2-5 This is the case where we use the relativistic form of Newton's second law, but unlike Problem 2-3 in which **F** is parallel to **v**, here **F** is perpendicular to **v** and $\mathbf{F}=\frac{d\mathbf{p}}{dt}$ so that

$$\mathbf{F}=q\mathbf{v}\times\mathbf{B}=\frac{d\mathbf{p}}{dt}=\frac{d}{dt}\left\{\frac{m\mathbf{v}}{\sqrt{1-(v/c)^2}}\right\}.$$

Assuming that **B** is perpendicular to the plane of the orbit of q, the force is radially inward, and we find

$$\mathbf{F}=qv\mathbf{B}\big|_{\text{radial}}=\frac{d}{dt}\left\{\frac{mv}{\sqrt{1-(v/c)^2}}\right\}.$$

As the force is perpendicular to **v**, it does no work on the charge and the magnitude (but not the direction) of **v** remains constant in time. Thus,

$$\frac{d}{dt}\left\{\frac{mv}{\sqrt{1-(v/c)^2}}\right\}=\frac{m}{\sqrt{1-(v/c)^2}}\frac{dv}{dt}.$$

Identifying $\left(\frac{dv}{dt}\right)$ as the centripetal acceleration where the scalar equation $\frac{dv}{dt}=\left(\frac{v^2}{r}\right)_{\text{radial}}$ gives $qvB\big|_{\text{radial}}=\left[\frac{m}{1-v^2/c^2}\right]^{1/2}\left(\frac{v^2}{r}\right)\bigg|_{\text{radial}}$ or $v=\left(\frac{qBr}{m}\right)\left(1-\frac{v^2}{c^2}\right)^{1/2}$. Finally, the period T is $\frac{2\pi r}{v}$ and $T=\frac{2\pi r}{(qBr/m)(1-v^2/c^2)^{1/2}}=\frac{2\pi m}{(qB)(1-v^2/c^2)^{1/2}}$. As $f=\frac{1}{T}$, $f=\left(\frac{qB}{2\pi m}\right)\left(1-\frac{v^2}{c^2}\right)^{1/2}$.

2-6 Using Equation 2.4 $p=e^-BR=(1.60\times10^{-19}\text{ C})BR\text{ kg}\cdot\text{m/C}\cdot\text{s}=1.60\times10^{-19}BR\text{ kg}\cdot\text{m/s}$. To convert $\text{kg}\cdot\text{m/s}$ to MeV/c, use $1\text{ MeV}/c=\frac{(10^6)(1.60\times10^{-19}\text{ C})(1\text{ J/C})}{3.00\times10^8\text{ m/s}}=5.34\times10^{-22}\text{ kg}\cdot\text{m/s}$, so that $p=\frac{(1.60\times10^{-19}BR\text{ kg}\cdot\text{m/s})(1\text{ MeV}/c)}{5.34\times10^{-22}\text{ kg}\cdot\text{m/s}}=300BR\text{ MeV}/c$.

2-7 $E=\gamma mc^2$, $p=\gamma mu$; $E^2=(\gamma mc^2)^2$; $p^2=(\gamma mu)^2$;

$$E^2-p^2c^2=(\gamma mc^2)^2-(\gamma mu)^2c^2=\gamma^2\left\{(mc^2)^2-(mc)^2u^2\right\}$$

$$=(mc^2)^2\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-1}=(mc^2)^2\text{ Q.E.D.}$$

$$E^2=p^2c^2+(mc^2)^2$$

2-8 (a) $E_R = mc^2 = (1.67\times10^{-27}\ \text{kg})(3\times10^8\ \text{m/s})^2 = 1.503\times10^{-10}\ \text{J} = 939.4\ \text{MeV}$ (Numerical round off gives a slightly larger value for the proton mass)

(b) $$E = \gamma mc^2 = \frac{1.503\times10^{-10}\ \text{J}}{\left(1-(0.95c/c)^2\right)^{1/2}} = 4.813\times10^{-10}\ \text{J} \approx 3.01\times10^3\ \text{MeV}$$

(c) $$K = E - mc^2 = 4.813\times10^{-10}\ \text{J} - 1.503\times10^{-10}\ \text{J} = 3.31\times10^{-10}\ \text{J} = 2.07\times10^3\ \text{MeV}$$

2-9 (a) When $K = (\gamma-1)mc^2 = 5mc^2$, $\gamma = 6$ and $E = \gamma mc^2 = 6(0.511\,0\ \text{MeV}) = 3.07\ \text{MeV}$.

