**PHYSICS 160: Stellar Structure**

Instructor: Dr. A. M. Wolfe (phone: 47435)
Homework no. 5
Due: Tues. Nov. 27

**1**

Suppose that within the H-burning core of the sun, the luminosity due to fusion reactions is initially given by.

$$\mathcal{L}_{\rm nuc} = 2\times\mathcal{L}_{\rm rad}$$

where $\mathcal{L}_{\rm rad}$ is the radiative luminosity. Suppose that the energy generation rate per unit mass per unit time on the pp cycle is

$$\epsilon_{\rm pp} = 1.07\times10^{-27}\rho X^2 (T/10^6)^4$$

(a) Compare the initial core temperature of the sun with the standard case in which $\mathcal{L}_{\rm nuc}= \mathcal{L}_{\rm rad}$. Assume the temperature and density throughout the co:wre are uniform.

(b)Apply the virial theorem to the core of the sun and derive an expression for the time rate of change of internal energy.

(c) Use the equation derived in (b) to solve a simple differential equation for temperature. Ignoring the change in density $\rho$, compute how long it would take for the initial core temperature to equal the standard core temperature computed in (a).

(d) Use the virial theorem again to show how the temperature varies on radius. Assume the potential energy $\Omega_{\rm g} = -(3/5)GM^2/R$,

**2**

Recall that radiative luminosity of a main sequence star is roughly give by

$$\mathcal{L}_{\rm rad} = \frac{(4\pi R^3/3)(aT^{4)}}{3R^2/lc}$$

where $R$ is the radius of the star, $T$ is the average interior temperature, $a$ is the radiation constant, and $l\equiv1/n\sigma$ is the photon mean-free-path. The latter is given by

$$l\propto T^{3.5}/\rho^2 : {\rm low-medium\ mass\ stars}$$

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$$l\propto 1/\rho : {\rm high-mass\ stars}$$

where $\rho$ is the mean density of the star.

(a) Use the equations of stellar stucutre to show that

$$\rho\propto M/R^3$$

$$P\propto GM^2/R^4$$

(b) In stars with low-medium mass ideal equation of state holds, but in massive stars, radiation pressure dominates. Therefore, show that

$$P\propto\rho T : {\rm low-high\ mass\ stars}$$

$$P \propto T^4 : \text{very} - \text{high mass stars}$$

(c) Combine the above results to show that

$$\mathcal{L}_{rad} \propto M^{5.5}/R^{0.5} : \text{low} - \text{medium mass stars}$$

$$\mathcal{L}_{rad} \propto M^3 : \text{high} - \text{mass stars}$$

$$\mathcal{L}_{rad} \propto M : \text{very high} - \text{mass stars}$$

For main sequence H burning we can assume $R \propto M$. So $\mathcal{L}_{rad} \propto M^5$ is a good approximation for the low-to-medium mass stars.

(d) Make a schematic plot for all the mass ranges and show on a $\mathcal{L}_{rad}$ versus $M$ plot that $\mathcal{L}_{rad} \propto M^4$ is a rough approximation for the entire mass range.

(e)Estimate the mass dependence of the H-burning main-sequence lifetime of the stars. Normalize your results to the 10 billion year lifetime of the sun to make estimates in years of stars in which $M$=$0.5M_{\odot}$, $M$=$3M_{\odot}$, and $M$=$20M_{\odot}$.