Absolute Measurements Versus Measurements of Variability

Objectives:
1. A manufacturer of racquet balls has discovered that another manufacturer is infringing his trademark. The illegal racquet balls are impossible to distinguish from the real ones by inspecting the outside. However, it has been learned that the illegal manufacturing process is not as uniform as the original one, so that the thickness of the shell of the hollow spheres varies from ball to ball by about 10%. You are to devise a simple, fast, and non-destructive method for measuring the variation in thickness of the shell of large numbers of the balls in each shipment arriving at a number of stores, to determine if the variation in thickness is much less than 10%.

The suggested method is to measure the moment of inertia and mass of the balls. The moment of inertia for a hollow sphere of inner radius, r_2, outer radius, r_1, and total mass, M, is:

$$I = \frac{2M\left(r_1^5 - r_2^5\right)}{5\left(r_1^3 - r_2^3\right)}$$

2. Expensive, structural rods for use on a special machine are made in the form of an outer tube and a solid inner cylinder each of a different material. It is important that the diameters of the inner and outer cylinders be the same from sample to sample to within better than 5%. You are to devise a method to measure the thickness and density of the outer and inner cylinders without damaging them so that rods not within specifications will not be used in the machine.

As in the above case, the recommended procedure is to measure the mass and the moment of inertia of the cylinder.
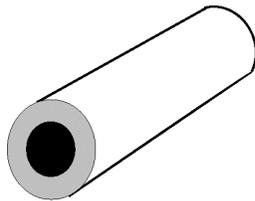
For a solid cylinder, $I_1 = \dfrac{M_1 R_1^2}{2}$

For a hollow cylinder, $I_2 = \dfrac{M_2}{2}\left(R_1^2 + R_2^2\right)$

(see textbook section 11-7)

Reference: Halliday, Resnick and Walker, 5th edition, Chapters 11, 12, and 14

Possible Methods:
Make a torsion pendulum.
The period of the pendulum, T is related to the torsion constant of the fiber, κ, and the moment of inertia of the pendulum, I, according to: $T = 2\pi\sqrt{\dfrac{I}{\kappa}}$ (see textbook sect. 16-5) Use a known moment of inertia such as a solid cylinder or sphere of uniform density and measured mass to determine the torsion constant of the wire from the above equation. Measure the total mass of the object of interest. Then measure the period of the torsion pendulum with the fiber attached along an axis of symmetry of the object (the axis of the cylinder).

Roll down an inclined plane

Measure the total mass, M, and the radius, R, of the object. Allow it to roll down an inclined plane of angle θ for a distance d (see text section 12-1). Use the photo gate timers to measure the time to travel distance d. From this determine the acceleration and final velocity, v_d, of the object. From energy conservation,

$$Mgd\sin(\theta) = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_d^2$$

where $\omega = v_d / R$. *This technique will not work well for the cylinder because it will not roll straight down the inclined plane without a constraint and the constraint will add friction.*