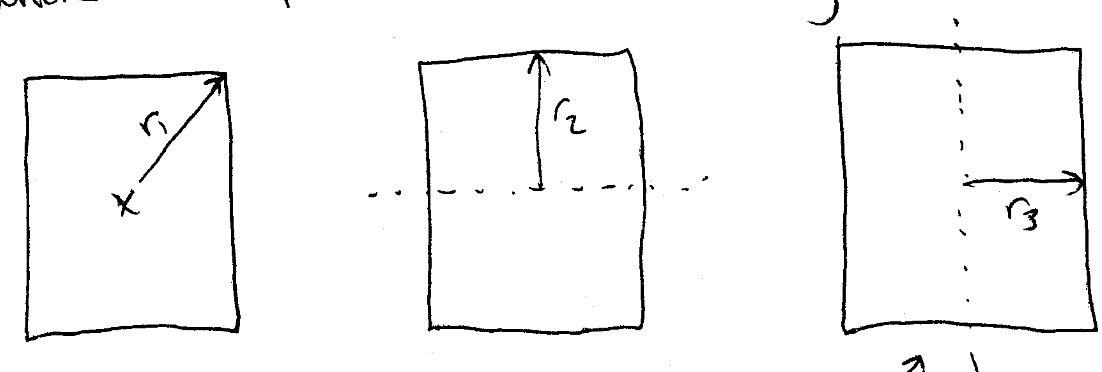
Homework #8

Ch 10

(Q3) (a) The child farther away from the center has to travel ~~the~~ a greater distance in the same amount of time so she must have a greater speed.

(b) Since both children travel through 2π rad per rotation in the same amount of time, their angular speed is the same.

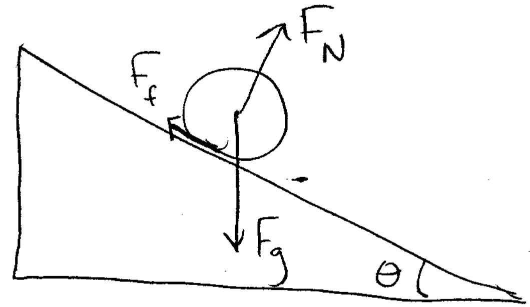
(Q10) Since the book has a relatively uniform density, you can tell which axis would have the least moment of inertia where each "piece" of the book is really close to the axis.



$I \propto r^2$

This axis will have the least moment of inertia.

(Q21) Mechanical energy is conserved, so



$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Since in both cases, the ball rolls without slipping, the linear and rotational kinetic energies are coupled.

$$v = \omega r \longrightarrow \omega = \frac{v}{r}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v^2}{r^2}\right)$$

$$\cancel{m}gh = \cancel{m}v^2\left(\frac{1}{2} + \frac{1}{5}\right)$$

$$v = \sqrt{\frac{10}{7}gh}$$ $\longrightarrow$ The linear speed does not depend on the angle. Both balls will have the same speed at the bottom.

$\rightarrow$ You may ask yourself "But the friction force that causes the ball to roll <u>does</u> depend on the angle of the incline (See Example 10-21). Doesn't that affect the energy of the ball?" There are a couple of arguments that address this:

(#1) This friction force does not change the total energy of the ball. The part of the ball that is in contact with the incline has no displacement while the friction force is acting on it. Therefore, the work is zero.

(#2) This force is causing a torque on the ball, giving it rotational kinetic energy. For the steeper incline, the ball takes less time to reach the bottom

$$t = \frac{d}{v} = \frac{H}{\sqrt{\frac{10}{7}gH}\,\sin\theta}$$ ← The bigger θ is, the smaller time is.

The torque is the rate of change of the angular momentum, and it is also dependent on θ

$$\tau = F_f \times r = \left(\frac{2}{7} mg\sin\theta\right) r$$

So, the steeper the incline, the larger the torque.

$$\tau = \frac{dL}{dt} \rightarrow \Delta L = \tau \Delta t$$

$$= \left(\frac{2}{7} mgr \cancel{\sin\theta}\right)\left(\frac{H}{\sqrt{\frac{10}{7}gH}\,\cancel{\sin\theta}}\right)$$

$$\Delta L = I\Delta\omega = \frac{\frac{2}{7} mgrH}{\sqrt{\frac{10}{7}gH}}$$ → Does not depend on the angle

Thefore, the change in linear speed must be the same for both inclines.

