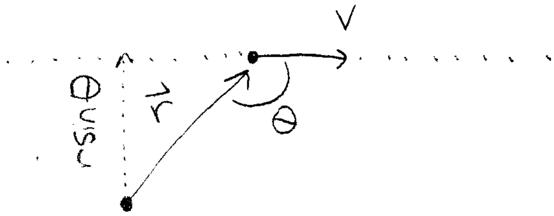
ll

(Q4) Angular momentum is calculated by $\vec{L} = \vec{r} \times \vec{p} =$
$|\vec{L}| = rp\sin\theta$



As r increases, so does θ. The component of $\vec{r}$ that contributes to the angular momentum is the component that is perpendicular to the linear momentum. This component is a constant for the situation that the particle moves in a straight line:

$$|\vec{L}| = (r\sin\theta)p = \text{constant}$$ for a particle that moves in a straight line at constant speed.

(Q8) The basic idea here is to decrease the moment of inertia of part of her body so that that part of her body can have increased angular speed, and that return to the larger moment of inertia to slow her down. For example:

a) To flip upside down, she should tuck her knees to her chest. Bring her knees up causes her upper body to rotate forward so that angular momentum is conserved. When her head is pointed 180° degrees from where it started, she should stick her legs out straight again. Again, because of angular momentum conservation, she will stop rotating.

b) To do an about face, first she should bring her arms in close to her upper body. Then she should twist so that her shoulders and hips rotate in opposite directions. Once she has stretched a bit at the waist, she should extend her arms so that her moment of inertia of her upper body increases. Now her upper body will not be able to rotate very quickly. She can now reverse the direction of her rotating hips so they turn the other way. The angular momentum of her upper body will of course balance so that the total angular momentum is zero, but the angular speed of her upper body will be slow, so that her hips will move through a larger angle than her shoulders. Now, if she brings her arms close to her upper body and stops twisting, she will come to a position different from her initial direction, rotated in the direction her shoulders were sent to move in the initial twisting. She can repeat until she moves through an angle of π radians.

(Q9) The second rotor keeps the body of the helicopter from rotating. When the helicopter is in the air, in order for the angular momentum to be zero, the body of the helicopter should rotate opposite the blades of the main rotor. The second rotor causes a force so that the torques on the body of the helicopter balance.

(Q14) When the axis is horizontal, you will start rotating. The change of direction of the angular momentum vector requires a torque, and because of Newton's 3rd Law, any force you exert on the wheel will cause the wheel to exert a force on you. If you then point the wheel axis downward, you will continue to rotate, and the speed of your rotation will increase. The angular momentum of the bicycle wheel will be half of your angular momentum so that the net angular momentum is equal to the original angular momentum.

P6) (a) Since $\vec{A}\times\vec{A}=0$ then

$$\hat{i}\times\hat{i}=0$$
$$\hat{j}\times\hat{j}=0$$
$$\hat{k}\times\hat{k}=0$$

And since $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ then

$$\hat{i}\times\hat{j} = -\hat{j}\times\hat{i} = \hat{k}$$
$$\hat{i}\times\hat{k} = -\hat{k}\times\hat{i} = -\hat{j}$$
$$\hat{j}\times\hat{k} = -\hat{k}\times\hat{j} = \hat{i}$$

so $\rightarrow$ $\vec{A}\times\vec{B} = \cancel{(A_x\hat{i}\times B_x\hat{i})}^{0} + (A_x\hat{i}\times B_y\hat{j}) + (A_x\hat{i}\times B_z\hat{k})$

$+ (A_y\hat{j}\times B_x\hat{i}) + \cancel{(A_y\hat{j}\times B_y\hat{j})}^{0} + (A_y\hat{j}\times B_z\hat{k})$

$+ (A_z\hat{k}\times B_x\hat{i}) + (A_z\hat{k}\times B_y\hat{j}) + \cancel{(A_z\hat{k}\times B_z\hat{k})}^{0}$

$= A_xB_y\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{k} + A_yB_z\hat{i}$

$+ A_zB_x\hat{j} - A_zB_y\hat{i}$

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_y$$

(b) Using the rules for evaluating a determinant

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}(A_yB_z - A_zB_y) - \hat{\jmath}(A_xB_z - A_zB_x$$

$$+ \hat{k}(A_xB_y - B_xA_y)$$

$$= (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - B_xA_y)$$

(P9) $\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y$

$$= A_xB_yC_z - A_xB_zC_y + A_yB_zC_x - A_yB_xC_z + A_zB_xC_y -$$

I can regroup these terms in two ways:

(1) $B_x(C_yA_z - C_zA_y) + B_y(C_zA_x - C_xA_z) + B_z(C_xA_y - C_yA_x) = \vec{B}\cdot(\vec{C}\times$

