PHYSICS 110A : CLASSICAL MECHANICS

1. Introduction to Dynamics
   motion of a mechanical system
   equations of motion : Newton's second law
   ordinary differential equations (ODEs)
   dynamical systems
   simple examples

2. Systems of Particles
   kinetic, potential, and interaction potential energies
   forces; Newton's third law
   momentum conservation
   torque and angular momentum
   kinetic energy and the work-energy theorem

3. Motion in $d = 1$: Two-Dimensional Phase Flows
   (x, v) phase space
   dynamical system $\frac{d}{dt}\left\{ \begin{matrix} x \\ v \end{matrix} \right\} = \left\{ \begin{matrix} v \\ a(x,v) \end{matrix} \right\}$
   two-dimensional phase flows
   examples: harmonic oscillator and pendulum
   fixed points in two-dimensional phase space; separatrices

4. Solution of the Equations of One-Dimensional Motion
   Potential energy $U(x)$
   Conservation of energy
   sketching phase flows from $U(x)$
   solution by quadratures
   turning points; period of orbit

5. Linear Oscillations
   Taylor's theory and the ubiquity of harmonic motion
   the damped harmonic oscillator: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$
      reduction to algebraic equation
      generalization to all autonomous homogeneous linear ODEs
   solution to the damped harmonic oscillator: underdamped and overdamped behavior

6. Forced Linear Oscillations
   $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t)$
   solution for harmonic forcing $f(t) = A\cos(\Omega t)$
      presence of homogeneous solution: transients
   amplitude resonance and phase lag; Q factor

7. Green's functions for autonomous linear ODEs
   Fourier transform
   physical meaning of $G(t - t')$; causality
   response to a pulse

8. `MIDTERM EXAMINATION #1`

9. Calculus of Variations I

Snell's law for refraction at an interface
    continuum limit of many interfaces
functionals
variational calculus: extremizing $\int dx\, L(y, y', x)$
preview: Newton's second law from $L = T - U$

10. Calculus of Variations II

Examples
    surfaces of revolution
    geodesics
    brachistochrone
generalization to several dependent and independent variables
Constrained Extremization
Lagrange undetermined multipliers in calculus: review
systems with integral constraints
    hanging rope of fixed length
holonomic constraints

11. Lagrangian Dynamics

generalized coordinates
action functional
equations of motion: Newton's second law
    examples: spring, pendulum, *etc.*
double pendulum: Lagrangian and equations of motion
Lagrangian for a charged particle interacting with an electromagnetic field
    Lorentz force law

12. Constrained Dynamical Systems

undetermined multipliers as forces of constraints
simple pendulum with $r = l$ or $x^2 + y^2 = l^2$ constraint
Examples

13. Noether's Theorem and Conservation Laws

continuous symmetries
"one-parameter family of diffeomorphisms" $q_i \to h_i^\lambda(q_1, \ldots, q_N)$
Noether's theorem and the conserved "charge" $Q = \sum_i \frac{\partial L}{\partial q_i} \frac{\partial h_i^\lambda}{\partial \lambda}\Big|_{\lambda=0}$
linear and angular momentum

14. `MIDTERM EXAMINATION #2`

15. The Two-Body Central Force Problem

    CM and relative coordinates
    angular momentum conservation and Kepler's law $\dot{\mathcal{A}}$ =const.
    energy conservation
    the effective potential
        radial equation of motion for the relative coordinate
        the effective potential and its interpretation
        phase curves
        solution for $r(t)$ and $\phi(t)$ by quadratures

16. The Shape of the Orbit

    equation for $r(\phi)$, the geometric shape of the orbit
        $s = 1/r$ substitution
    examples
    almost circular orbits: bound *versus* closed motion, precession

17. Coupled Oscillations I: The Double Pendulum

    review: Lagrangian for the double pendulum
    equations of motion
    linearization
    solution of two coupled linear equations
    normal modes

18. Coupled Oscillations II: General Theory

    harmonic potentials
    T and V matrices
    normal modes
    the mathematical problem: simultaneous diagonalization of T and V