(Q12) The reason that increasing the mass of the tires has more of an effect than increasing the mass of the frame is because you have to consider the rotational energy of the tires. To change the speed of the bicycle, the work require is

$$W = \Delta K = \frac{1}{2} M_{frame} v^2 + \frac{1}{2} m_{tire} v^2 + \frac{1}{2} I_{tire} \omega^2$$

$$I = mr^2 \quad , \quad v = r\omega$$

$$W = \frac{1}{2} M_{frame} v^2 + \frac{1}{2} m_{tire} v^2 + \frac{1}{2} mr^2 \frac{v^2}{r^2}$$

$$W = \frac{1}{2} v^2 \left[M_{frame} + 2 m_{tires} \right]$$

This factor of two because there are two kinetic energies that change → linear and rotational

$$v^2 = \frac{2W}{M_{frame} + 2 m_{tires}}$$

So, to get the same change in speed, and increase in M_{frame} needs a smaller increase in W than the same increase in m_{tires} would. Therefore, increasing the mass of the tires makes it harder to acceleration the bicycle than the changing the mass of any other part of the bicycle by the same amount.

(P12.) $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega - \omega_0}{t}$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\left(\frac{\omega-\omega_0}{t}\right)t^2$$

$$= \omega_0 t + \frac{1}{2}\omega t - \frac{1}{2}\omega_0 t$$

$$= \frac{(\omega + \omega_0)t}{2} \rightarrow \omega_{Ave}\, t$$

$$= \frac{1}{2}(380 \text{ rad} + 210 \text{ rpm})(6.5\text{ s})$$

$$\Delta\theta = 1900 \text{ rad}$$

(P16) (a) $w(t) = \frac{d\theta(t)}{dt} = 6.0 - 16t + 18t^3$

(b) $\alpha(t) = \frac{d\,w(t)}{dt} = \frac{d^2\theta(t)}{dt^2} = -16 + 54t^2$

(c) $w(3.0s) = 6.0 - 16(3.0) + 18(3.0)^3 = 440 \text{ rad/s}$

$\alpha(3.0s) = -16 + 54(3.0)^2 = 470 \text{ rad/s}^2$

(d) $$W_{AVE} = \frac{w(2.0s) + w(3.0s)}{2}$$

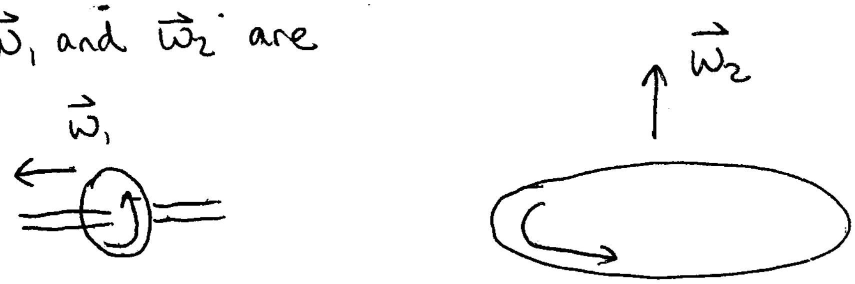
$$= \frac{118 \text{ rad/s} + 444 \text{ rad/s}}{2} = 280 \text{ rad/s}$$

(e) $$\alpha_{AVE} = \frac{\alpha(2.0s) + \alpha(3.0s)}{2} = \frac{200 \text{ rad/s}^2 + 470 \text{ rad/s}^2}{2}$$

$$= 340 \text{ rad/s}^2$$

(P19.)