(b) $$\frac{1}{\gamma} = \left[1-\left(\frac{v}{c}\right)^2\right]^{1/2} \text{ and } v = c\left[1-\left(\frac{1}{\gamma}\right)^2\right]^{1/2} = c\left[1-\left(\frac{1}{6}\right)^2\right]^{1/2} = 0.986c$$

2-10 $$E = \gamma mc^2;\ 1.5mc^2 = \gamma mc^2;\ \gamma = 1.5 \Rightarrow 1.5 = \frac{1}{\left[1-(v^2/c^2)\right]^{1/2}};\ v = c\left[1-\left(\frac{1}{1.5}\right)^2\right]^{1/2} = 0.75c$$

2-11 (a) $K = 50\times10^9\ \text{eV}$; $mc^2 = 938.27\ \text{MeV}$;

$$E = K + mc^2 = (50\times10^9\ \text{eV}) + (938.27\times10^6\ \text{eV}) = 50\,938.3\ \text{MeV}$$

$$E^2 = p^2c^2 + m^2c^4 \Rightarrow p = \left[\frac{E^2 - m^2c^4}{c^2}\right]^{1/2}$$

$$p = \frac{\left[(50\,938.3\ \text{MeV})^2 - (938.27\ \text{MeV})^2\right]^{1/2}}{c} = 5.09\times10^{10}\ \text{eV}/c$$

$$= \frac{5.09\times10^{10}\ \text{eV}}{3\times10^8\ \text{m/s}}(1.6\times10^{-19}\ \text{J/eV}) = 2.71\times10^{-17}\ \text{kg}\cdot\text{m/s}$$

(b) $$E = \gamma mc^2 = \frac{mc^2}{\left[1-(v/c)^2\right]^{1/2}} \Rightarrow v = c\left[1-\left(\frac{mc^2}{E}\right)^2\right]^{1/2}$$

$$= (3\times10^8\ \text{m/s})\left[1-\left(\frac{938.27\ \text{MeV}}{50\,938.3\ \text{MeV}}\right)^2\right]^{1/2} = 2.999\,5\times10^8\ \text{m/s}$$

2-12 (a) When $K_e = K_p$, $m_ec^2(\gamma_e - 1) = m_pc^2(\gamma_p - 1)$. In this case $m_ec^2 = 0.511\,0\ \text{MeV}$ and $m_pc^2 = 938\ \text{MeV}$, $\gamma_e = \left[1-(0.75)^2\right]^{1/2} = 1.511\,9$. Substituting these values into the first equation, we find $\gamma_p = 1 + \frac{m_ec^2(\gamma-1)}{m_pc^2} = 1 + \frac{(0.511\,0)(1.511\,9-1)}{939} = 1.000\,279$. But $\gamma_p = \frac{1}{\left[1-(u_p/c)^2\right]^{1/2}}$; therefore $u_p = c\left(1-\gamma_p^{-2}\right)^{1/2} = 0.023\,6c$.

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $u_p = \left(\frac{\gamma_e}{\gamma_p}\right)\left(\frac{m_e}{m_p}\right)u_e$,

$$u_p = \left(\frac{1.5119}{1.000\,279}\right)\left[\frac{0.5110/c^2}{939/c^2}\right](0.75c) = 6.17\times10^{-4}c.$$

2-13 (a) $E = 400mc^2 = \gamma mc^2$

$$\gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2} = 400$$

$$\left(1-\frac{v^2}{c^2}\right) = \left(\frac{1}{400}\right)^2$$

$$\frac{v}{c} = \left[1-\frac{1}{400^2}\right]^{1/2}$$

$$v = 0.999\,997c$$

(b) $K = E - mc^2 = (400-1)mc^2 = 399mc^2 = (399)(938.3\text{ MeV}) = 3.744\times10^5\text{ MeV}$

2-14 (a) $E = mc^2$

$$m = \frac{E}{c^2} = \frac{4\times10^{26}\text{ J}}{(3.0\times10^8\text{ m/s})^2} = 4.4\times10^9\text{ kg}$$

(b) $t = \frac{(2.0\times10^{30})\text{ kg}}{4.4\times10^9\text{ kg/s}} = 4.5\times10^{20}\text{ s} = 1.4\times10^{13}\text{ years}$

2-15 (a) $K = \gamma mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1+\frac{Vq}{mc^2}\right)^2$ and $\frac{v}{c} = \left\{1-\left(1+\frac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{1-\frac{1}{1+(5.0\times10^4\text{ eV}/0.511\text{ MeV})^2}\right\}^{1/2} = 0.4127$$

or $v = 0.413c$.

(b) $K = \frac{1}{2}mv^2 = Vq$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2(5.0\times10^4\text{ eV})}{0.511\text{ MeV}/c^2}\right\}^{1/2} = 0.442c$$

(c) The error in using the classical expression is approximately $\frac{3}{40}\times100\%$ or about 7.5% in speed.