(2) $C_x(B_zA_y - B_yA_z) + C_y(B_xA_z - B_zA_x) + C_z(B_xA_y - B_xA_y) = \vec{C}\cdot(\vec{B}\times$

This is possible because of the distributive and commutative properties of multiplying scalars.

to get the total angular momentum

$$\int d\vec{L} = \int \vec{r} \times \vec{v}\, dm = \int rv\, dm$$

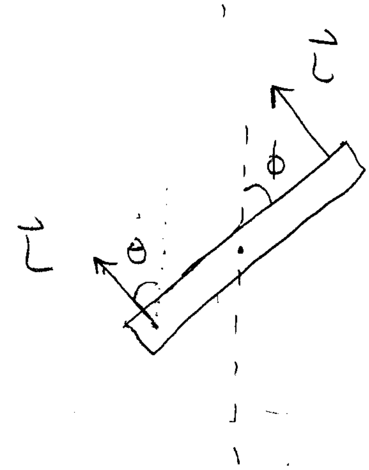
$$= 2 \int_0^{L/2} r\,(r \sin\phi\, \omega)\, \frac{M}{L}\, dr$$

$$= \frac{2M\omega \sin\phi}{L} \int_0^{L/2} r^2\, dr$$

$$= \frac{2M\omega \sin\phi}{L} \left. \frac{r^3}{3} \right|_0^{L/2}$$

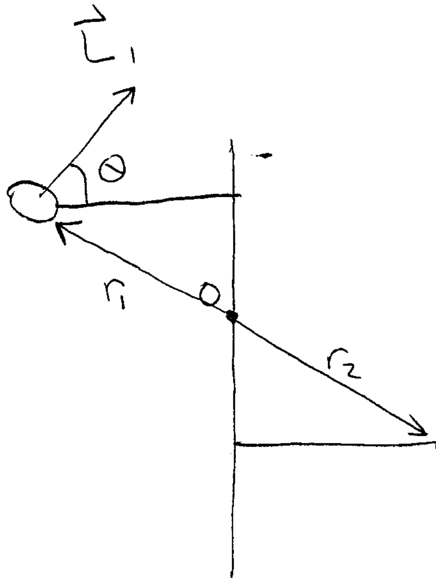
$$= \frac{2M\omega \sin\phi}{L} \frac{L^3}{24}$$

$$= \frac{ML^2}{12} \omega \sin\phi \rightarrow$$ As $\phi \rightarrow 90°$, $L \rightarrow \frac{ML^2}{12} \omega$ which is the angular momentum for a long uniform rod rotating with angular speed ω about an axis through the center.



By the right hand rule, the angular momentum vector makes a $90°$ angle with the rod, so the angular momentum vector makes an angle $\theta = 90° - \phi$ with respect to the axis of rotation.

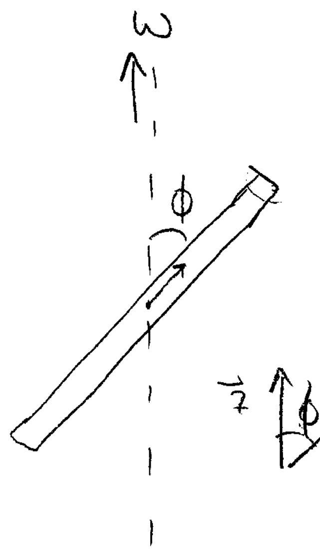
(b) Pick the origin to be at the point on the axle midway between the masses.



Use the right-hand rule to find the direction of the angular momentum of each ball.

Both balls have the angular momentum acting in the same direction $\theta = 45°$ from the axle.

(P28) Calculate the angular momentum with respect to the center of the rod



Each little mass element has angular speed ω around the axis of rotation.

For each little mass piece dm

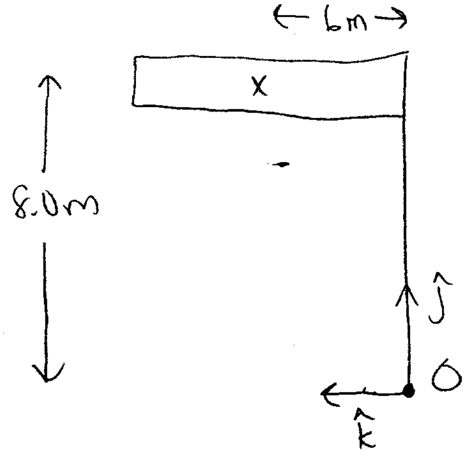
$$d\vec{L} = \vec{r} \times \vec{v}\, dm$$

$$dm = \frac{M}{L}\, dr$$

$$v = r \sin\phi\, \omega$$

Both $\vec{v}$ and $\vec{r}$ are in the opposite direction for the bottom half of the rod, so the angular momentum will be the same for the bottom half as it is for the top half. Therefore, I'll calculate the angular momentum for the top half and then multiply by two

