \ll c$, it is okay to use $p = mv$ instead of $p = \gamma mv$.

5-5 (a) $\lambda = \frac{h}{p}$ or $p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1\,240\ \text{eV nm}}{(10\ \text{nm})(c)} = \frac{124\ \text{eV}}{c}$. As

$$K = E - mc^2 = \left[p^2c^2 + \left(mc^2\right)^2\right]^{1/2} - mc^2,$$

we must use the relativistic expression for K, when $pc \approx mc^2$. Here $pc = 124\ \text{eV} \ll mc^2 = 0.511\ \text{MeV}$, so we can use the classical expression for K, $K = \frac{p^2}{2m}$.

$$K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(124\ \text{eV})^2}{2(0.511\ \text{MeV})} = 0.150\ \text{eV}$$

(b) Electrons with $\lambda = 0.10$ nm $p = \frac{hc}{\lambda c} = \frac{12\,400\ \text{eV}}{c}$ as in (a). As $pc \ll mc^2 = 0.511\ \text{MeV}$, use $K = \frac{p^2}{2m} = p^2c^2 = \frac{(12\,400)^2(\text{eV})^2}{(2)(0.511\times10^{6}\ \text{eV})} = 150\ \text{eV}$.

(c) Electrons with $\lambda = 10\ \text{fm} = 10\times10^{-15}$ m, $p = \frac{hc}{\lambda c} = \frac{1.24\times10^{3}\ \text{MeV}}{c}$. As $pc \gg mc^2 = 0.511\ \text{MeV}$, use

$$K = \left[p^2c^2 + \left(mc^2\right)^2\right]^{1/2} - mc^2 = pc - mc^2 = 1\,240\ \text{MeV} - 0.511\ \text{MeV} = 1\,239\ \text{MeV}.$$

For alphas with $mc^2 = 3\,726$ MeV:

(a) p still is $\frac{124\ \text{eV}}{c}$. As $pc \ll 3\,726$ MeV, we use $K = \frac{p^2}{2m}$:

$$K = \frac{p^2c^2}{2mc^2} = \frac{(124\ \text{eV})^2}{(2)(3\,726\ \text{MeV})} = 2.06\times10^{-6}\ \text{eV}.$$

(b) For alphas with $\lambda = 0.10$ nm, $p = \frac{12\,400\ \text{eV}}{c}$. As $pc \ll mc^2 = 3\,726$ MeV, $K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(12\,400\ \text{eV})^2}{(2)(3\,726\ \text{MeV})} = 0.020\,6\ \text{eV}$.

(c) $p = \frac{1.24 \times 10^3 \text{ MeV}}{c}$ and $pc = 1\,240 \text{ MeV} \sim mc^2 = 3\,726 \text{ MeV}$. We use

$$K = \left[p^2c^2 + \left(mc^2\right)^2\right]^{1/2} - mc^2 = \left[(1\,240 \text{ MeV})^2 + (3\,726 \text{ MeV})^2\right]^{1/2} - 3\,726 \text{ MeV}$$
$$= 201 \text{ MeV}.$$

5-6 From Problem 5-2, a 50 keV electron has $\lambda = 5.36 \times 10^{-3}$ nm. A 50 keV proton has $K = 50 \text{ keV} << 2mc^2 = 1\,877 \text{ MeV}$ so we use $p = (2mK)^{1/2}$:

$$\lambda = \frac{h}{p} = \frac{h}{\left[(2)\left(\frac{938.3 \text{ MeV}}{c^2}\right)(50 \text{ keV})\right]^{1/2}} = \frac{hc}{[(2)(938.3 \text{ MeV})(50 \text{ keV})]^{1/2}}$$
$$= \frac{1\,240 \text{ eV nm}}{\left[(2)\left(938.3 \times 10^3 \text{ eV}\right)\left(50 \times 10^3 \text{ eV}\right)\right]^{1/2}} = 1.28 \times 10^{-4} \text{ nm}$$

5-7 A 10 MeV proton has $K = 10 \text{ MeV} << 2mc^2 = 1\,877 \text{ MeV}$ so we can use the classical expression $p = (2mK)^{1/2}$. (See Problem 5-2)

$$\lambda = \frac{h}{p} = \frac{hc}{[(2)(938.3 \text{ MeV})(10 \text{ MeV})]^{1/2}} = \frac{1\,240 \text{ MeV fm}}{\left[(2)(938.3)(10)(\text{MeV})^2\right]^{1/2}} = 9.05 \text{ fm} = 9.05 \times 10^{-15} \text{ m}$$

