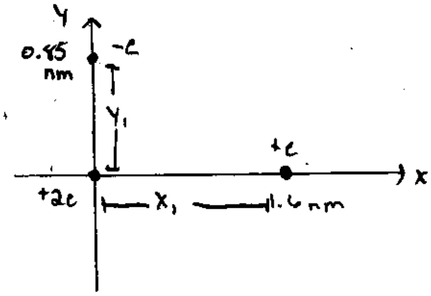
Physics 2B Lecture 1 Assigned Problem Solutions

Chapter 23

11. 

Find the force on a charge of +2e located at the origin.

$x_1 = 1.6$ nm
$y_1 = 0.85$ nm

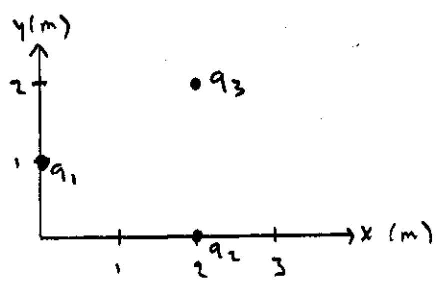
The force due to the electron is $\vec{F}_e = \dfrac{k(2e)(-e)}{y_1^2}(-\hat{j}) = \dfrac{2ke^2}{y_1^2}\hat{j} = 0.638 \text{ nN } \hat{j}$

the force due to the proton is $\vec{F}_p = \dfrac{k(2e)(e)}{x_1^2}(-\hat{i}) = -\dfrac{2ke^2}{x_1^2}\hat{i} = -0.180 \text{ nN } \hat{i}$

the total coulomb force is

$$\vec{F} = \vec{F}_e + \vec{F}_p$$

$$\boxed{\vec{F} = -0.180 \text{ nN } \hat{i} + 0.638 \text{ nN } \hat{j}}$$

19. 

$q_1 = 68\,\mu C$ $q_2 = -34\,\mu C$, $q_3 = 15\,\mu C$

Find the electric force on q_3. ($\vec{F}_3$)

Let $\vec{r}_1$ be the position vector of charge q_1, $\vec{r}_2$ for q_2, and $\vec{r}_3$ for q_3.

From the diagram we see $\vec{r}_1 = \hat{j}$, $\vec{r}_2 = 2\hat{i}$, $\vec{r}_3 = 2\hat{i} + 2\hat{j}$

All the position vectors have units of meters.

We can use the principle of superposition to solve this problem.

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$\vec{F}_{13} = \dfrac{kq_1q_3}{|r_{13}|}\hat{r}_{13}$ where $\vec{r}_{13} = \vec{r}_3 - \vec{r}_1$, $\hat{r}_{13} = \dfrac{\vec{r}_{13}}{|r_{13}|}$

likewise

$\vec{F}_{23} = \dfrac{kq_2q_3}{|r_{23}|}\hat{r}_{23}$ where $\vec{r}_{23} = \vec{r}_3 - \vec{r}_2$, $\hat{r}_{23} = \dfrac{\vec{r}_{23}}{|r_{23}|}$

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19. (con't) Now it's just a matter of arithmetic.

$$\vec{r}_{13} = \vec{r}_3 - \vec{r}_1 = 2\hat{i} + \hat{j} \quad , \quad |\vec{r}_{13}| = \sqrt{4+1} = \sqrt{5}$$

$$\vec{r}_{23} = \vec{r}_3 - \vec{r}_2 = 2\hat{j} \quad , \quad |\vec{r}_{23}| = 2$$

Then

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

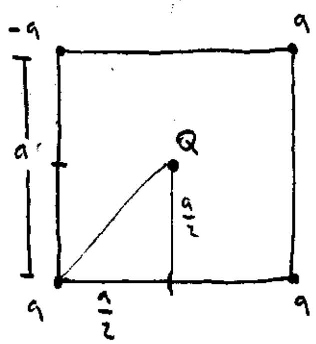
$$= \frac{kq_1q_3}{5\sqrt{5}}(2\hat{i}+\hat{j}) + \frac{kq_2q_3}{8}(2\hat{j})$$

$$= \frac{2kq_1q_3}{5\sqrt{5}}\hat{i} + \left(\frac{kq_1q_3}{5\sqrt{5}} + \frac{2kq_2q_3}{8}\right)\hat{j}$$

$$= 1.64\,N\,\hat{i} - 0.326\,N\,\hat{j}$$

$$\boxed{\vec{F}_3 = (1.64\,\hat{i} - 0.326\,\hat{j})\,N}$$

22.