(P13) We will get contribution to the torque in all three directions.



$$\vec{\tau} = (\vec{r} \times \vec{F})$$

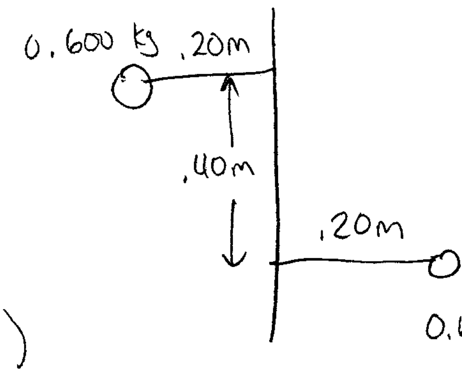
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8.0 & 6.0 \\ \pm 2.4 & -3.0 & 0 \end{vmatrix}$$

$$= [0-(-18)]\hat{i} - [0-(\pm 14)]\hat{j} + [0-(\pm 19)]\hat{k}$$

$$\vec{\tau} = (18\hat{i} \pm 14\hat{j} \mp 19\hat{k}) \text{ kN}\cdot\text{m}$$

(P24)

(a) $L = I\omega$



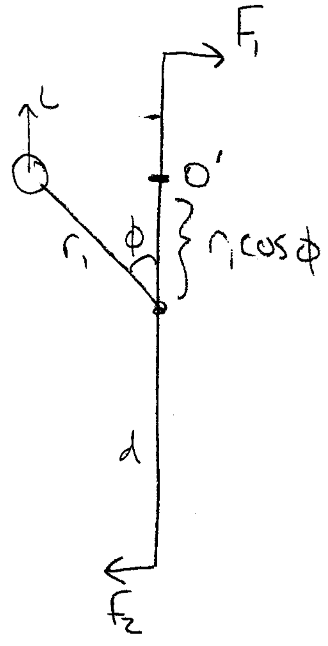
$$= 2(mr^2)\omega$$

$$= 2(0.600\text{kg})(0.20\text{m})^2(30 \text{ rad/s})$$

$$= 1.44 \text{ kgm}^2/\text{s}$$

Using the right hand rule, all the angular momentum lies along the axle for each mass

(P31) Choose the origin to be the center of the circle so that $\vec{L}$ is parallel to $\vec{\omega}$



$$L = \vec{r} \times \vec{p} = r_1 \sin\phi \, m_1 r_1 \sin\phi \, \omega$$

$$= m_1 r_1^2 \sin^2\phi \, \omega$$

The circular motion is provided by the radial force in the rod:

$$F_c = \frac{m_1 v^2}{r_1 \sin\phi} = \frac{m_1 (\omega r_1 \sin\phi)^2}{r_1 \sin\phi} = m_1 \omega^2 r_1 \sin\phi$$

Since the axle is not translating, the net force on it must be zero.

$$\Sigma F \rightarrow \quad F_1 - F_2 + F_c = 0$$

Since the angular momentum is not changing, the net torque on the system must be zero

$$F_1 (d - r_1 \cos\phi) + F_2 (d + r_1 \cos\phi) = 0$$

now, we have two equations with our two unknowns.

$$F_1 = F_2 - F_c = F_2 - m_1 \omega^2 r_1 \sin\phi.$$

$$F_1 = -F_2 \frac{(d + r_1 \cos\phi)}{d - r_1 \cos\phi}$$

$$F_2 - m_1 \omega^2 r_1 \sin\phi = -F_2 \left(\frac{d + r_1 \cos\phi}{d - r_1 \cos\phi} \right)$$

$$F_2 \left(1 + \frac{d + r_1 \cos\phi}{d - r_1 \cos\phi} \right) = m_1 \omega^2 r_1 \sin\phi$$

$$F_2 = \frac{m_1 \omega^2 r_1 \sin\phi \, (d - r_1 \cos\phi)}{d - \cancel{r_1 \cos\phi} + d + \cancel{r_1 \cos\phi}}$$

$$\boxed{F_2 = \frac{m_1 \omega^2 r_1 \sin\phi}{2d} (d - r_1 \cos\phi)}$$

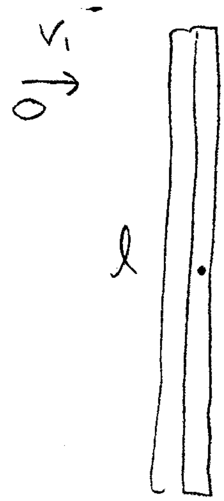
$$F_1 = F_2 - m_1 \omega^2 r_1 \sin\phi$$

$$= m_1 \omega^2 r_1 \sin\phi \left[\frac{d - r_1 \cos\phi}{2d} - 1 \right]$$

$$= m_1 \omega^2 r_1 \sin\phi \left[\frac{d - r_1 \cos\phi - 2d}{2d} \right]$$

$$\boxed{F_1 = \frac{m_1 \omega^2 r_1 \sin\phi}{2d} \left[r_1 \cos\phi - d \right]}$$