5-8 $$\lambda = \frac{h}{p} = \frac{h}{(2mK)^{1/2}} = \frac{h}{(2meV)^{1/2}} = \left[\frac{h}{(2me)^{1/2}}\right] V^{-1/2}$$

$$\lambda = \left[\frac{6.626 \times 10^{-34} \text{ Js}}{\left(2 \times 9.105 \times 10^{-31} \text{ kg} \times 1.602 \times 10^{-19} \text{ C}\right)^{1/2}}\right] V^{-1/2}$$

$$\lambda = \left[\frac{1.226 \times 10^{-9} \text{ kg}^{1/2}\text{m}^2}{\text{sC}^{1/2}}\right] V^{-1/2}$$

5-9 $m = 0.20 \text{ kg}: mgh = \frac{mv^2}{2}: v = (2gh)^{1/2}$

$$p = mv = m(2gh)^{1/2} = (0.20)[2(9.80)(50)]^{1/2} = 6.261 \text{ kg} \cdot \text{m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{6.261 \text{ kg} \cdot \text{m/s}} = 1.06 \times 10^{-34} \text{ m}$$

5-10 As $\lambda = 2a_0 = 2(0.052\,9) \text{ nm} = 0.105\,8 \text{ nm}$ the energy of the electron is nonrelativistic, so we can use

$$p = \frac{h}{\lambda} \text{ with } K = \frac{p^2}{2m};$$

$$K = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{2\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.058 \times 10^{-10} \text{ m}\right)^2} = 21.5 \times 10^{-18} \text{ J} = 134 \text{ eV}$$

This is about ten times as large as the ground-state energy of hydrogen, which is 13.6 eV.

5-11 (a) In this problem, the electron must be treated relativistically because we must use relativity when $pc \approx mc^2$. (See problem 5-5). the momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 6.626 \times 10^{-20}\ \text{kg}\cdot\text{m/s}$$

and $pc = 124\ \text{MeV} >> mc^2 = 0.511\ \text{MeV}$. The energy of the electron is

$$\begin{aligned} E &= \left(p^2c^2 + m^2c^4\right)^{1/2} \\ &= \left[\left(6.626 \times 10^{-20}\ \text{kg}\cdot\text{m/s}\right)^2\left(3 \times 10^8\ \text{m/s}\right)^2 + \left(0.511 \times 10^6\ \text{eV}\right)^2\left(1.602 \times 10^{-19}\ \text{J/eV}\right)^2\right]^{1/2} \\ &= 1.99 \times 10^{-11}\ \text{J} = 1.24 \times 10^8\ \text{eV} \end{aligned}$$

so that $K = E - mc^2 \approx 124\ \text{MeV}$.

(b) The kinetic energy is too large to expect that the electron could be confined to a region the size of the nucleus.

5-12 Using $p = \frac{h}{\lambda} = mv$, we find that $v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(1 \times 10^{-10}\ \text{m}\right)} = 7.27 \times 10^6\ \text{m/s}$. From the principle of conservation of energy, we get

$$eV = \frac{mv^2}{2} = \frac{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(7.27 \times 10^6\ \text{m/s}\right)^2}{2} = 2.41 \times 10^{-17}\ \text{J} = 151\ \text{eV}.$$

Therefore $V = 151\ \text{V}$.

5-13 A canceling wave will be produced when the path length difference between the surface reflection and the reflection from the nth plane below the surface equals some whole number of wavelengths plus $\frac{\lambda}{2}$ As the path length difference between a surface reflection and a reflection from plane n is given by $(n)(1.01\lambda)$, we find that a reflection from the 50th plane has a path difference of 50.5λ with the surface reflection, and consequently cancels the surface reflection. Essentially all waves reflected at θ will cancel as the wave reflected from the second plane will be cancelled by a reflection from the 51st plane and so on.

