4-8 (a) From Equation 4.16 we have $\Delta n \propto \left(\frac{\sin\phi}{2}\right)^{-4}$ or $\Delta n_2 = \Delta n_1 \frac{\left(\frac{\sin\phi_1}{2}\right)^4}{\left(\frac{\sin\phi_2}{2}\right)^4}$. Thus the number of α's scattered at 40 degrees is given by

$$\Delta n_2 = (100 \text{ cpm})\frac{(\sin\frac{20}{2})^4}{(\sin\frac{40}{2})^4} = (100 \text{ cpm})\left(\frac{\sin 10}{\sin 20}\right)^4 = 6.64 \text{ cpm}.$$

Similarly

4-8 a)
It should read "sin(phi/2)",
not "sin(phi)/2"

$$\Delta n \text{ at } 60 \text{ degrees} = 1.45 \text{ cpm}$$
$$\Delta n \text{ at } 80 \text{ degrees} = 0.533 \text{ cpm}$$
$$\Delta n \text{ at } 100 \text{ degrees} = 0.264 \text{ cpm}$$

(b) From 4.16 doubling $\left(\frac{1}{2}\right)m_\alpha v_\alpha^2$ reduces Δn by a factor of 4. Thus Δn at 20 degrees $= \left(\frac{1}{4}\right)(100 \text{ cpm}) = 25 \text{ cpm}$.

(c) From 4.16 we find $\frac{\Delta n_{\text{Cu}}}{\Delta n_{\text{Au}}} = \frac{Z_{\text{Cu}}^2 N_{\text{Cu}}}{Z_{\text{Au}}^2 N_{\text{Au}}}$, $Z_{\text{Cu}} = 29$, $Z_{\text{Au}} = 79$.

$$\begin{aligned} N_{\text{Cu}} &= \text{number of Cu nuclei per unit area} \\ &= \text{number of Cu nuclei per unit volume} * \text{foil thickness} \\ &= \left[(8.9 \text{ g/cm}^3)\left(\frac{6.02\times10^{23} \text{ nuclei}}{63.54 \text{ g}}\right)\right]t = 8.43\times10^{22}t \\ N_{\text{Au}} &= \left[(19.3 \text{ g/cm}^3)\left(\frac{6.02\times10^{23} \text{ nuclei}}{197.0 \text{ g}}\right)\right]t = 5.90\times10^{22}t \end{aligned}$$

$$\text{So } \Delta n_{\text{Cu}} = \Delta n_{\text{Au}}(29)^2\frac{8.43\times10^{22}}{(79)^2}(5.90\times10^2) = (100)\left(\frac{29}{79}\right)^2\left(\frac{8.43}{5.90}\right) = 19.3 \text{ cpm}.$$

4-9 The initial energy of the system of α plus copper nucleus is 13.9 MeV and is just the kinetic energy of the α when the α is far from the nucleus. The final energy of the system may be evaluated at the point of closest approach when the kinetic energy is zero and the potential energy is $k(2e)\frac{Ze}{r}$ where r is approximately equal to the nuclear radius of copper. Invoking conservation of energy $E_i = E_f$, $K_\alpha = (k)\frac{2Ze^2}{r}$ or

$$r = (k)\frac{2Ze^2}{K_\alpha} = \frac{(2)(29)(1.60\times10^{-19})^2(8.99\times10^9)}{(13.9\times10^6 \text{ eV})(1.60\times10^{-19} \text{ J/eV})} = 6.00\times10^{-15} \text{ m}.$$

4-11 $\frac{1}{\lambda}=R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$. For the Balmer series, $n_f=2$; $n_i=3, 4, 5, \ldots$. The first three lines in the series have wavelengths given by $\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$ where $R=1.097\,37\times10^{7}\ \text{m}^{-1}$.

4-12
The right hand side of the first equation should be R(1 - 1/n^2), as in 4-13 (a)

$$\text{1st line: } \frac{1}{\lambda}=R\left(\frac{1}{4}-\frac{1}{9}\right)=\left(\frac{5}{36}\right)R;\ \lambda=\frac{36}{5R}=656.112\ \text{nm}$$

$$\text{2nd line: } \frac{1}{\lambda}=R\left(\frac{1}{4}-\frac{1}{16}\right)=\left(\frac{3}{16}\right)R;\ \lambda=\frac{16}{3R}=486.009\ \text{nm}$$

$$\text{3rd line: } \frac{1}{\lambda}=R\left(\frac{1}{4}-\frac{1}{25}\right)=\left(\frac{21}{100}\right)R;\ \lambda=\frac{100}{21R}=433.937\ \text{nm}$$