(a) Use the right hand rule to figure out where $\vec{\omega}_1$ and $\vec{\omega}_2$ are



(b) $\omega_x = \omega_1$

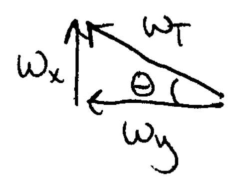
$\omega_y = \omega_2$

$$\omega_T = \sqrt{\omega_x^2 + \omega_y^2} = \sqrt{(50.0\ \text{rad/s})^2 + (35.0\ \text{rad/s})^2}$$

$$\omega_T = 61.0\ \text{rad/s}$$

$$\theta = \tan^{-1}\left(\frac{\omega_y}{\omega_x}\right) = \tan^{-1}\left(\frac{35.0\ \text{rad}}{50.0\ \text{rad}}\right)$$

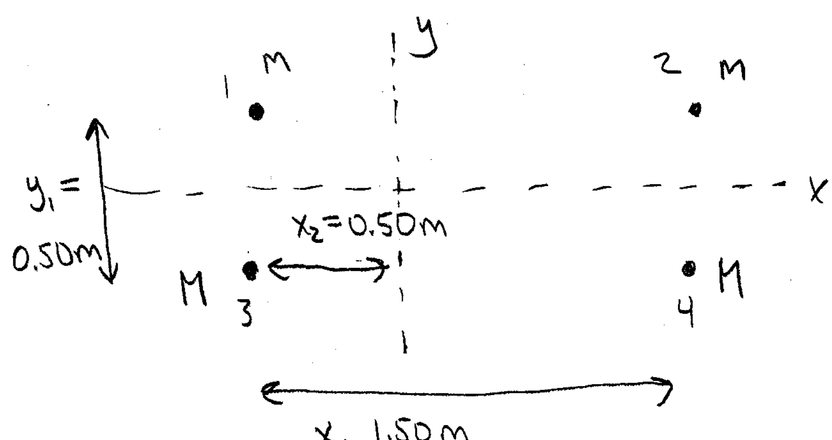
$\theta = 35.0°$ up from horizontal



(P30)

(a) Moment of inertia about y-axis

$$I = \sum_{i=1}^{4} m_i r_i^2$$



$$I_y = m x_2^2 + m(x_1 - x_2)^2 + M x_2^2 + M(x_1 - x_2)^2$$

$$= m\left[x_2^2 + (x_1 - x_2)^2\right] + M\left[x_2^2 + (x_1 - x_2)^2\right]$$

$$= (m + M)\left[x_2^2 + (x_1 - x_2)^2\right]$$

$$= (1.8\,kg + 3.1\,kg)\left[(0.50\,m)^2 + (1.0\,m)^2\right]$$

$$I_y = 6.1\ kgm^2$$

(b) $$I_x = m\left(\frac{y_1}{2}\right)^2 + m\left(\frac{y_1}{2}\right)^2 + M\left(\frac{y_1}{2}\right)^2 + M\left(\frac{y_1}{2}\right)^2$$

$$= 2(m + M)\frac{y_1^2}{4}$$

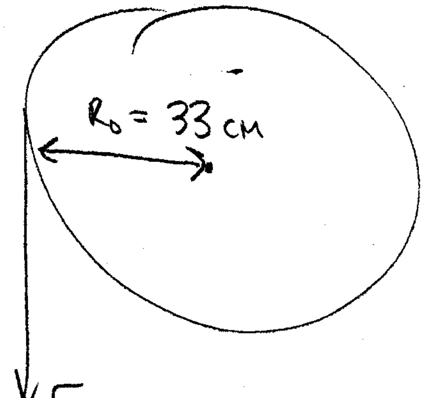
$$= \frac{(1.8\,kg + 3.1\,kg)(0.50\,m)^2}{2}$$

$$= 0.61\ kgm^2$$

→ It would be harder to accelerate the array around the vertical axis, because the moment of inertia is bigger.

(P35) The linear speed is related to the angular speed.

Fig 10-22



$F_T = 3.00t - 0.20t^2$

$M = 4.00$ kg

$I = 0.385$ kgm²

$\tau_{fr} = 1.10$ N·m

$$v(t) = R_0\,\omega(t)$$

Since the tension force varies with time the angular acceleration varies with time so to find the angular speed, we integrate

$$\omega(t) - \cancel{\omega_0}^{\,0} = \int_{t=0}^{t} \alpha(t')\,dt'$$

Use torque considerations to relate force to angular acceleration

$$\Sigma\tau = \tau_T - \tau_{Fr} = I\alpha$$

$$\tau_T = \vec{F}\times\vec{r} = F_T R_0 = (3t - 0.2t^2)R_0$$

$$v(t) = R_0\,\omega(t) = R_0 \int_{t=0}^{t} \frac{[3t' - 0.2(t')^2]R_0 - \tau_{Fr}}{I}\,dt'$$

