Physics 100 A — Final — Fall 2007
Professor Clifford Surko — December 14, 2007

Useful formulas:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\rho(\vec{r}')\,\hat{r}\,d\tau'}{r^2} \qquad V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\rho(\vec{r}')\,d\tau'}{r}$$

$$W = \frac{1}{2}\int \rho V d\tau \qquad W = \frac{\varepsilon_0}{2}\int E^2 d\tau$$

A separable solution of Laplace's equation in Cartesian coordinates:

$$X(x) = A\exp\left[\left(k^2 + l^2\right)^{1/2} x\right] + B\exp\left[-\left(k^2 + l^2\right)^{1/2} x\right]$$

$$Y(y) = C\sin(ky) + D\cos(ky)$$

$$Z(z) = E\sin(lz) + F\cos(lz)$$

A separable solution of Laplace's equation in spherical coordinates is:

$$V(r, \theta) = (1/4\pi\varepsilon_0)\sum_{l=0}^{\infty}(A_l r^l + B_l r^{-(l+1)})P_l[\cos(\theta)];$$

where

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = (3x^2 - 1)/2$
$P_3(x) = (5x^3 - 3x)/2$
$P_4(x) = (35x^4 - 30x^2 + 3)/8$
$P_5(x) = (63x^5 - 70x^3 + 15x)/8$

A multipole-expansion solution of Laplace's equation can be written:

$$V(\vec{r}) = (1/4\pi\varepsilon_0)\sum_{n=0}^{\infty} r^{-(n+1)}\int (r')^n P_n[\cos(\theta')]\rho(\vec{r'})d\tau'$$

In an insulating dielectric material:

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

In a sphere of uniform dipole moment per unit volume , the electric field is:

$$\vec{E} = -(1/3\varepsilon_0)\,\vec{P}$$