4-12 $\frac{1}{\lambda}=R\left(\frac{1-1}{n^2}\right)$ where $n=2, 3, 4, \ldots$ and $R=1.097\,373\,2\times10^{7}\ \text{m}^{-1}$;

$$\text{For } n=2\text{: } \lambda=R^{-1}\left(1-\frac{1}{2^2}\right)^{-1}=1.215\,02\times10^{-7}\ \text{m}=121.502\ \text{nm (UV)}$$

$$\text{For } n=3\text{: } \lambda=R^{-1}\left(1-\frac{1}{3^2}\right)^{-1}=1.025\,17\times10^{-7}\ \text{m}=102.517\ \text{nm (UV)}$$

$$\text{For } n=4\text{: } \lambda=R^{-1}\left(1-\frac{1}{4^2}\right)^{-1}=1.972\,018\times10^{-7}\ \text{m}=97.201\,8\ \text{nm (UV)}$$

4-13 (a) $$\lambda=102.6\ \text{nm};\ \frac{1}{\lambda}=R\left(1-\frac{1}{n^2}\right)\Rightarrow n=\frac{R}{\left(R-\frac{1}{\lambda}\right)^{1/2}}=\frac{R}{\left(R-\frac{1}{102.6\times10^{-9}\ \text{m}}\right)^{1/2}}=2.99\approx3$$

(b) This wavelength cannot belong to either series. Both the Paschen and Brackett series lie in the IR region, whereas the wavelength of 102.6 nm lies in the UV region.

4-14 (a) $r_n=\frac{n^2\hbar^2}{m_eke^2}$; where $n=1, 2, 3, \ldots$

$$r_n=n^2\frac{(1.055\times10^{-34}\ \text{Js})^2}{(9.11\times10^{-31}\ \text{kg})(9.0\times19^{9}\ \text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}=(0.052\,9\ \text{nm})n^2$$

For $n=1$: $r_n=0.052\,9$ nm
For $n=2$: $r_n=0.212\,1$ nm
For $n=3$: $r_n=0.477\,2$ nm

(b) From Equation 4.26, $v=\left(\frac{ke^2}{m_er}\right)^{1/2}$

$$v_1=\left[\frac{(8.99\times10^{9}\ \text{Nm}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(0.052\,9\times10^{-9}\ \text{m})}\right]^{1/2}=2.19\times10^{6}\ \text{m/s}$$

$v_2=1.09\times10^{6}$ m/s
$v_3=7.28\times10^{5}$ m/s

(c) As $c=3.0\times10^{8}$ m/s, $v\ll c$ and no relativistic correction is necessary.

4-15 (a) The energy levels of a hydrogen-like ion whose charge number is 2 is given by $E_n = (-13.6\text{ eV})\dfrac{Z^2}{n^2} = \dfrac{-54.4\text{ eV}}{n^2}$ for $Z = 2$. (He^+)

0
$E_3 = -6.04\text{ eV}$
$E_2 = -13.6\text{ eV}$

So $E_1 = -54.4\text{ eV}$
$E_2 = -13.6\text{ eV}$
$E_3 = -6.04\text{ eV}$, etc.

$E_1 = -54.4\text{ eV}$

(b) For He^+, $Z = 2$ so we see that the ionization energy (the energy required to take the electron from the state $n = 1$ to the state $n = \infty$ is $E = (-13.6\text{ eV})\dfrac{2^2}{1^2} = \dfrac{-54.4\text{ eV}}{n^2}$ for $Z = 2$. (He^+)

4-16 For Li^{2+}, $Z = 3$ from Equation 4.36

0
$E_3 = -13.6\text{ eV}$

$$E_n = -\frac{13.6Z^2}{n^2\text{ eV}} = -\frac{122.4}{n^2\text{ eV}}$$

$E_2 = -30.6\text{ eV}$

So $E_1 = -122.4\text{ eV}$
$E_2 = -30.6\text{ eV}$
$E_3 = -13.6\text{ eV}$, etc.

$E_1 = -122.4\text{ eV}$

4-17 $r = \dfrac{n^2\hbar^2}{Zm_eke^2} = \left(\dfrac{n^2}{Z}\right)\left(\dfrac{\hbar^2}{m_eke^2}\right)$; $n = 1$

$$r = \frac{1}{Z}\left[\frac{\left(1.055\times10^{-34}\text{ Js}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(9\times10^{9}\text{ Nm}^2/\text{C}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2}\right] = \frac{5.30\times10^{-11}\text{ m}}{Z}$$

(a) For He^+, $Z = 2$, $r = \dfrac{5.30\times10^{-11}\text{ m}}{2} = 2.65\times10^{-11}\text{ m} = 0.026\,5\text{ nm}$

(b) For Li^{2+}, $Z = 3$, $r = \dfrac{5.30\times10^{-11}\text{ m}}{3} = 1.77\times10^{-11}\text{ m} = 0.017\,7\text{ nm}$

