Lecture 1.2

Plasma oscillations

The electron slab is displaced relative the ion slab at distance Δx and released.

Electric field:

$$E_x = 4\pi e n_0 \Delta x \qquad (14)$$

The electron slab will move toward the ion slab. Because electrons acquire some speed, they will overshoot the ion slab. Now we again have electric field but in opposite direction. As a result, harmonic oscillations appear.

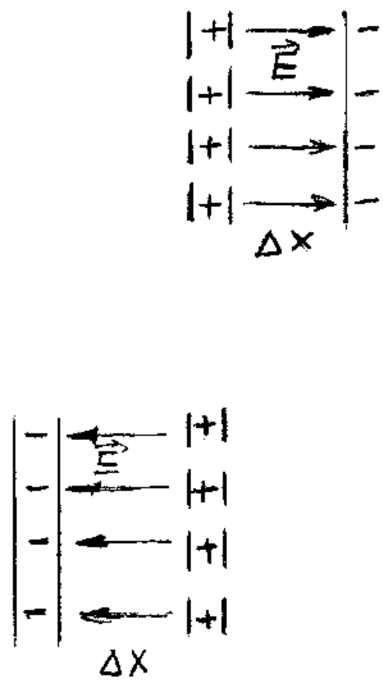


Fig. 1

Equation of motion:

$$m_e \frac{d^2 \Delta x}{dt^2} = -eE_x = -4\pi e^2 n_0 \Delta x$$

$$m_e \frac{d^2 \Delta x}{dt^2} + \omega_{pe}^2 \Delta x = 0 \qquad (15)$$

Here

$$\omega_{pe} = \sqrt{4\pi e^2 n_0 / m_e} \qquad (16)$$

is electron plasma frequency (Langmuir frequency). $\omega_{pe} \approx 5.6 \times 10^4 n_0^{1/2}$ rad/s.

The phase velocity is $v_{ph} = \omega_{pe}/k$, where k is the wave number $k = 2\pi/\lambda$, λ is the wavelength.

One can see that

$$\lambda_{De} = \sqrt{k_B T/m_e} / \omega_{pe} . \qquad (17)$$

For small wavelength a "sound effects" should be taken into account. As is well known the sound wave is connected with change of the gas pressure. Here we should take into account change of the electron gas pressure:

$$\frac{\omega^2}{k^2} = \frac{\omega_{pe}^2}{k^2} + \frac{\partial P_e}{\partial \rho_e}$$

or

$$\omega^2 = \omega_{pe}^2 + k^2 \frac{\partial P_e}{\partial \rho_e} \quad (18)$$

P_e is the pressure of the electron "gas", $\rho_e = nm_e$ is the electron mass density. Langmuir was the first who obtained this formula using analogy of electron plasma oscillations with sound waves.

For adiabatic compression $P \sim n^\gamma$ with $\gamma = 3$ (Vlasov has proven this in kinetic theory) the electron plasma wave dispersion relation has the form (see also Figure 2);

$$\omega^2 = \omega_{pe}^2 + 3k^2 \frac{k_B T}{m_e} \quad (19)$$

For the long wavelength oscillations $\lambda >> \lambda_{De}$, second term is much smaller than first. In this case

$$\omega = \omega_{pe}\left(1 + \frac{3}{2}k^2\lambda_{De}^2\right) \quad (19')$$

Electron plasma waves in this case have nonzero group velocity

$$v_g = 3k\lambda_{De}v_{Te}$$

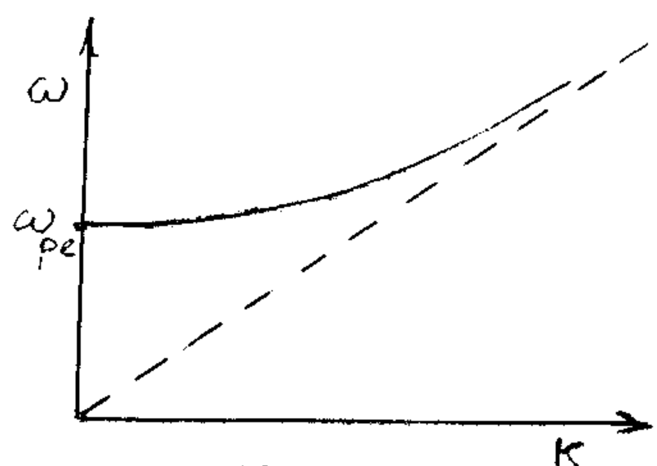


Fig. 2

Landau damping

This is a collisionless damping of plasma waves.

If an electron plasma wave is excited in an equilibrium plasma with electric field

$$E(t,x) = E(t)\sin(kx - \omega t + \varphi) \tag{20}$$

then resonant interaction of this wave with electrons leads to exponential damping of the amplitude of the wave electric field with time

$$E(t) = E(0)e^{\gamma t}$$

where

$$\gamma = \frac{2\pi^2 e^2}{m_e k^2}\omega_{pe}\left.\frac{\partial f_0}{\partial v_x}\right|_{v_x=\omega/k} \tag{21}$$

is Landau decay rate.

In (21), $f_0(v_x)$ is an electron equilibrium distribution function integrated over two velocity components perpendicular to the direction of the wave propagation.