$$= \frac{R_0^{\,2}}{I}\left[\frac{3}{2}t^2 - \frac{0.2}{3}t^3 - \frac{\tau_{Fr}\,t}{R_0}\right]_{t=0}^{8s}$$

$$= \frac{(0.33\,m)^2}{0.385\,kg\,m^2}\left[\frac{3}{2}(8)^2 - \frac{0.2}{3}(8)^3 - \frac{(1.10)(8)}{.33}\right] - 0$$

$$v(t) = 10.\ m/s$$

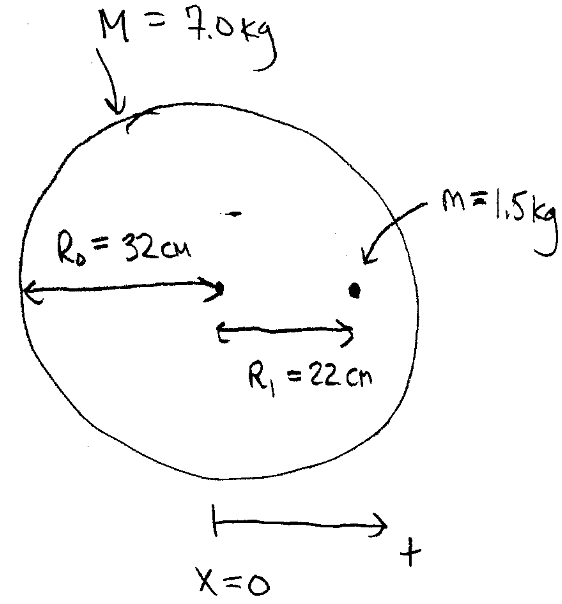
(P48)

a) $$X_{CM} = \frac{\sum_{i=1}^{2} m_i x_i}{\sum_{i=1}^{2} m_i}$$

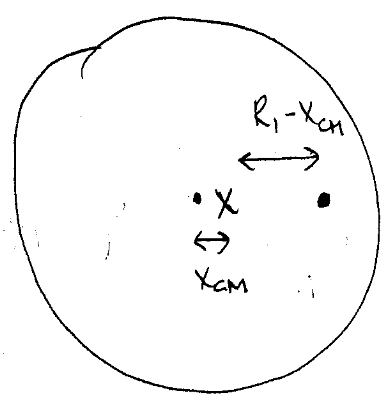
$$= \frac{M \cancelto{0}{x_{wheel}} + m x_{weight}}{M+m}$$

$$= \frac{m}{M+m} x_{weight} = \frac{1.5\,kg}{7.0\,kg + 1.5\,kg}(0.22\,m)$$

$= 0.039$ m from the center



(b) The moment of inertia will be the sum of the moments of inertia for the wheel and the weight through the center of mass



Wheel: Use parallel-axis theorem

$$I_{wheel} = I_{CM} + Mh^2$$

$$I(X_{cm}, system) = I_{CM,wheel} + M X_{CM}^2$$

$$= \frac{MR_0^2}{2} + M X_{cm}^2$$

Weight: $I_{weight} = m(R_1 - x_{cm})^2$

$$\rightarrow I(x_{cm}) = I_{wheel} + I_{weight}$$

$$= M\left(\frac{R_0^2}{2} + x_{cm}^2\right) + m(R_1 - x_{cm})^2$$

$$= 7.0\,kg\left(\frac{(0.32\,m)^2}{2} + (0.039\,m)^2\right) + 1.5\,kg(.22\,m - 0.039\,m)^2$$

$$= 0.42\ kg\,m^2$$

(P64) The work the motor must do should balance the work done by friction on the platform.