(a) The charge Q is equidistant from all four charges forming the square.

The magnitude of the force of a single charge will be

$$F = \frac{kqQ}{(a/\sqrt{2})^2} = \frac{2kqQ}{a^2}$$

Now, the forces due to the diagonally opposite positive charges will cancel. They have the same charge but opposite directions, resulting in a relative minus sign.

The forces due to the positive and negative charges diagonally opposite will add. They have opposite charge and opposite direction.

Thus, the magnitude of the charge will be $F_{tot} = \frac{4kqQ}{a^2}$

(b) for $Q > 0$, the force is directed towards $-q$

for $Q < 0$, the force is directed away $-q$

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24. Initially, we have $F_1 = 2.5N = \frac{-kq_1q_2}{r_1^2}$, $r_1 = 1m$

After the spheres touch, they each have half the total charge $q = \frac{1}{2}(q_1+q_2)$, $q^2 = \frac{1}{4}(q_1+q_2)^2 = \frac{1}{4}q_1^2 + \frac{1}{4}q_2^2 + \frac{1}{2}q_1q_2$

then $F_2 = 2.5N = \frac{kq^2}{r_1^2}$

Equating F_1 and F_2, we get $q^2 = -q_1q_2$

$$\Rightarrow q_1^2 + q_2^2 + 2q_1q_2 = -4q_1q_2$$

$$\Rightarrow q_1^2 + q_2^2 + 6q_1q_2 = 0$$

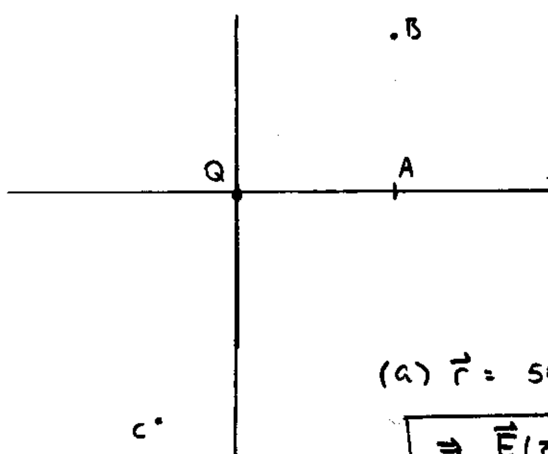
Solutions to this quadratic equation are $q_1 = (-3 \pm \sqrt{8})q_2$

Both are possible solutions, but they just represent a relabelling of the charges.

Using the fact that $-q_1q_2 = \frac{2.5N}{k}$ and $q_1 = (-3 \pm \sqrt{8})q_2$

we can find that $\boxed{q_1 = \pm 40.2\,\mu C \text{ and } q_2 = \mp 6.90\,\mu C}$

30.



$Q = 65\,\mu C$

The electric field due to a point charge at the origin is

$$\vec{E}(\vec{r}) = \frac{kQ}{|r|^2}\hat{r} = \frac{kQ\vec{r}}{|r|^3}$$

(a) $\vec{r} = 50\,cm\,\hat{i} = 0.5\,m\,\hat{i}$

$$\boxed{\Rightarrow \vec{E}(\vec{r}) = 2.34 \times 10^6 \frac{N}{C}\,\hat{i}}$$

(b) $\vec{r} = 0.5m\,\hat{i} + 0.5m\,\hat{j}$

$$\boxed{\Rightarrow \vec{E}(\vec{r}) = \left(827 \times 10^3 \frac{N}{C}\right)(\hat{i} + \hat{j})}$$

(c) $\vec{r} = -0.25m\,\hat{i} - 0.75m\,\hat{j}$

$$\boxed{\Rightarrow \vec{E}(\vec{r}) = -296 \times 10^3 \frac{N}{C}\,\hat{i} + 888 \times 10^3 \frac{N}{C}\,\hat{j}}$$

