Homework # 6

Ch8

(Q4) A superball cannot rebound to a greater height because that would require that energy be added to the earth-ball system.

$U_i \geq U_f$ in general for a dropped ball

for a superball $U_i = U_f$

(Q11) Yes, the spring can leave the table. By pressing in with your hand, you are doing work on the spring (adding energy to it). That energy is stored as elastic potential energy. When you ~~removed~~ remove your hand, the elastic potential energy is released. The spring will push against the table, but the table can't go anywhere, so the spring's elastic potential energy will be converted to kinetic energy and an increase is gravitational potential energy.

(Q17) The gravitational potential energy goes like $U \propto \frac{-1}{r}$ so it will be greatest when r is the largest (the minus sign is important here). Since the earth is closest to the sun during the winter, the gravitational potential energy is greatest during the summer (Northern Hemisphere).

(P6) $\vec{F} = -\frac{dU}{d\vec{r}} = -\frac{dU}{dx}\hat{i} - \frac{dU}{dy}\hat{j} - \frac{dU}{dz}\hat{k}$

$-\frac{dU}{dx} = -6x - 2y$ $\qquad U = 3x^2 + 2xy + 4y^2 z$

$-\frac{dU}{dy} = -2x - 8yz$

$-\frac{dU}{dz} = -4y^2$

$\vec{F} = (-6x - 2y)\hat{i} + (-2x - 8yz)\hat{j} + (-4y^2)\hat{k}$

(P10) Use conservation of energy:

$K_i + \cancel{U_i}^{0} = \cancel{K_f}^{0} + U_f$

$K_i = U_f$

$\frac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh_f$

$h_f = \frac{v_i^2}{2g} = \frac{(5.0\,m/s)^2}{2(9.8\,m/s^2)} = 1.3\ m$

The maximum height the rope would allow is 8 m (twice the radius of a circle), so as long as the final height is less than 8 m, the length of the vine doesn't matter.

(P19)

(a) When the mass reaches its maximum speed, all the elastic potential energy will have turned into kinetic energy

$$U_{e_i} + K_i = K_f$$

$$\frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}mV_{max}^2$$

$$V_{max} = \sqrt{\frac{k}{m}x_0^2 + V_0^2}$$

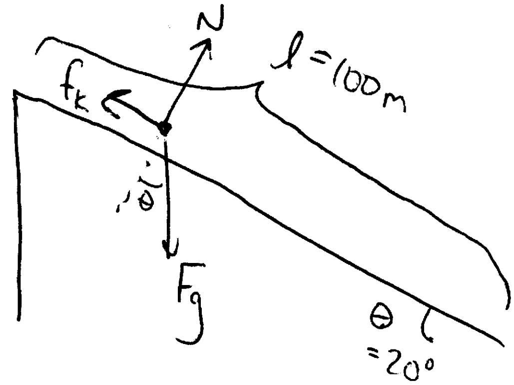
(b) When the mass reaches its maximum displacement, all the kinetic energy will be turned into elastic potential energy.

$$U_{e_i} + K_i = U_{e_f}$$

$$\frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}k\,x_{max}^2$$

$$x_{max} = \sqrt{x_0^2 + \frac{m}{k}v_0^2}$$

(P27) $W_{net} = \Delta K$



(a)

$$W_{gravity} - W_{friction} = \frac{1}{2}m\left(v_f^2 - \cancel{v_i^2}^{\,0}\right)$$

$$W = \vec{F}\cdot\vec{d}$$

$$|W_{gravity}| = mg(l\sin\theta)$$

$$|W_{friction}| = (\mu_k mg\cos\theta)l$$

$$W_{normal} = 0 \text{ (because } \vec{N} \text{ is } \perp \text{ to } \vec{d})$$

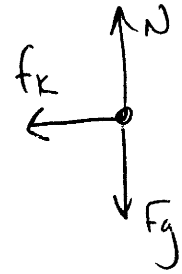
$$\cancel{m}gl(\sin\theta - \mu_k\cos\theta) = \frac{1}{2}\cancel{m}v_f^2$$

$$v_f = \sqrt{2gl(\sin\theta - \mu_k\cos\theta)}$$

$$= \sqrt{2(9.8\,m/s^2)(100m)[\sin 20° - (0.090)\cos 20°]}$$

$$\boxed{v_f = 22\,m/s}$$

(b) $W_{net} = \Delta K$



$$-W_{friction} = \frac{1}{2}m\left(\cancel{v_f^2}^{\,0} - v_i^2\right)$$

$$-\mu_k \cancel{m}g\,d = -\frac{1}{2}\cancel{m}v_i^2$$

$$d = \frac{v_i^2}{2\mu_k g} = \frac{(22\,m/s)^2}{2(0.090)(9.8\,m/s^2)} = \boxed{270m = d}$$