$$W_{friction} = \Delta K = \frac{1}{2} I \Delta(\omega^2) = -\frac{1}{2}\left(\frac{1}{2}MR_0^2\right)\left(3.8\,rev/s\ \frac{2\pi\,rad}{rev}\right)^2$$

$$W_{friction} = -\frac{1}{4}(280\,kg)(5.5\,m)^2(23.9\,rad/s)^2$$

$$= -1.2\times10^6\ J$$

$$\tau_{friction} = \frac{W}{\Delta\theta} = \frac{W}{\frac{-\omega_i^2}{2\alpha}} \rightarrow$$ Assume constant α

$$\alpha = \frac{-\omega_i}{t} = \frac{-3.8(2\pi)}{18}$$

$$\alpha = 1.3\ rad/s^2$$

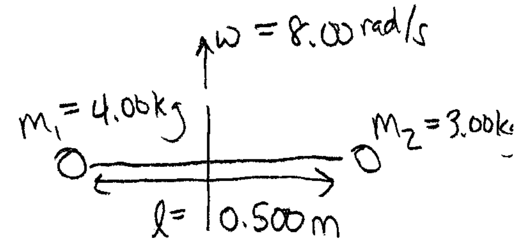
$$= \frac{-1.2\times10^6\,J\,(2)(-1.3\,rad/s^2)}{-[(2\pi)(3.8\,rev/s)]^2}$$

$$\tau_{friction} = 5500\ N\cdot m$$

$$P = \vec{\tau} \cdot \vec{\omega} = (5500\ \mathrm{N \cdot m})(2\pi)(3.8\ \mathrm{rev/s})$$

$$P = 1.3 \times 10^5\ \mathrm{W} = 180\ \mathrm{hp}$$

(P68)



(a) $K = \frac{1}{2} I \omega^2$

$$I = \sum_i m_i r_i^2 = m_1 \left(\frac{\ell}{2}\right)^2 + m_2 \left(\frac{\ell}{2}\right)^2$$

$$I = \frac{\ell^2}{4}(m_1 + m_2) = \frac{(0.500\ \mathrm{m})^2}{4}(7.00\ \mathrm{kg})$$

$$I = 0.438\ \mathrm{kg\,m^2}$$

$$K = \frac{1}{2}(0.438\ \mathrm{kg\,m^2})(8.00\ \mathrm{rad/s})^2$$

$$K = 14.0\ \mathrm{J}$$

(b) Each mass is moving in a circle, so the net force must be toward the center

$$F_c = \frac{mv^2}{r} = \frac{m(r\omega)^2}{r} = mr\omega^2$$

$$F_{net\,1} = (4.00\ \mathrm{kg})\left(\frac{0.500\ \mathrm{m}}{2}\right)(8.00\ \mathrm{rad/s})^2 = 64.0\ \mathrm{N}$$

$$F_{net\,2} = (3.00\ \mathrm{kg})\left(\frac{0.500\ \mathrm{m}}{2}\right)(8.00\ \mathrm{rad/s})^2 = 48.0\ \mathrm{N}$$

(C) First we have to find the location of the center of mass.

$$\vec{x}_{cm} = \frac{m_1\left(-\frac{l}{2}\right) + m_2\left(\frac{l}{2}\right)}{m_1+m_2} = \frac{l}{2}\left(\frac{m_2-m_1}{m_1+m_2}\right)$$

$$= -\frac{0.500m}{2}\left(\frac{1kg}{7kg}\right)$$

$$\vec{X}_{cm} = -0.0357\text{ m from the center.}$$

Now, use the parallel axis theorem to find the new moment of inertia

$$I_{old} = I_{cm} + Mx_{cm}^2$$

$$I_{cm} = I_{old} - Mx_{cm}^2$$

$$= 0.438\ kgm^2 - (7kg)(0.0357m)^2$$

$$= 0.429\ kgm^2$$

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(0.429\ kgm^2)(8\ rad/s)^2$$

$$K = 13.7\ J$$

$$F_{net_1} = m_1 r_1 \omega^2 = m\left(\frac{l}{2} - x_{cm}\right)\omega^2 = 4kg(0.25-0.0357)(8)^2$$