(c) For Be^{3+}, $Z = 4$, $r = \dfrac{5.30\times10^{-11}\text{ m}}{4} = 1.32\times10^{-11}\text{ m} = 0.013\,2\text{ nm}$

4-18 (a) Just use energy conservation
4-18 (b) You must consider the possible intermediate transitions, i.e. a transition either from n=3 to n=1 directly, or from n=3 to n=2 and then from n=2 to n=1

4-18 (a) $\Delta E = (13.6\text{ eV})\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.1\text{ eV}$

(b) Either $\Delta E = 12.1\text{ eV}$ or $\Delta E = (13.6\text{ eV})\left(\frac{1}{1} - \frac{1}{2^2}\right) = 10.2\text{ eV}$ and

$$\Delta E = (13.6\text{ eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89\text{ eV}.$$

4-19 (a) $\Delta E = (-13.6\text{ eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = (-13.6\text{ eV})\left(\frac{1}{9} - \frac{1}{4}\right) = 1.89\text{ eV}$

(b) $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \left(4.14 \times 10^{-15}\text{ eV s}\right)\frac{3 \times 10^8\text{ m/s}}{1.89\text{ eV}} = 658\text{ nm}$

(c) $c = \lambda f \Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8\text{ m/s}}{657 \times 10^{-9}\text{ m}} = 4.56 \times 10^{14}\text{ Hz}$

4-20 $\Delta E = (-13.6\text{ eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

(a) $\Delta E = (-13.6\text{ eV})\left(\frac{1}{25} - \frac{1}{16}\right) = 0.306\text{ eV}$

(b) $\Delta E = (-13.6\text{ eV})\left(\frac{1}{36} - \frac{1}{25}\right) = 0.166\text{ eV}$

4-21 (a) For the Paschen series; $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$; the maximum wavelength corresponds to $n_i = 4$, $\frac{1}{\lambda_{\max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$; $\lambda_{\max} = 1\,874.606\text{ nm}$. For minimum wavelength, $n_i \to \infty$, $\frac{1}{\lambda_{\min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$; $\lambda_{\min} = \frac{9}{R} = 820.140\text{ nm}$.

(b) $\frac{hc}{\lambda_{\min}} = \frac{\left(\frac{hc}{1\,874.606\text{ nm}}\right)}{1.6 \times 10^{-19}\text{ J/eV}} = 0.662\,7\text{ nm}$

$\frac{hc}{\lambda_{\min}} = \frac{\left(\frac{hc}{820.140\text{ nm}}\right)}{1.6 \times 10^{-19}\text{ J/eV}} = 1.515\text{ nm}$

4-22 $E = K + U = \frac{mv^2}{2} - \frac{ke^2}{r}$. But $\frac{mv^2}{2} = \left(\frac{1}{2}\right)\frac{ke^2}{r}$. Thus $E = \left(\frac{1}{2}\right)\left(\frac{-ke^2}{r}\right) = \frac{U}{2}$, so $U = 2E = 2(-13.6\text{ eV}) = -27.2\text{ eV}$ and $K = E - U = -13.6\text{ eV} - (-27.2\text{ eV}) = 13.6\text{ eV}$.

4-23 (a) $r_1 = (0.0529 \text{ nm})n^2 = 0.0529 \text{ nm (when } n = 1)$

4-23 (b) You may find the velocity either by the quantization condition on angular momentum, or by setting the electrostatic force equal to the centripetal force on the electron.

(b) $m_e v = m_e\left(\frac{ke^2}{m_e r}\right)^{1/2}$

$$m_e = \left[\frac{(9.1\times10^{-31}\text{ kg})(9\times10^{9}\text{ Nm}^2/\text{C}^2)}{5.29\times10^{-11}\text{ m}}\right]^{1/2}\times(1.6\times10^{-19}\text{ C})$$

$M_e v = 1.99\times10^{-24}\text{ kg m/s}$

(c) $L = m_e vr = (1.99\times10^{-24}\text{ kg m/s})(5.29\times10^{-11}\text{ m}),\ L = 1.05\times10^{-34}(\text{kg m}^2/\text{s}) = \hbar$

(d) $K = |E| = 13.6 \text{ eV}$

(e) $U = -2K = -27.2 \text{ eV}$

(f) $E = K + U = -13.6 \text{ eV}$

4-23 (*d*) *and* (*e*) *These equations may be found* by setting the electrostatic force equal to the centripetal force on the electron, and then using the definitions of kinetic and electrostatic potential energy.