5-14 (a) $n\lambda = d\sin\phi$ or

$$\sin\phi = \frac{n\lambda}{d} = \frac{n}{d}\frac{h}{p} = \frac{n}{d}\left(\frac{h}{(2m_eK)^{1/2}}\right) = \left(\frac{nhc}{d}\right)\left(2m_ec^2K\right)^{1/2}$$

(b)

$$d_1 = \frac{nhc}{(\sin\phi)\left(2m_ec^2K\right)^{1/2}} = \frac{(1)\left(12.40 \times 10^{-7}\ \text{eV}\cdot\text{m}\right)}{(\sin 24.1°)\left(2 \times 0.511 \times 10^6\ \text{eV} \times 100\ \text{eV}\right)^{1/2}} = 3.00 \times 10^{-10}\ \text{m}$$

$$= 3.00\text{Å}$$

$$d_2 = \frac{(2)\left(12.40 \times 10^{-7}\ \text{eV}\cdot\text{m}\right)}{(\sin 54.9°)\left(2 \times 0.511 \times 10^6\ \text{eV} \times 100\ \text{eV}\right)^{1/2}} = 3.00 \times 10^{-10}\ \text{m}$$

As we obtain the same spacing in both cases, 24.1° must correspond to $n = 1$ and 54.9° to $n = 2$.

5-15 For a free, non-relativistic electron $E=\frac{m_e v_0^2}{2}=\frac{p^2}{2m_e}$. As the wavenumber and angular frequency of the electron's de Broglie wave are given by $p=\hbar k$ and $E=\hbar\omega$, substituting these results gives the dispersion relation $\omega=\frac{\hbar k^2}{2m_e}$. So $v_g=\frac{d\omega}{dk}=\frac{\hbar k}{m_e}=\frac{p}{m_e}=v_0$.

5-16 $v_p=\left(\frac{2\pi S}{\lambda\rho}\right)^{1/2}$ means that individual harmonic waves (ripples) with short wavelength travel fastest. A surface pulse, such as that generated by a stone, is composed of a spread of wavelengths or wavenumbers centered on k_0. The pulse travels with an average velocity v_g while individual ripples propagate through the pulse through the pulse at, v_p. Writing

$$v_p=k^{1/2}\left(\frac{S}{\rho}\right)^{1/2}$$

$$v_g=v_p\big|_{k_0}+k\left(\frac{dv_p}{dk}\right)_{k_0}=k^{1/2}\left(\frac{S}{\rho}\right)^{1/2}+(k_0)\left(\frac{1}{2}\right)\left(\frac{S}{\rho}\right)^{1/2}=\frac{3}{2}k_0^{1/2}\left(\frac{S}{\rho}\right)^{1/2}$$

Assuming that the dominant individual wave has $k=k_0$, $v_p=k_0^{1/2}\left(\frac{S}{\rho}\right)^{1/2}$. Hence $v_g=v_p$ and the individual ripples move inward through the disturbance.

5-17 $E^2=p^2c^2+\left(m_ec^2\right)^2$

$E=\left[p^2c^2+\left(m_ec^2\right)^2\right]^{1/2}$. As $E=\hbar\omega$ and $p=\hbar k$

$\hbar\omega=\left[\hbar^2k^2c^2+\left(m_ec^2\right)^2\right]^{1/2}$ or

$$\omega(k)=\left[k^2c^2+\frac{\left(m_ec^2\right)^2}{\hbar^2}\right]^{1/2}$$

$$v_p=\frac{\omega}{k}=\frac{\left[k^2c^2+\left(m_ec^2/\hbar\right)^2\right]^{1/2}}{k}=\left[c^2+\left(\frac{m_ec^2}{\hbar k}\right)^2\right]^{1/2}$$

$$v_g=\frac{d\omega}{dk}\bigg|_{k_0}=\frac{1}{2}\left[k^2c^2+\left(\frac{m_ec^2}{\hbar}\right)^2\right]^{-1/2}2kc^2=\frac{kc^2}{\left[k^2c^2+\left(m_ec^2/\hbar\right)^2\right]^{1/2}}$$

$$v_pv_g=\left\{\frac{\left[k^2c^2+\left(m_ec^2/\hbar\right)^2\right]^{1/2}}{k}\right\}\left\{\left[k^2c^2+\left(m_ec^2/\hbar\right)^2\right]^{1/2}\right\}=c^2$$

Therefore, $v_g<c$ if $v_p>c$.

