PHYSICS 2B - Lecture Notes

Ch. 29: The Magnetic Field

Preliminaries

One of the earliest references to magnetism was recounted by the historian *Pliny* and had to do with a shepherd named *Magnes* in Asia Minor (part of modern Turkey) around 800 BCE. *Magnes* followed his flocks to a field strewn with rocks that attracted the iron tip of his staff and the nails in his sandals. The shepherd's name was subsequently given to the region (Magnesia), the rocks (*magnetite*, also called *lodestone*) and to the phenomenon itself. There are fleeting references that indicate that the Egyptians, Chinese and Mayans were also aware of this phenomenon. There also indications that the early Chinese (200-100 BCE) were aware of the property of a lodestone to align itself in a north-south direction. A compass that was modern in every respect was recorded in China in 1088 AD. *Gilbert* (1600 AD) wrote the first western text about magnetism and in 1820, *Oersted* discovered the connection between electrical current and magnetism.

The type of magnet with which we are most familiar is the bar-magnet. Iron filings sprinkled around a bar-magnet form a suggestive pattern similar to the lines of force about an electric dipole. The end of a bar-magnet that points itself toward the north geographical pole is labeled the north pole of the magnet and opposite end is called the south pole. It is readily seen the like poles repel and unlike poles attract. The tendency of a magnet to align itself in a north-south direction indicates that the Earth acts as if a bar magnet is located at the center of the Earth and aligned (almost) with the Earth's rotation axis and with the north geographical pole coinciding with the Earth's south magnetic pole.

While there seem to be many similarities electrical and magnetic forces, there is one very fundamental difference. If we cut a bar magnet in half we do not get isolated north and south poles. Instead we get two bar magnets each with a north and south pole. This observation has never been violated experimentally. That is, we have never observed a *magnetic monopole.* We will later formulate this observation into the second of *Maxwell's equations.*

As mentioned above, the field lines about a bar-magnet resemble those about an electric dipole. We may arbitrarily assign a direction to these field lines choosing the direction of the field lines as pointing away from a north pole and toward a south pole.

Magnetic Force on a Straight Current Carrying Wire

If we place a current carrying wire in the field of a magnet, we quickly find that if the current flows in the direction of the field, the force on the wire will be zero. However, if the current flows perpendicularly to the field, there will be a force on the wire. Furthermore, the force will be proportional to the current I and to the length of the wire, ●. In analogy with the procedure followed in the case of the electric field, we define the strength of the magnetic field by,

$$B \equiv \frac{F}{I\ell},$$

so that the force on the wire will be $F = BI$●. In general, if the wire makes an angle θ with the field, the force on the wire will be,

$$F = BI\ell \sin\theta \ .$$

Units

In the SI system in which force is expressed in newtons, current in amperes and length in meters, the unit for the magnetic field is the *tesla*. A smaller unit, which is more typical of everyday field is the *gauss*, where

$$1 \text{ gauss} = 10^{-4} \text{ tesla.}$$

Direction

What sets the magnetic force apart from other forces encountered to this point is its direction; the magnetic force is perpendicular to the magnetic field and the current direction. The sense of the direction is given by the *right-hand rule*:

> *Using one's right hand, if the index finger points in the direction of the current and the middle finger points in the direction of the magnetic field, the thumb will point in the direction of the force on the wire.*

We can write this in vector form as,

$$\vec{F} = I\vec{\ell} \times \vec{B} \ .$$

General Curves and Variable Field

For the case of a conductor in a variable field, the above generalizes to

$$\vec{F} = I\int_C d\vec{\ell} \times \vec{B} \ .$$

Magnetic Force on a Moving Charge

From the force on a current carrying wire, we can infer the magnetic force on a moving charge. Recall the microscopic picture of current,

$$I = nqAv$$

Then the magnetic force on a wire of length ● is actually the total force acting on all the moving charges in the conductor. This is

$$\vec{F}_{tot} = I\vec{\ell} \times \vec{B} = (nA\ell)q\vec{v} \times \vec{B} \ ,$$

where we have taken the direction of the velocity to be parallel to the wire. The quantity in parentheses is the total number of carriers with charge q in the segment of wire so that the force on the wire is the total of the forces on the individual charges, each of which is

$$\vec{F} = q\vec{v} \times \vec{B} \, .$$

Note: the direction of the magnetic force on a positively charged particle is that given by the right-hand rule. For a negatively charged particle, however, the force direction will be opposite that for a positive particle. As a consequence, the direction of deflection of a charged particle in a magnetic field enables to determine its charge.

