PHYSICS 2B - Lecture Notes

Ch. 31: Electromagnetic Induction

Preliminaries

Michael Faraday (1791-1867) of Britain is regarded as one of the greatest of experimenters. He had little formal education and no training in mathematics, but he had remarkable physical intuition. He originated the concept of a field and is credited with inventing the electric motor and the dynamo. He knew that an electric field in a conductor gives rise to an electric current, which in turn produces a magnetic field. He reasoned that it must also be possible to produce an electric field from a magnetic field. We now consider the process called *induction.*

Faraday's Law

A very dramatic lecture demonstration consists of a solenoid connected to a galvanometer. As the north pole of a magnet is inserted into the solenoid, the galvanometer will deflect, say, to the left. With the magnet inserted, the galvanometer reading returns to zero. If the magnet is withdrawn, the galvanometer will deflect to the right. If the magnet is now reversed and the south pole inserted, the deflection will be to the right and to the left on withdrawal. The current through the galvanometer is said to be *induced.* Faraday reasoned that the induced emf in the galvanometer circuit was related to the change in magnetic flux through the circuit.

Recall, the magnetic flux through an area A is,

$$\phi_B = \int_A \vec{B} \cdot d\vec{A} .$$

Faraday inferred that the results of his observations could be summarized as,

$$\mathcal{E} = -\frac{d\phi_B}{dt} .$$

This is Faraday's law of induction and is the integral form of the fourth of Maxwell's equations.

It is important to remember the directionality of the integrals in this expression. Since the area A is two-sided, there is an ambiguity in the choice of direction of the normal to the area. However, once the normal direction is chosen, the positive sense for the integral implied in the emf is given by the right-hand screw rule. The negative sign above indicates, that if the area lies in the plane of the page and the normal direction is chosen to be out of the page, the positive sense of the line integral would be counter-clockwise, so that if the flux integral is positive, the induced emf would drive a current in the clock-wise direction.

It is interesting to consider ways in which the magnetic flux integral might change:

1. the magnitude of the magnetic field can change
2. the magnitude of the area can change
3. the relative orientation of the magnetic field and the area can change.

Any of these can give rise to an induced emf and examples of each are given in your text.

Lenz's Law

Consider wire loop of area *A* in the plane of the paper with the normal direction taken as pointing out of the page. A magnetic field is applied also pointing out of the page so that the magnetic flux is positive. Allow the magnetic field to increase with time so that the time derivative of the flux is positive. By Faraday's law the induced emf is clockwise as is the induced current in the wire. This induced current produces a magnetic field of its own that points *into* the page. That is,

the induced current produces a magnetic field that <u>*opposes*</u> *the change in the applied field.*

This statement is called *Lenz's Law* and is represented by the negative sign in Faraday's law. It may be thought of as an expression of the Conservation of Energy. If Lenz's law did not hold, the induced field could add to the applied field so that the total field could increase without limit.

Non-Conservative Electric Fields

We saw that the electric field produced by a static array of charges is conservative, that is

$$\oint_C \vec{E} \cdot d\vec{\ell} = 0,$$

for any closed path *C*. A different situation arises, however, in the case of induced emfs. We write Faraday's law as

$$\mathcal{E} = \oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}\,.$$

Clearly, if there is a changing magnetic flux through the area encircled by *C*, the induced electric field will not be conservative.

Betatron and Tokamak

This principle is the basis of the operation of an early particle accelerator called the *betatron* which was used to accelerate electrons. A strong magnetic field was applied perpendicular to the plane of a torus. The induced electric field inside the torus served to accelerate a beam of electrons. A similar arrangement provides part of the heating of the plasma confined in a *tokamak*. This is a toroidal device to induce nuclear fusion as potential source of energy.

Superconductivity and Perfect Diamagnetism

At low temperatures some materials have the property that their electrical resistance becomes exactly zero below a critical temperature. This is called *superconductivity* and is a totally quantum mechanical phenomenon that cannot be explained classically. As a result of having zero resistance, once a current is induced in a superconductor, it persists indefinitely; in some cases, for years. When a magnet is brought close to a superconductor, it induces eddy currents that produce a magnetic field that exactly cancels the applied field, so that the magnetic field inside the superconductor remains exactly zero. This is called the *Meissner effect* and is an example of perfect diamagnetism for which the susceptibility of the material is -1.

Examples

Eddy Currents

Consider a magnetic field directed into the page. A conductor is moved into the field from the left. Taking the current carriers to be positive, the magnetic force on these carriers will be upward, inducing an upward current. The magnetic force on this current will be to the left, tending to resist the motion of the conductor. This current will be bent into a counter-clockwise path, inducing a magnetic field that opposes the applied field, in accordance with Lenz's law. This induced current is called an *eddy current* and the retarding force provides the braking action for the magnetic brakes used on *mag-lev* trains.

Satellite Tethers

A space station deploys, to a somewhat higher orbit, a small weight that is attached to the station by a tether. The weight will maintain the orbital angular velocity of the station, which is much heavier than the weight. To maintain this motion, the tether will exert a force on the weight directed toward the station. If the tether contains a conductor, an emf will be induced by its passage through the Earth's magnetic field. The electrons from ionized atmospheric molecules will provide a return path, completing a circuit. Electrical devices on the station could then be powered by this emf. A proof of concept test was carried out on the 1996 Space Shuttle flight and was successful until mechanical difficulties ended the test prematurely.