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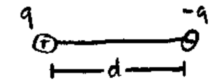
39. A dipole consists of two opposite charges a distance d apart

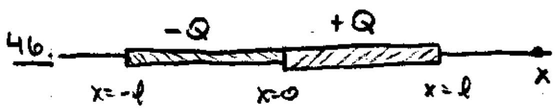


The dipole moment is $|p| = qd$

In our case, $|p| = 6.2\times10^{-30}\ C\cdot m$, $q = e = 1.6\times10^{-19}\ C$

$$d = \frac{|p|}{q} = \frac{|p|}{e} = \frac{6.2\times10^{-30}\ C\cdot m}{1.6\times10^{-19}\ C} = \boxed{3.88\times10^{-11}\ m = 3.88\ pm = d}$$

46.



(a) Please look at example 23-7 for how to find the field due to a charged rod.

Following this formulation, we find the field due to the positively charged rod is

$$E_+ = \frac{kQ}{x(x-l)} \text{ to the right,}$$

and the field due to the negatively charged rod is $E_- = \dfrac{kQ}{x(x+l)}$ to the left

then

$$E = E_+ - E_- \quad \text{(because the fields are in different directions)}$$

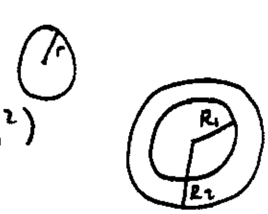
$$= \frac{kQ}{x}\left[\frac{1}{x-l} - \frac{1}{x+l}\right] = \boxed{\frac{2kQl}{x(x^2-l^2)} = E}$$

(b) for $x \gg l$, $x^2 - l^2 \approx x^2$ then $E = \dfrac{2kQl}{x(x^2-l^2)} \approx \dfrac{2kQl}{x^3}$

(c) Comparing with equation 23-7(b), we see that $p = Ql$

Please read example 23-6

48. (a) The area of a circle is πr^2



The area of an annulus is $\pi(R_2^2 - R_1^2)$

for a thin ring, $R_1 = r$
$R_2 = r + dr$

then the area of a thin ring is $\pi\left[r^2 + 2rdr + dr^2 - r^2\right]$

↓

48. (con't)

then area = $dA = \pi[2r\,dr + dr^2]$ since dr is small, dr^2 is negligible

then $\boxed{dA = 2\pi r\,dr}$

(b) A surface charge density is charge over area, i.e. $\sigma = \frac{dq}{dA}$

then $dq = \sigma dA = 2\pi\sigma r\,dr$

(c) Please read example 23-8.

Using the results from example 23-8, we find

$$dE_x = \frac{kx}{(x^2+r^2)^{3/2}}\,dq = \frac{2\pi k\sigma x r}{(x^2+r^2)^{3/2}}\,dr$$

where x is our distance from the center of the disk

(d) We find $E_x = \int_0^R dE_x = 2\pi k\sigma x \int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$

$$= 2\pi\sigma k x \left(\frac{-1}{\sqrt{x^2+r^2}}\right)\Bigg|_0^R$$

$$= 2\pi\sigma k\left[\frac{x}{|x|} - \frac{x}{\sqrt{x^2+R^2}}\right]$$

for $x>0$, $|x| = x$, so

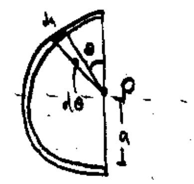
$$E_x = 2\pi\sigma k\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

for $x<0$, $|x| = -x$, so

$$E_x = 2\pi\sigma k\left[-1 + \frac{|x|}{\sqrt{x^2+R^2}}\right]$$

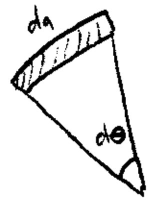
this answer is consistent with symmetry on the axis, because $E_x(x) = -E_x(-x)$

50.



Let the charge distribution be denoted λ.

