Physics 110A: Problem Set #3

Due Monday, October 29 by 12:30 pm

Reading: MT chapter 5 ; lecture notes #4

[1] Find the differential equation which results from extremization of the following functionals. Specify whatever boundary conditions are required.

$$(a) \qquad \mathcal{F}[y(x)] = \int_a^b dx \, \left[\tfrac{1}{2}(y')^2 - e^{-yy'} \right]$$

$$(b) \qquad \mathcal{F}[y(x)] = \int_a^b dx \, \left[\tfrac{1}{2} \frac{(y')^2}{1+y^2} + \tfrac{1}{2}(y'')^2 \, e^{-y} \right]$$

[2] A cylinder has height L and radius a. Maximize its volume subject to the constraint $L^4 + a^4 = 1$. (Find a, L, and V.)

[3] The speed of an object moving along a plane depends on the object's distance from a single point (the origin): $v = v(r)$. Construct the time functional $T[\phi(r)]$, where $\phi(r)$ is the object's path represented by azimuthal angle as a function of radius. Find the equation of the extremizing path connecting the points $(r = a, \phi = 0)$ and $(r = b, \phi = \frac{1}{2}\pi)$ for each of the following $v(r)$:

(a) $v(r) = v_0$ (v_0 a constant)

(b) $v(r) = r/\tau$ (τ a constant).

[4] A bumpy cylinder is defined by $\rho(z) = a + b \sin(z/\lambda)$, where $0 < b < a$. What differential equation must be solved to obtain geodesics $\phi(z)$ on such a surface?