5-18 $\Delta x\Delta p\geq\frac{\hbar}{2}$ where $\Delta p=m\Delta v=(0.05\text{ kg})(10^{-3}\times 30\text{ m/s})=1.5\times10^{-3}\text{ kg}\cdot\text{m/s}$. Therefore,

$\Delta x=\frac{\hbar}{2\Delta p}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(1.5\times10^{-3}\text{ kg}\cdot\text{m/s})}=3.51\times10^{-32}\text{ m}$.

5-19 $K=\frac{mv^2}{2}=\frac{p^2}{2m}$: $(1\times10^6\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})=\frac{p^2}{2(1.67\times10^{-27}\ \text{kg})}\Rightarrow p=2.312\times10^{-20}\ \text{kg}\cdot\text{m/s}$,

$\Delta p=0.05p=1.160\times10^{-21}\ \text{kg}\cdot\text{m/s}$ and $\Delta x\Delta p=\frac{h}{4\pi}$. Thus

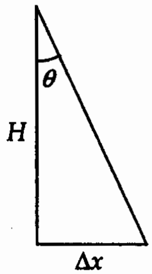
$$\Delta x=\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.16\times10^{-21}\ \text{kg}\cdot\text{m/s})(4\pi)}=4.56\times10^{-14}\ \text{m}.$$

Note that non-relativistic treatment has been used, which is justified because the kinetic energy is only $\frac{(1.6\times10^{-13})\times100\%}{1.50\times10^{-10}}=0.11\%$ of the rest energy.

5-20 $p=\frac{h}{\lambda}$; $\frac{\Delta p}{\Delta\lambda}\approx\frac{dp}{d\lambda}=-\frac{h}{\lambda^2}$, so $\Delta p\approx\frac{-h\Delta\lambda}{\lambda^2}$. As $(\Delta p\Delta x)_{\text{min}}=\frac{\hbar}{2}$,

$$|\Delta x_{\text{min}}|=\left|\frac{\hbar}{2\Delta p}\right|=\frac{\hbar}{2(h\Delta\lambda/\lambda^2)}=\frac{h\lambda}{2(h\Delta\lambda/\lambda)}=\frac{\lambda}{4\pi(\Delta\lambda/\lambda)}=\frac{6\,000\ \text{Å}}{4\pi(10^{-6})}=4.78\times10^8\ \text{Å}=4.78\times10^{-2}\ \text{m}.$$

5-21 (a) The woman tries to hold a pellet within some horizontal region Δx_i and directly above the spot on the floor. The uncertainty principle requires her to give a pellet some x velocity at least as large as $\Delta v_x=\frac{\hbar}{2m\Delta x_i}$. When the pellet hits the floor at time t, the total miss distance is $\Delta x_{\text{total}}=\Delta x_i+\Delta v_x t=\Delta x_i+\left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}}$. The minimum value of the function Δx_{total} occurs for $\frac{d(\Delta x_{\text{total}})}{d(\Delta x_i)}=0$ or $1-\frac{\hbar}{2m}\sqrt{\frac{2H}{g}}(\Delta x_i)^{-2}=0$.



We find $\Delta x_i=\left(\frac{\hbar}{2m}\right)^{1/2}\left(\frac{2H}{g}\right)^{1/4}$.

(b) For $H=2.0$ m, $m=0.50$ g, $\Delta x_{\text{total}}=5.2\times10^{-16}$ m.

5-24 (a) $\Delta x \Delta p = \hbar$ so if $\Delta x = r$, $\Delta p \approx \frac{\hbar}{r}$

(b) $$K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \frac{\hbar^2}{2m_e r^2}$$
$$U = -\frac{ke^2}{r}$$
$$E = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}$$

(c) To minimize E take $\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{ke^2}{r^2} = 0 \Rightarrow r = \frac{\hbar^2}{m_e ke^2}$ = Bohr radius $= a_0$. Then
$$E = \left(\frac{\hbar}{2m_e}\right)\left(\frac{m_e ke^2}{\hbar^2}\right)^2 - ke^2\left(\frac{m_e ke^2}{\hbar^2}\right) = \frac{m_e k^2 e^4}{2\hbar^2} = -13.6 \text{ eV}.$$

5-25 To find the energy width of the γ-ray use $\Delta E \Delta t \geq \frac{\hbar}{2}$ or

$$\Delta E \geq \frac{\hbar}{2\Delta t} \geq \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{(2)(0.10 \times 10^{-9} \text{ s})} \geq 3.29 \times 10^{-6} \text{ eV}.$$

As the intrinsic energy width of $\sim \pm 3 \times 10^{-6}$ eV is so much less than the experimental resolution of ± 5 eV, the intrinsic width can't be measured using this method.