(P36) It's a collision, so angular momentum must be conserved.



$$L_{before} = m v_1 \frac{l}{4}$$

$$L_{After} = I\omega + m v_2 \frac{l}{4}$$

$$m v_1 \frac{l}{4} = \frac{M l^2}{12}\omega + m v_2 \frac{l}{4}$$

$$m \frac{l}{4}(v_1 - v_2) = \frac{M l^2}{12}\omega$$

$$\omega = \frac{3m}{l M}(v_1 - v_2) = \frac{3(0.003\,kg)}{(1.0\,m)(0.300\,kg)}(250\,m/s -$$

$$\omega = 2.7\ rad/s$$

(P38) a) Angular momentum must be conserved:

$$L_i = L_f$$

$$I_{platform}\,\omega_i = (I_{platform} + I_{person})\,\omega_f$$

$$\omega_f = \frac{I_{platform}}{I_{platform} + I_{person}}\,\omega_i$$

$$= \frac{670\,kg\,m^2}{670\,kg\,m^2 + (55\,kg)(2.5\,m)^2}(2.0\,rad/s)$$

$$\omega_f = 1.3\ rad/s$$

b) $KE_i = \frac{1}{2} I \omega_i^2 = \frac{1}{2}(670\ kgm^2)(2.0\ rad/s)^2$

$= 1300 J$

$KE_f = \frac{1}{2}(I_{platform} + I_{person})\ \omega_f^2$

$= \frac{1}{2}\left(670\ kgm^2 + (55kg)(2.5m)^2\right)(1.3\ rad/s)^2$

$= 860 J$

$\%\ lost = \frac{KE_i - KE_f}{KE_i} = 34\%$ lost

(P42) $\Omega = \frac{Mgr}{L}$

$T = \frac{2\pi}{\Omega} = \frac{2\pi L}{Mgr} = \frac{2\pi I \omega}{Mgr}$

$T = \frac{2\pi\left(\frac{1}{2}\cancel{M} r_0^2\right)\omega}{\cancel{M} g r} = \frac{\pi r_0^2 \omega}{g r}$

$= \frac{\pi (0.055m)^2 (70\ rev/s)\left(\frac{2\pi rad}{1 rev}\right)}{9.8 m/s^2\ (.17m/2)}$

$T = 5.0 s$

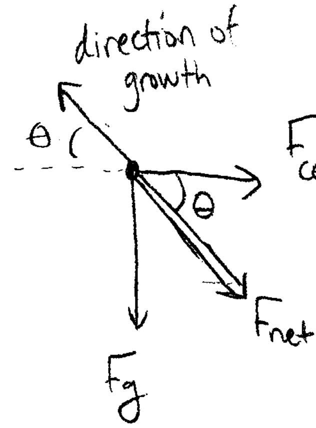
(P45) Put yourself in the rotating reference frame.

You feel two forces → gravity and centrifugal force

$$F_c = \frac{mv^2}{r} \qquad F_g = mg$$

You want to grow opposite these forces (towards the sun)

So



$$\theta = \tan^{-1}\left(\frac{\cancel{m}g}{\cancel{m}\omega^2 r}\right)$$

$$\theta = \tan^{-1}\left(\frac{g}{\omega^2 r}\right)$$ above the horizontal

The reason may be because in order to rotate, there must be a force pulling the plant towards the center. The whole plant must rotate with the same angular speed for structural integrity. So as the distance from the center decreases, the force exerted on the plant decreases, making it easier to remain in tact.

Also notice that the angle gets more vertical the closer the plant gets to the middle.

P59) Use conservation of energy to find the speed

$$mg\left(\frac{l}{2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

The rod is rotating about its center so

$$I = \frac{ml^2}{12}$$

$$mg\left(\frac{l}{2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}\frac{ml^2}{12}\frac{v^2}{(l/2)^2}$$

$$\frac{\cancel{m}gl}{2} = \cancel{m}v^2\left(\frac{1}{2} + \frac{1}{6}\right)$$

$$v^2 = \frac{3}{2}\left(\frac{gl}{2}\right)$$

$$v = \sqrt{\frac{3gl}{4}}$$