Physical meaning of Landau damping: It is a result of the energy exchange between the wave and resonant electrons whose velocities in the direction of the wave propagation are close to the wave phase velocity.

The resonance condition has the form

$$\omega = \vec{k}\vec{v} \tag{22}$$

The resonance condition (22) can be also obtained using quantum mechanics consideration. In this approach, radiation of a wave with frequency ω and wave vector $\vec{k}$ by an electron leads to change of its energy and momentum

$$\Delta w = \hbar\omega, \quad \Delta\vec{p} = \hbar\vec{k} \tag{23}$$

Using the relationship $\Delta w = \vec{v}\Delta\vec{p}$ one can obtain the resonance condition (22).

The width of the resonance

In the wave reference frame, all electrons (excluding very small number of trapped particles) fly over the potential profile

$$\phi(t)\cos(kx'+\theta), \qquad x' = x - v_{ph}t \tag{24}$$

periodically coming across accelerating or decelerating phases.

For electrons to be resonant ones, their velocity should be small in the wave reference frame, so the time of their flight over the distance of the order of the wavelength was longer or of the order of the characteristic time scale of the change of the amplitude of the wave potential

$$\frac{\lambda}{|v_x - v_{ph}|} > \frac{1}{\gamma} \qquad \text{or} \qquad |v_x - v_{ph}| < 2\pi\frac{\gamma}{k} \tag{25}$$

In this case electrons interact with the wave with different amplitudes during acceleration on one half of the wavelength and the deceleration on another, and at an average, exchange energy with the wave.

If the particle velocity in the wave reference frame is large, so the flight time over the distance of the order of the wavelength is much smaller than $1/\gamma$, this particle doesn't, at an average, exchange energy with waves (so-called nonresonant particles).

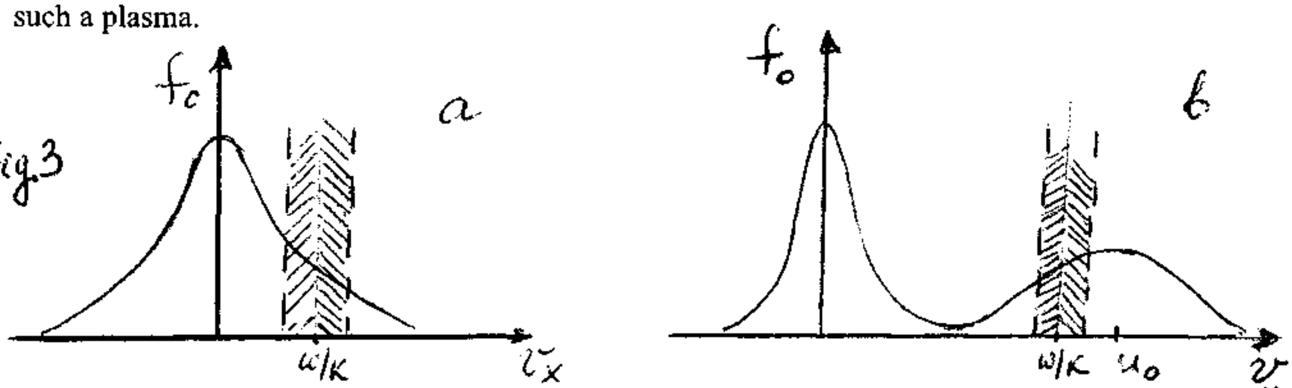
The relation (25) determines by order of magnitude the interval of resonant particles from the distribution function $f_0(v_x)$. Resonant particles with velocities less than the wave phase speed take energy away from the wave. Resonant particles with $v_x > v_{ph}$ give energy back to the wave.

Since the damping rate is determined by the balance of these two types of resonant particles, it is proportional to the derivative of the distribution function at $v_x = v_{ph}$ (see

(21)) and its value strongly depends on details of the distribution function in resonant region.

In an equilibrium plasma $\partial f_0/\partial v_x|_{v_x=\omega/k} < 0$ (the number of backward resonant particles is larger than that of surpassing particles (see Fig. 3a) and the wave damps in such a plasma.

Fig.3



If there is an electron beam in a plasma, the derivative of distribution function can be positive in some velocity interval (see Fig. 3b). Then the same mechanism of interaction with resonant particles leads to increase with time of the amplitude of the wave with phase velocity in this interval. This is so-called beam-plasma instability.

The expression (21) for Landau damping of Langmuir waves is relevant for conditions when the wavelength λ is much larger than the Debye length λ_D. In this case, the wave phase velocity is large in comparison with the electron thermal velocity and number of resonant particles is small. The wave frequency is close to electron plasma frequency. When the wave phase velocity decreases, the number of resonant particles increases and the wave damping rate increases as well. If the wavelength is comparable with or become less than λ_D, the magnitude of the damping rate is on the order of electron plasma frequency and such waves don't in fact propagate in plasma.

The foregoing consideration was related to Landau damping of the wave with a small amplitude when the width of the trapping region over velocities that is on the order of

$$\Delta v_x^{tr} \approx \sqrt{e\phi/m_e} \qquad (26)$$

is much smaller than the width of resonance

$$\frac{\gamma}{k\sqrt{e\phi/m_e}} >> 1 \qquad (27)$$