(P32)

(a)

$$\Delta K + \Delta U = 0$$

$$K_f - \cancel{K_i}^{\,0} + \cancel{U_f}^{\,0} - U_i = 0$$

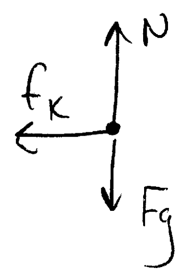
$$U_i = K_f$$

$$\cancel{m} g h_A = \frac{1}{2} \cancel{m} v_B^2$$

$$v_B = \sqrt{2gh_A} = \sqrt{2(9.8\,m/s^2)(2m)} = 6.3\,m/s$$

(b)

$$W = \vec{F} \cdot \vec{d}$$



$$W_{friction} = -f_K d$$

$$= -\mu_K m g d$$

$$= -(0.25)(1.0kg)(9.8\,m/s^2)(3.0m)$$

$$= -7.4\,J$$

(negative sign means work is being done on the block).

(c)

$$W_{net} = \Delta K$$

$$W_{friction} = \frac{1}{2} m (v_C^2 - v_B^2)$$

$$\sqrt{\frac{2W_{friction}}{m} + v_B^2} = v_f = v_c$$

$$\sqrt{\frac{2(-7.4J)}{1.0kg} + (6.3 m/s)^2} = \boxed{5.0 m/s = V_c}$$

(d) $\Delta K + \Delta U = 0$

$$\cancel{K_f}^{0} - K_i + U_f - \cancel{U_i}^{0} = 0$$

$$U_f = K_i$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mV_c^2$$

$$k = \frac{mV_c^2}{x^2} = \frac{(1.0kg)(5.0 m/s)^2}{(0.20m)^2}$$

$$k = 625 \text{ N/m}$$

(P41) $\Delta U = U(r = r_E) - U(r = h)$

$$\Delta U = -GmM_E\left(\frac{1}{r_E} - \frac{1}{r_E + h}\right)$$

for little "g"

$$F_g = mg = \left(\frac{-GM_E}{R_E^2}\right)m \rightarrow g = \frac{-GM_E}{r_E^2}$$

$$\Delta U = -GM_E m \left(\frac{r_E + h - r_E}{r_E (r_E + h)} \right)$$

$$= -GM_E m \left(\frac{h}{r_E^2 (1 + h/r_E)} \right)$$

$$= \left(\frac{-GM_E}{r_E^2} \right) \frac{mh}{(1 + h/r_E)}$$

$$\Delta U = \frac{mgh}{(1 + h/r_E)}$$

(#52) Let's pretend like we have a solid sphere with the same density as our uniform shell with a radius of $r = r_1$. The total mass is

$$M_T = M + M'$$

The gravitational potential is $U_1(r) = -\frac{G(M+M')m}{r}$

Now we add on top of that another sphere with a density opposite of that of our big sphere, now with radius r_2

the potential is:

$$U_2(r) = -\frac{G(-M')m}{r}$$

Now, we recognize that if I superpose these two spheres so that the density adds to zero from $r=0$ to $r=r_2$ (the densities cancel). The total gravitational potential is just the sum of the potentials from my two spheres.

$$U_T = U_1(r) + U_2(r) = -\frac{Gm}{r}(M+M'-M')$$

$$U_T = -\frac{GmM}{r}$$

So its just like the gravitational potential of a sphere with mass M.