19. Coupled Oscillations III: The Recipe

    eigenvalues: $\det(\omega^2 T - V) = 0$
    eigenvectors: $(\omega_s^2 T_{ij} - V_{ij})a_j^{(s)} = 0$
    normalization: $a_i^{(s)} T_{ij} a_j^{(s')} = \delta_{ss'}$
    modal matrix: $A_{js} = a_j^{(s)}$
    examples

- COMPREHENSIVE FINAL EXAMINATION

PHYSICS 110B : CLASSICAL MECHANICS

1. <u>Accelerated Coordinate Systems</u>

   $\frac{d\vec{A}}{dt}\Big|_{\text{inertial}} = \frac{d\vec{A}}{dt}\Big|_{\text{body}} + \vec{\omega} \times \vec{A}$
   acceleration and fictitious forces
   translations

2. <u>Fictitious Forces</u>

   centrifugal force
   - spinning bucket of water

   Coriolis force
   - motion near the earth's surface

3. <u>Foucault's Pendulum</u>

   choice of coordinate system
   approximate solution of the equations of motion
   precession of the plane of motion

4. <u>Elements of Rigid Body Motion</u>

   kinetic energy and angular momentum
   center of mass, principal axes, and the inertia tensor
   - computing the center of mass
   - computing the inertia tensor

5. <u>Euler's Equations</u>

   principal axes of inertia
   Euler's equations
   - torque-free motion of an axisymmetric body
   - "tennis racket theorem"

6. <u>Euler's Angles</u>

   The rotation group O(3)
   Euler's angles
   Tops
   - torque-free symmetric top
   - symmetric top with one point fixed
   - rotation, precession, and nutation
   - friction and skidding of real tops

7. <u>Thinking Geometrically: Phase Flows in One Dimension</u>

   $\dot{u} = F(u)$; graphical analysis
   fixed points and stability
   physical examples (reexamine systems from first lecture)
   logistic equation $\dot{N} = rN(1 - N/K)$
   uniqueness of solutions; impossibility of oscillations

8. <u>Bifurcations</u>

   saddle-node bifurcation
   transcritical bifurcation
   pitchfork bifurcation

overdamped bead on a hoop
imperfect bifurcation

9. Flows on the Circle

   vector fields on the circle
   linear oscillator
   nonlinear oscillator
   fireflies and Josephson junctions

10. Two-Dimensional Flows

    phase portraits
    classification of fixed points
    computation of phase flows
    Lotka-Volterra model

11. Limit Cycles

    classification of limit cycles
    examples
    relaxation oscillations
    weakly nonlinear oscillators

12. Bifurcations Revisited

    saddle-node, transcritical, and pitchfork bifurcations
    Hopf bifurcations
    oscillating chemical reactions
    global bifurcations
    Josephson junction

13. `MIDTERM EXAMINATION`

14. Continuum Mechanics

    continuum limit of a chain of masses and springs
    field theory
      - Lagrangian density
      - Euler-Lagrange equations
      - boundary conditions

15. Helmholtz Equation I

    the Helmholtz equation
    d'Alambert's solution
    reflection and transmission at an interface
    reflection and transmission at a concentrated load

16. Helmholtz Equation II

    boundary conditions on finite strings
    Bernoulli's solution: Fourier series
    normal modes

17. Helmholtz Equation III

    field theory for a drumhead
    equations of motion
    separation of variables
    normal modes
        circular drumheads: Bessel functions

18. Dispersion

    Schrödinger equation for a free particle in $d = 1$
    general solution by Fourier integral
    wave packets
    dispersion
    phase and group velocity

19. Special Relativity

    principle of relativity
    intervals
        spacelike, timelike, and lightlike classification
        the light cone
    proper time

20. Lorentz Transformations I

    Galilean transformations
    Lorentz transformations
        rotations and boosts
        examples

21. Lorentz Transformations II

    transformation of velocities
    four-vectors
        four-velocity
        proper time for a particle with constant acceleration

22. Examples and Paradoxes

    Doppler effect
    the Twin Paradox

23. Relativistic Mechanics

    action for a relativistic particle
    energy and momentum
    relativistic invariants

24. Relativistic Kinematics

    CM energy and velocity
    decay of particles
    relativistic cross section

- COMPREHENSIVE FINAL EXAMINATION