Then $\lambda = \frac{Q}{\pi a}$



$dq = \lambda dl$, $dl = a d\theta$

then $dq = \lambda a d\theta$

thus, the magnitude $dE = \frac{k\,dq}{a^2} = \frac{k\lambda d\theta}{a}$

the direction of dE is

$$\vec{r} = \sin\theta\,\hat{i} - \cos\theta\,\hat{j}$$

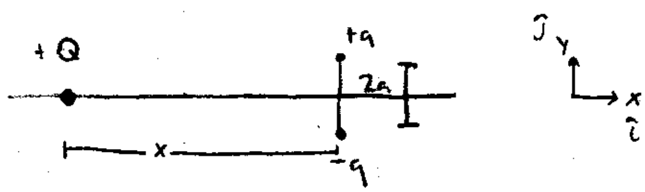
then

$$\vec{E} = \int_0^{\pi} d\vec{E} = \frac{k\lambda}{a}\int_0^{\pi}(\sin\theta\,\hat{i} - \cos\theta\,\hat{j})\,d\theta$$

$$= \frac{k\lambda}{a}\left(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)\Big|_0^{\pi} = \frac{k\lambda}{a}\left[\,2\hat{i} + 0\hat{j}\,\right] = \frac{2k\lambda}{a}\,\hat{i}$$

thus $\boxed{\vec{E} = \frac{2kQ}{\pi a^2}\,\hat{i}}$

68.



(a) In the limit $x \gg a$, the torque on the dipole is $\vec{p}\times\vec{E}$, where $\vec{E}$ is the field due to the point charge

$$\vec{p} = 2aq\,\hat{j} \quad , \quad \vec{E} = \frac{kQ}{x^2}\,\hat{i}$$

then $\boxed{\vec{p}\times\vec{E} = \frac{2aqkQ}{x^2}\,\hat{j}\times\hat{i} = -\frac{2kaqQ}{x^2}\,\hat{k} \text{ , i.e. into the page}}$

(b) Please read example 23-6.

The net force on Q due to the dipole is $Q\vec{E}_{dip} = -\frac{2kQqa}{x^3}\,\hat{j}$

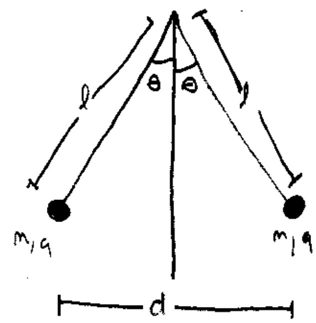
↓

65. (con't) Since there is no acceleration in the system, we know that the force **on** the dipole must be equal and opposite to the force exerted **by** the dipole.

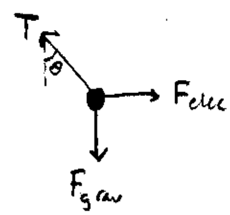
$$\boxed{\text{Thus, the force on the dipole is } \vec{F} = \frac{2kQqa}{x^3}\hat{j} \quad \text{the direction is parallel to the dipole moment}}$$

(c) see above

78.



Draw a free body diagram of one of the spheres.



Since the system is in equilibrium, we know that $\Sigma F_x = \Sigma F_y = 0$

$\Sigma F_x = F_{elec} - T\sin\theta = 0 \qquad \Sigma F_y = T\cos\theta - mg = 0$

Then $T\sin\theta = F_{elec}$, $T\cos\theta = mg$

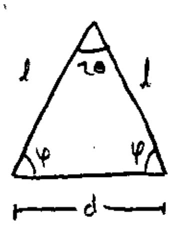
$$\Rightarrow \quad \tan\theta = \frac{F_{elec}}{mg} = \frac{kq^2}{d^2}\cdot\frac{1}{mg}$$

from trigonometry we see $d = 2l\sin\theta$

then

$$\tan\theta = \frac{kq^2}{mg}\cdot\frac{1}{(2l\sin\theta)^2} \quad \Rightarrow \quad q^2 = \left(\frac{mg\tan\theta}{k}\right)(2l\sin\theta)^2$$

$$\boxed{\Rightarrow \quad q = \pm 2l\sin\theta\sqrt{\frac{mg}{k}\tan\theta}}$$



Using the law of sines

$$\frac{d/2}{\sin\theta} = \frac{l}{\sin 90^\circ} \quad \Rightarrow \quad d = 2l\sin\theta$$