2-16 (a) In S the speed of the particle is u and $p=\frac{mu}{\left(1-v^2/c^2\right)^{1/2}}$, $E=\frac{mc^2}{\left(1-u^2/c^2\right)^{1/2}}$, and $\left(E^2-p^2c^2\right)=m^2c^4$. In S', $u'=\frac{u-v}{1-uv/c^2}$, and

$$p'=\frac{mu'}{\sqrt{1-(u'/c)^2}}=\frac{m\left[(u-v)/\left(1-uv/c^2\right)\right]}{\sqrt{1-\left[(u-v)/\left(1-uv/c^2\right)\right]^2\left(1/c^2\right)}}=\frac{m\left[(u-v)/\left(1-uv/c^2\right)\right]}{\sqrt{1-(u-v)^2/\left(1-uv/c^2\right)^2}}$$

$$E'=\frac{mc^2}{\sqrt{1-(u'/c)^2}}=\frac{mc^2}{\sqrt{1-\left[(u-v)/\left(1-uv/c^2\right)\right]^2\left(1/c^2\right)}}=\frac{mc^2}{\sqrt{1-(u-v)^2/\left(1-uv/c^2\right)^2}}$$

(b) Using these expressions for E' and p', one obtains $\left(E'^2-p'^2c^2\right)=m^2c^4$, and since $E^2-p^2c^2=m^2c^4$, it follows that, $E'^2-p'^2c^2=E^2-p^2c^2$.

2-17 $\Delta m=m_{\text{Ra}}-m_{\text{Rn}}-m_{\text{He}}$ (an atomic unit of mass, the u, is one-twelfth the mass of the ^{12}C atom or $1.660\,54\times10^{-27}$ kg)

$$\Delta m=(226.025\,4-22.017\,5-4.002\,6)\text{ u}=0.005\,3\text{ u}$$
$$E=(\Delta m)(931\text{ MeV/u})=(0.005\,3\text{ u})(931\text{ MeV/u})=4.9\text{ MeV}$$

2-18 (a) The mass difference of the two nuclei is

$$\Delta m=54.927\,9\text{ u}-54.924\,4\text{ u}=0.003\,5\text{ u}$$
$$\Delta E=(931\text{ MeV/u})(0.003\,5\text{ u})=3.26\text{ MeV}.$$

(b) The rest energy for an electron is 0.511 MeV. Therefore,

$$K=3.26\text{ MeV}-0.511\text{ MeV}=2.75\text{ MeV}.$$

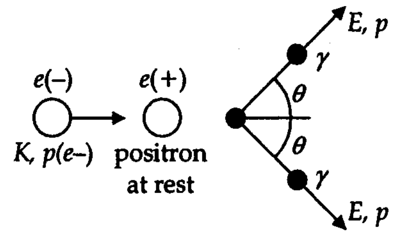
2-19 $\Delta m=6m_p+6m_n-m_{\text{C}}=\left[6(1.007\,276)+6(1.008\,665)-12\right]\text{ u}=0.095\,646\text{ u}$,

$$\Delta E=(931.49\text{ MeV/u})(0.095\,646\text{ u})=89.09\text{ MeV}.$$

Therefore the energy per nucleon $=\frac{89.09\text{ MeV}}{12}=7.42\text{ MeV}$.

2-20 $\Delta m=m\;\;-m_p-m_e=1.008\,665\text{ u}-1.007\,276\text{ u}-0.000\,548\,5\text{ u}=8.404\times10^{-4}\text{ u}$
$E=c^2\left(8.404\times10^{-4}\text{ u}\right)=\left(8.404\times10^{-4}\text{ u}\right)(931.5\text{ MeV/u})=0.783\text{ MeV}$.

2-21



Conservation of mass-energy requires $K + 2mc^2 = 2E$ where K is the electron's kinetic energy, m is the electron's mass, and E is the gamma ray's energy.

$$E = \frac{K}{2} + mc^2 = (0.500 + 0.511)\text{ MeV} = 1.011\text{ MeV}.$$

Conservation of momentum requires that $p_{e^-} = 2p\cos\theta$ where p_{e^-} is the initial momentum of the electron and p is the gamma ray's momentum, $\frac{E}{c} = 1.011\text{ MeV}/c$. Using $E_{e^-}^2 = p_{e^-}^2 c^2 + \left(mc^2\right)^2$ where E_{e^-} is the electron's total energy, $E_{e^-} = K + mc^2$, yields

$$p_{e^-} = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{\sqrt{(1.00)^2 + 2(1.00)(0.511)}\text{ MeV}}{c} = 1.422\text{ MeV}/c.$$

Finally, $\cos\theta = \frac{p_{e^-}}{2p} = 0.703$; $\theta = 45.3°$.