(#54)

$$P = \frac{\Delta E}{t} = \frac{\Delta U}{t}$$

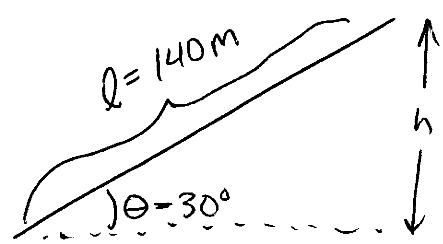
$$t = \frac{\Delta U}{P} = \frac{mgh}{P} = \frac{(285\,kg)(9.8\,m/s^2)(16.0\,m)}{1750\,W}$$

$$t = 25.5\,s$$

(#63)

$$P = \frac{\Delta E}{t} = \frac{\Delta U}{t} = \frac{mgh}{t} = \frac{mg\,l\sin\theta}{t}$$

$$= \frac{(105\,kg)(9.8\,m/s^2)(140\,m)\sin 30^\circ}{61\,s}$$

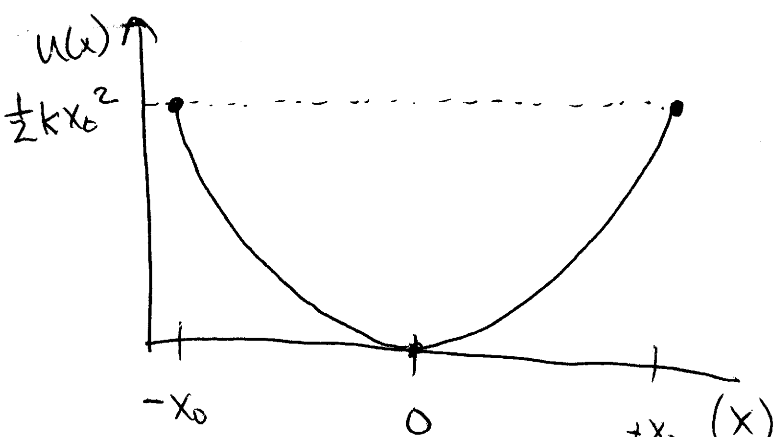


$$P = 1200\,W$$

(#68) for a spring:

$$F = -kx$$

$$U = -\int F\,dx = \frac{1}{2}kx^2$$



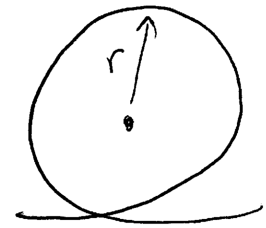
The potential energy is quadratic in x. The spring will have maximum potential energy $U_{max} = \frac{1}{2}kx_0^2$ when the displacement is the biggest (where the speed/kinetic energy is zero).

(#82)(a) The ~~maximum~~ minimum release height should give the box enough energy so that the speed of the box is enough to maintain contact with the track: That is, the centripetal force at the top of the loop is due only to the weight of the box.

$$\cancel{m}g = \frac{\cancel{m}v_T^2}{r}$$

$$v_T = \sqrt{rg}$$

The ~~top~~ total energy at the top of the loop is



$$E_T = K + U = \frac{1}{2}mv_T^2 + mgh$$

$$= \frac{1}{2}mrg + mg(2r)$$

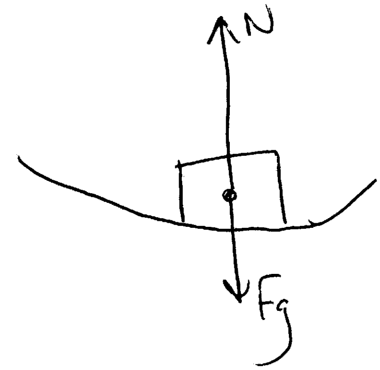
$$E_T = \frac{5}{2}mrg$$

At the top of the track, there is kinetic energy, so the height should provide a gravitational potential energy of E_T

$$\cancel{m}\cancel{g}h_i = \frac{5}{2}\cancel{m}r\cancel{g}$$

$$h_i = \frac{5}{2}r = 2.5\,r$$

(b) At the bottom of the loop, the normal force must provide the centripetal force necessary to move in a circle.



→ The speed at the bottom of the loop

$U_i = K_B$ (setting $U=0$ at the bottom of the loop)

$$\cancel{m}g(2h) = \frac{1}{2}\cancel{m}v_B^2$$

$$v_B = \sqrt{2(2gh)}$$

$$= 2\sqrt{gh}$$

→ $$\frac{mv^2}{r} = N - mg$$

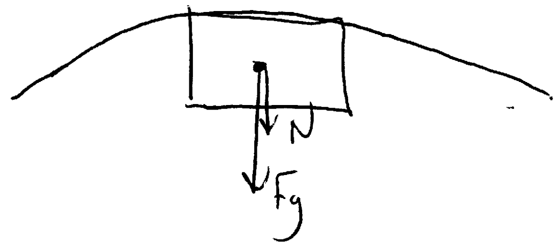
$$N = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

$$= m\left(\frac{4gh}{r} + g\right)$$

$$= mg\left(\frac{4h}{r} + 1\right)$$

(c) Similarly, at the top of the ~~track~~ loop, the normal force must provide the centripetal force necessary to move in a circle.