4-24 (a) $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) = R\left[(1)^2-\left(\frac{1}{3}\right)^2\right] = \frac{8R}{9};\ \lambda = \frac{9}{8R} = \frac{9}{(8)(1.09737\times10^{7}\text{ m}^{-1})} = 102.517 \text{ nm}$

(b) As the atom is initially at rest, the momentum of the photon, $\frac{h}{\lambda}$, must equal the recoil momentum of the atom:

$$P_{\text{atom}} = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\text{ J/s}}{102.517\text{ nm}} = 6.47\times10^{-27}\text{ kg m/s}$$

$$K_{\text{atom}} = \frac{p^2}{2m} = \frac{(6.47\times10^{-27}\text{ kg m/s})}{(2)(1.67\times10^{-27}\text{ kg})} = 1.25\times10^{-26}\text{ J} = 7.83\times10^{-8}\text{ eV}$$

This tiny amount of energy comes from the photon's energy making λ slightly longer than 102.517 nm.

4-25 (a) $\Delta E = hf = (13.6\text{ eV})\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$ or $f = (13.6\text{ eV})\left(\frac{\frac{1}{9}-\frac{1}{16}}{4.14\times10^{-15}\text{ eV s}}\right) = 1.60\times10^{14}\text{ Hz}$

(b) $T = \frac{2\pi r_n}{v}$ so $f_{\text{rev}} = \frac{1}{T} = \frac{v}{2\pi r_n}$. Using $v = \left(\frac{ke^2}{m_e r_n}\right)^{1/2}$, $f_{\text{rev}} = \left(\frac{ke^2}{mr_n}\right)^{1/2}$. For $n = 3$, $r_3 = (3)^2 a_0$ and

4-25 (b) The period T is defined simply as the time it takes for the electron to complete one revolution around the proton. The frequency of rotation is defined accordingly as the inverse of T.

$$f_{\text{rev}} = \frac{(8.99\times10^{9}\text{ Nm}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{\frac{[(9.11\times10^{-31}\text{ kg})(9)(5.29\times10^{-11}\text{ m})]^{1/2}}{(2)(3.14)(9)(5.29\times10^{-11}\text{ m})}}$$

$f_{\text{rev}} = 2.44\times10^{14}\text{ Hz } (n = 3)$

$f_{\text{rev}} = 1.03\times10^{14}\text{ Hz } (n = 4)$

Thus the photon frequency is about halfway between the two frequencies of the revolution.

4-26 Lyman series has $n_f = 1$, λ_{max} has $n_i = 2$; λ_{min} has $n_i = \infty$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R\left(\frac{1-1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda_{max} = \frac{4}{3R} = \frac{4}{(3)\left(1.097\times10^7 \text{ m}^{-1}\right)} = 121.5 \text{ nm}$$

$$\lambda_{min} = \frac{1}{R} = 91.16 \text{ nm}$$

As shown on the inside front cover, the visible spectrum begins at about 350 nm, so the Lyman series is in the UV.

4-27 (a) $\lambda = \frac{C_2 n^2}{n^2 - 2^2}$ so $\frac{1}{\lambda} = \left(\frac{1}{C_2}\right)\frac{n^2 - 2^2}{n^2} = \left(\frac{1}{C_2}\right)\left(\frac{1-2^2}{n^2}\right)$ or $\frac{1}{\lambda} = \left(\frac{2^2}{C_2}\right)\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ where $R = \frac{2^2}{C_2}$.

(b) $R = \frac{2^2}{36\,545.6\times10^{-8} \text{ cm}} = 109\,720 \text{ cm}^{-1}$

4-28 Call the energy available from the $n = 2$ to $n = 1$ transition ΔE:

$$\Delta E(13.6 \text{ eV})Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = (13.6)(24)^2\left(\frac{1}{1} - \frac{1}{4}\right) = 5\,875 \text{ eV} = 5.875 \text{ keV}.$$

The kinetic energy K of the Auger electron is equal to 5.875 keV minus the energy required to ionize an electron in the $n = 4$ state, $E_{ionization}$. Thus,

$$K = 5.875 \text{ keV} - E_{ionization} = 5.875 \text{ keV} - (13.6 \text{ eV})\frac{Z^2}{4} = 5.385 \text{ keV}.$$

4-29 (a) Both momentum and energy must be conserved. Momentum: $Mv = \frac{E_{photon}}{c}$, energy: $E = E_{photon} + \frac{1}{2}Mv^2$ where M is the atom's mass, v is its recoil velocity, E_{photon} is the photon's energy and E is the energy difference between the $n = 3$ and $n = 1$ states. Combining equations, $v^2 + 2cv - \frac{2E}{M} = 0$ and using the quadratic formula,

$$v = \frac{-2c \pm \left(\frac{4c^2 + 8E}{M}\right)^{1/2}}{2} = c\left[-1 \pm \left(\frac{1+2E}{Mc^2}\right)^{1/2}\right].$$

For $\frac{2E}{Mc^2} \ll 1$, $v \cong \frac{E}{Mc}$ or $v \cong \frac{-2c - E}{Mc}$, the second of which is non physical. Thus in general, $v \cong \frac{E}{Mc}$. For the $n = 3$ to $n = 1$ transition in particular,