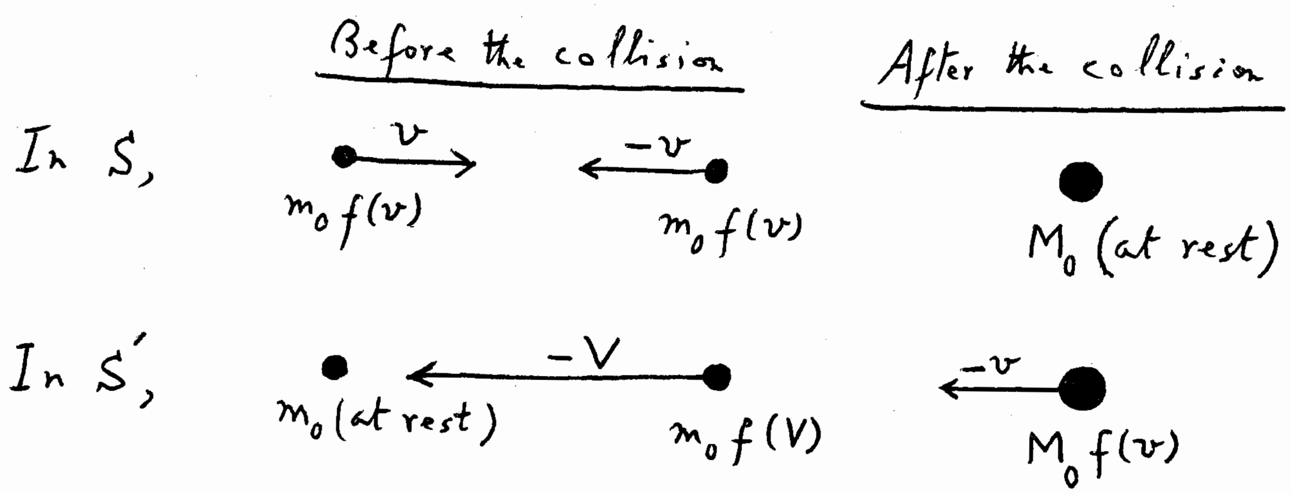
HANDOUT 1

To show that $m(u) = m_0 f(u)$, where $f(u) = \dfrac{1}{\sqrt{1-u^2/c^2}}$.

We consider a <u>totally inelastic</u> collision of two <u>identical</u> balls of rest mass m_0.



By <u>conservation of total mass</u> in S, we have

$$M_0 = 2m_0 f(v), \tag{1}$$

and in S'

$$M_0 f(v) = m_0 + m_0 f(V). \tag{2}$$

By <u>conservation of total momentum</u> in S, we have the trivial result $0=0$; however, in S' we have the non-trivial result

$$M_0 f(v)\, v = m_0 f(V)\, V. \tag{3}$$

By (1) & (3),

$$\frac{M_0}{m_0} = 2f(v) = \frac{f(V)\,V}{f(v)\,v}, \quad \therefore\ f(V) = \frac{2f^2(v)\,v}{V}. \tag{4}$$

By (2) & (3),

$$\frac{M_0}{m_0} = \frac{1+f(V)}{f(v)} = \frac{f(V)\,V}{f(v)\,v}, \quad \therefore f(V) = \frac{v}{V-v}. \tag{5}$$

Equating (4) & (5), we get

$$f^2(v) = \frac{V}{2(V-v)}. \tag{6}$$

Now, what is V ?

In Newtonian mechanics, $V = 2v$; hence, $f(v) = 1$.

In relativistic mechanics, $V = \dfrac{2v}{1+v^2/c^2}$; hence

$$f^2(v) = \frac{\dfrac{2v}{1+v^2/c^2}}{2\left[\dfrac{2v}{1+v^2/c^2} - v\right]}$$

$$= \frac{1}{\left[2 - \left(1 + \frac{v^2}{c^2}\right)\right]} = \frac{1}{1 - \frac{v^2}{c^2}}.$$

It follows that

$$f(v) = \frac{1}{\sqrt{1 - v^2/c^2}}.$$ ✓