→ The speed at the top of the loop



$$U_i = K_T + U_T$$

$$\cancel{m}g(2h) = \frac{1}{2}\cancel{m}v_T^2 + \cancel{m}g(2r)$$

$$\sqrt{2g(2h-2r)}$$

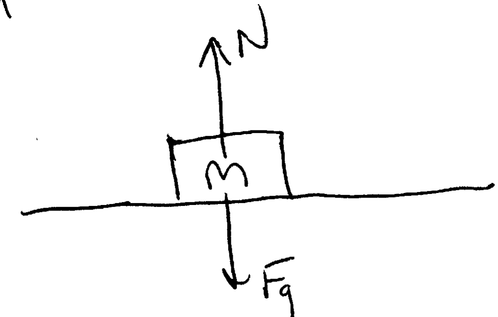
$$V_T = 2\sqrt{g(h-r)}$$

$$\frac{mv_T^2}{r} = N + mg$$

$$N = m\left(\frac{v_T^2}{r} - g\right) = m\left(\frac{4g(h-r)}{r} - g\right)$$

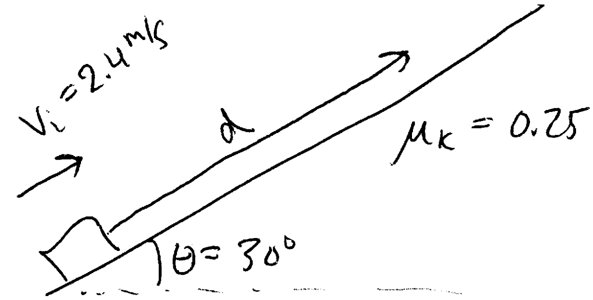
$$N = mg\left(\frac{4(h-r)}{r} - 1\right)$$

(d) On the flat section, the normal force is equal to the weight



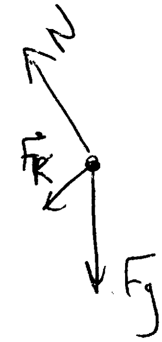
(#90)

(a) $W_{net} = \Delta K$



$W_{net} = W_{gravity} + W_{friction} + \cancel{W_{Normal}}^{0}$

$= (-mg\sin\theta\, d) + (-\mu_K mg\cos\theta\, d)$



$-\cancel{m}gd(\sin\theta + \mu_K\cos\theta) = \frac{1}{2}\cancel{m}(\cancel{v_f^2}^{0} - v_i^2)$

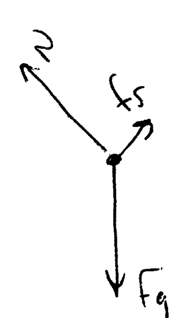
$gd(\sin\theta + \mu_K\cos\theta) = \frac{1}{2}v_i^2$

$$d = \frac{v_i^2}{2g(\sin\theta + \mu_K\cos\theta)}$$

$$= \frac{(2.4\,m/s)^2}{2(9.8\,m/s^2)(\sin 30° + 0.25\cos 30°)}$$

$d = 0.41\ m$

(b) The condition on the coefficient of static friction should be that it is small enough so that there is a net force down the slope



$\cancel{m}g\sin\theta > F_{s,max} = \mu_s \cancel{m}g\cos\theta$

$\mu_s < \tan\theta = \tan 30°$

$\mu < 0.58$

(c) $W_{net} = \Delta KE$

$W_{net} = (-F_k d)_{up} + (-F_k d)_{down}$

$= -2 \mu_k mg \cos\theta \, d$

$\frac{1}{2} \cancel{m}(v_f^2 - v_i^2) = -2\mu_k \cancel{m} g \cos\theta \, d$

$v_f = \sqrt{-4\mu_k g \cos\theta \, d + v_i^2}$

$= \sqrt{-4(0.25)\cos 30^\circ (0.41\,m)(9.8) + (2.4\,m/s)^2}$

$v_f = 1.5\,m/s$