2-22 (a) Using the results of Problem 2-6 and substituting numerical values

$$p(\text{in MeV}/c) = 300BR = (300)(2.00\text{ T})(0.343\text{ m}) = 206\text{ MeV}/c.$$

Since the momentum of the K^0 is zero before the decay, conservation of momentum requires the pion momenta to be equal in magnitude and opposite in direction. The pion's speed u may be found by noting that $\frac{p}{E} = \frac{mu/\sqrt{1-u^2/c^2}}{mc^2/\sqrt{1-u^2/c^2}} = \frac{u}{c^2}$ or $\frac{u}{c} = \frac{pc}{E}$ where p is the pion momentum and E is the pion's total energy. Thus for either pion, $\frac{u}{c} = \frac{pc}{E} = \frac{pc}{\left[p^2c^2 + \left(mc^2\right)^2\right]^{1/2}}$ where m is the pion's mass. Finally,

$$\frac{u}{c} = \frac{206\text{ MeV}}{\sqrt{(206\text{ MeV})^2 + (104\text{ MeV})^2}} = 0.827.$$

(b) Conservation of mass-energy requires that $E_{K^0} = 2E$ where E_{K^0} is the total energy of a pion. As the K^0 pion decays at rest,

$$E_{K^0} = m_{K^0}c^2 = 2\sqrt{p^2c^2 + \left(mc^2\right)^2} = 2\sqrt{(206)^2 + (140)^2}\text{ MeV} = 498\text{ MeV},$$

or $m_{K^0} = 498\text{ MeV}/c^2$.

2-23 In this problem, M is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2 = \frac{m_l c^2}{\sqrt{1 - v_l^2/c^2}} + \frac{m_h c^2}{\sqrt{1 - v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987c)(6.22) = (m_h)(0.868c)(2.01)$$

$$m_l = \frac{(m_h)(0.868c)(2.01)}{(0.987)(6.22)} = 0.284 m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains $3.34 \times 10^{-27}\ \text{kg} = 6.22 m_l + 2.01 m_h$ which in turn gives

$3.34 \times 10^{-27}\ \text{kg} = (6.22)(0.284) m_l + 2.01 m_h$ or $m_h = \dfrac{3.34 \times 10^{-27}\ \text{kg}}{3.78} = 8.84 \times 10^{-28}\ \text{kg}$ and

$m_l = (0.284) m_h = 2.51 \times 10^{-28}\ \text{kg}$.

2-24 The moving observer sees the charge as stationary, so she says it feels no magnetic force.

$$q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q(\mathbf{E}' + \text{zero}),\ \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}.$$

2-25 (a) The x component of the gravitational force between a light particle of mass m and the Sun is given by $F_x = \dfrac{GM_S m}{r^2} \sin\phi = \dfrac{GM_S mb}{\left(b^2 + y^2\right)^{3/2}}$. The change in momentum in the x direction is given by $\Delta p_x = \int_{-\infty}^{\infty} F_x dt = \int_{-\infty}^{\infty} \dfrac{GM_S mb}{\left(b^2 + y^2\right)^{3/2}} dt$. To convert dt to dy, assume the deflection is very small and that the position of the light particle is given by $y = -ct$ for $x = 0$. Thus $dt = -\dfrac{dy}{c}$ and we get

$$\Delta p_x = -\frac{GM_S mb}{c} \int_{+\infty}^{-\infty} \frac{dy}{\left(b^2 + y^2\right)^{3/2}} = \frac{2GM_S mb}{c} \int_0^{+\infty} \frac{dy}{\left(b^2 + y^2\right)^{3/2}} = \frac{2GM_S mb}{c} \left. \frac{y}{b^2\left(y^2 + b^2\right)^{1/2}} \right|_0^{\infty}$$

$$= \frac{2GM_S mb}{c}\left(\frac{1}{b^2}\right) = \frac{2GM_S m}{cb}$$

From Figure P2.25(b), $\theta \cong \dfrac{\Delta p_x}{mc}$ so we find $\theta \cong \dfrac{2GM_S m}{cb(mc)} = \dfrac{2GM_S}{bc^2}$.

(b) For $b = R_S = 6.96 \times 10^8$ m and $M_S = 1.99 \times 10^{30}$ kg

$$\theta = \frac{2\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(1.99 \times 10^{30}\ \text{kg}\right)}{\left(6.96 \times 10^8\ \text{m}\right)\left(3.00 \times 10^8\ \text{m/s}\right)^2} = 4.24 \times 10^{-6}\ \text{rad} = 2.43 \times 10^{-4}\ \text{deg}$$