$$F_{net_1} = 54.9\ N$$

$$F_{net2} = m\left(\frac{l}{2} + x_{cm}\right)\omega^2 = 54.9\ N$$

(P.74) We need to consider both forces and torques

(a) Relative to the car, a_{ball}

$$m(a_{car} - a_{ball}) = F_s$$

$$\tau = F_s r = I\alpha = I\left(\frac{a_{ball}}{r}\right)$$

$$F_s = \frac{I}{r^2} a_{ball}$$

$$F_s = \frac{\frac{2}{5} M \cancel{r^2}}{\cancel{r^2}} a_{ball}$$

$$F_s = \frac{2}{5} M a_{ball}$$

Plugging back into Newton's 2nd Law

$$\cancel{m}(a_{car} - a_{ball}) = \frac{2}{5}\cancel{m} a_{ball}$$

$$a_{car} = \left(1 + \frac{2}{5}\right) a_{ball}$$

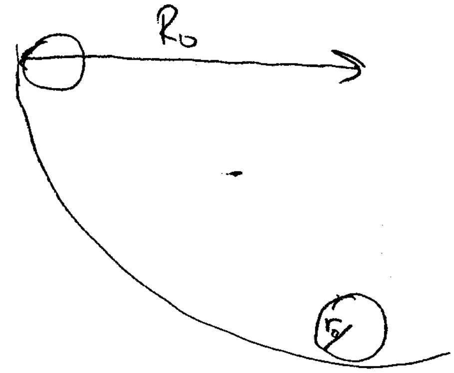
$$a_{ball} = \frac{a_{car}}{1 + 2/5} = \frac{5 a_{car}}{7} \quad \text{backward}$$

(b) Relative to the ground $\rightarrow$ $a_{ground} = (a_{car} - a_{ball})$

$$a_{ground} = a_{car} - \frac{5}{7} a_{car}$$

$$= \frac{2 a_{car}}{7} \quad \text{forward}$$

(P75) Energy is conserved so



$$mg(R_0 - r_0) = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

since the ball rolls without slipping

$$v = r_0 \omega \quad , \quad I = \frac{2}{5} m r_0^2$$

$$mg(R_0 - r_0) = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{5} m r_0^2\right)\left(\frac{v}{r_0}\right)^2$$

$$= m v^2 \left(\frac{1}{2} + \frac{1}{5}\right)$$

$$g(R_0 - r_0) = \frac{7}{10} v^2$$

$$v = \sqrt{\frac{10}{7}(R_0 - r_0) g}$$

(P96) Angular momentum is conserved

$$L_i = L_f$$

$$(I \omega_0)_{disk} = (I \omega)_{Disk} + (I \omega)_{rod}$$

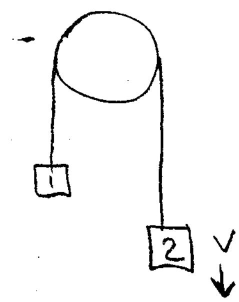
$$= (I_{Disk} + I_{rod}) \omega$$

$$\omega = \frac{I_{Disk}}{I_{Disk} + I_{rod}} \omega_0 = \frac{\frac{1}{2} \cancel{m r_0^2}}{\frac{1}{2} \cancel{m r_0^2} + \frac{1}{12} \cancel{m} (2 \cancel{r_0})^2} \omega_0$$

$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} \omega_0 = \frac{3}{5} \omega_0 = \frac{3}{5}(7.0 \text{ rev/s})$$

$$\omega = 4.2 \text{ rev/s}$$

(P102) Since m_1 and m_2 are connected by a rope, their accelerations will be the same

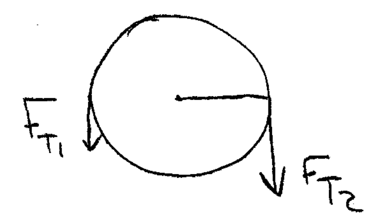


$$a_1 = a_2 = a$$

If the rope doesn't slip on the pulley, then

$$a = R\alpha$$

The net torque on the pulley will be



$$\Sigma\tau = \tau_2 - \tau_1 = I\alpha$$

$$\tau_2 = F_{T_2}R \quad , \quad \tau_1 = F_{T_1}R$$

$$m_2 a = m_2 g - F_{T_2} \rightarrow \tau_2 = m_2(g-a)R$$

$$m_1 a = F_{T_1} - m_1 g \rightarrow \tau_1 = m_1(g+a)R$$

$$m_2(g-a)R - m_1(g+a)R = I\alpha = I\frac{a}{R}$$

$$m_2 g - m_2 a - m_1 g - m_1 a = \frac{Ia}{R^2}$$

$$g(m_2 - m_1) = \frac{I}{R^2}a + (m_1+m_2)a$$

$$a = g\left(\frac{m_2 - m_1}{\frac{I}{R^2} + m_1 + m_2}\right)$$

If $I = 0 \rightarrow$

$$a = g\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$ which would be larger.