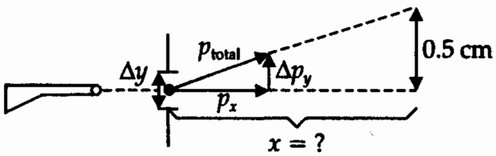
5-26 The full width at half-maximum (FWHM) is 110 MeV. So $\Delta E = 55$ MeV and using $\Delta E_{min} \Delta t_{min} = \frac{\hbar}{2}$,

$$\Delta t_{min} = \frac{\hbar}{2\Delta E} = \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{2(55 \times 10^6 \text{ eV})} \cong 6.0 \times 10^{-24} \text{ s}$$
$$\tau = \text{lifetime} \sim 2\Delta t_{min} = 1.2 \times 10^{-23} \text{ s}$$

5-27 For a single slit with width a, minima are given by $\sin\theta = \frac{n\lambda}{a}$ where $n = 1, 2, 3, \ldots$ and $\sin\theta \approx \tan\theta = \frac{x}{L}$, $\frac{x_1}{L} = \frac{\lambda}{a}$ and $\frac{x_2}{L} = \frac{2\lambda}{a} \Rightarrow \frac{x_2 - x_1}{L} = \frac{\lambda}{a}$ or

$$\lambda = \frac{a\Delta x}{L} = \frac{5 \text{ Å} \times 2.1 \text{ cm}}{20 \text{ cm}} = 0.525 \text{ Å}$$
$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} = \frac{(1.24 \times 10^4 \text{ eV} \cdot \text{Å})^2}{2(5.11 \times 10^5 \text{ eV})(0.525 \text{ Å})^2} = 546 \text{ eV}$$

5-31



$\Delta y \Delta p_y \sim \hbar \quad \Delta p_y = \frac{\hbar}{\Delta y}$. From the diagram, because the momentum triangle and space triangle are similar, $\frac{\Delta p_y}{p_x} = \frac{0.5 \text{ cm}}{x}$;

$$x = \frac{(0.5 \text{ cm})p_x}{\Delta p_y} = \frac{(0.5 \text{ cm})p_x \Delta y}{\hbar} = \frac{(0.5\times10^{-2} \text{ m})(0.001 \text{ kg})(100 \text{ m/s})(2\times10^{-3} \text{ m})}{1.05\times10^{-34} \text{ J}\cdot\text{s}}$$
$$= 9.5\times10^{27} \text{ m}$$

Once again we see that the uncertainty relation has no observable consequences for macroscopic systems.

5-32 (a) $f = \frac{E}{h} = \frac{(1.8)(1.6\times10^{-19} \text{ J})}{6.63\times10^{-34} \text{ J}\cdot\text{s}} = 4.34\times10^{14} \text{ Hz}$

(b) $\lambda = \frac{c}{f} = 691 \text{ nm}$

(c) $\Delta E \geq \frac{\hbar}{\Delta t} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{2\pi(2\times10^{-6} \text{ s})}$

$\Delta E \geq 5.276\times10^{-29} \text{ J} = 3.30\times10^{-10} \text{ eV}$

5-33 From the uncertainty principle, $\Delta E \Delta t \sim \hbar \qquad \Delta mc^2 \Delta t = \hbar$. Therefore,

$$\frac{\Delta m}{m} = \frac{h}{2\pi c^2 \Delta t m} = \frac{h}{2\pi \Delta t E_{rest}} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{2\pi(8.7\times10^{-17} \text{ s})(135\times10^{6} \text{ eV})(1.6\times10^{-19} \text{ J/eV})} = 5.62\times10^{-8}.$$

5-34 (a) $g(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} V(t)(\cos\omega t - i\sin\omega t)dt$, $V(t)\sin\omega t$ is an odd function so this integral vanishes leaving $g(\omega) = 2(2\pi)^{-1/2} \int_0^{\tau} V_0 \cos\omega t dt = \left(\frac{2}{\pi}\right)^{1/2} V_0 \frac{\sin\omega\tau}{\omega}$. A sketch of $g(\omega)$ is given below.