Lorentz Force

We can now write down the force on a charge q moving through a mix of electric and magnetic fields as the sum of the electric and magnetic forces acting on the charge as

$$\vec{F} = \vec{F}_{elec} + \vec{F}_{mag} = q\vec{E} + q\vec{v} \times \vec{B}$$

This is called the *Lorentz force* and represents the fundamental interaction of a point charge q with a general electromagnetic field.

Work Done on a Moving Particle by a Magnetic Force

The work done by the magnetic force on a charge q, traveling with a velocity $\vec{v}$ over a time interval dt is,

$$dW = \vec{F} \cdot d\vec{\ell} = q\vec{v} \times \vec{B} \cdot d\vec{\ell} = q(\vec{v} \times \vec{B}) \cdot \vec{v}dt = 0 \, .$$

That is, the magnetic force does no work on the particle, so that its kinetic energy is unchanged. Only the direction of the velocity changes, not its magnitude.

Particle Motion in a Magnetic Field

Consider a positively charged particle, q, with mass m traveling upward in the plane of the page with a magnetic field directed into the page. The particle will experience a force to the left with magnitude,

$$F = qvB.$$

Since this force changes the direction of the velocity but not its magnitude, this is a centripetal force and bends the particle path into a circle of radius r. By Newton's second law we have,

$$qvB = m\frac{v^2}{r} \qquad \Rightarrow \qquad \frac{qB}{m} = \frac{v}{r} = \omega = 2\pi f \, .$$

where ω is the angular velocity of rotation and f is the frequency of rotation, called the *cyclotron frequency.*

If the particle also has a velocity component parallel to the magnetic field, that component will be unchanged and the path of the particle will be a spiral. When charged particles arrive at Earth following a solar eruption, they follow such spiral paths in the Earth's magnetic field.

Torque on a Current Loop

For simplicity, consider a rectangular loop, with sides *a* and *b*, carrying a current *I*, in a uniform field, *B*. The forces on parallel sides are equal and opposite so that the net force on the loop is zero. As we shall see, however, the net torque is not. We allow the rectangle to rotate about an axis parallel to the *a* sides. The force on each of these sides is

$$F = IaB.$$

The sense of the torque is to bring the area of the loop perpendicular to the magnetic field. If the axis bisects the *b* sides and the area of the loop makes an angle θ with the equilibrium orientation, the net torque about the axis will be

$$\tau = 2\left(\frac{b\sin\theta}{2} IaB\right) = IAB\sin\theta$$

where $A = ab$, is the area of the loop. In vector form, this is,

$$\vec{\tau} = (IA\hat{n}) \times \vec{B}\ .$$

The unit vector is perpendicular to the plane of the loop in the direction given by the right-hand screw rule. That is, if we walk around loop in the direction of the current, keeping the area to our left, the unit vector points upward. The quantity in parentheses is the *magnetic dipole moment* of the loop,

$$\vec{\mu} \equiv IA\hat{n}\ .$$

Although derived for a rectangular loop, this result is generally true.

With the addition of some means if reversing the current in the loop (sliding contacts called *brushes*), we have the makings of a rudimentary electric motor.

Potential Energy of a Magnetic Dipole

The work necessary to rotate the loop about the axis through the angle $d\theta$ is

$$dW = \tau d\theta = \mu B\sin\theta d\theta = -d(\mu B\cos) = dU\ .$$

Where,

$$U = -\vec{\mu}\cdot\vec{B}\ ,$$

is the potential energy of the magnetic dipole in a magnetic field.