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Confusion by representation: On student's comprehension of the electric field concept

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It is argued that university students' diffuse ideas about the two related concepts of force and force field could be due to lack of mastery of the graphical representation of these and related concepts.

I. INTRODUCTION

The idea of interaction at a distance is one of the great contributions to physics made by Isaac Newton. When Michael Faraday introduced the concept of a field that can be graphically represented by field lines we were given a powerful tool for our thinking and communicating about interaction at a distance. But Faraday seemed to have attributed more reality to the field lines than we nowadays find acceptable. Maxwell writes in *Preface to A Treatise on Electricity and Magnetism* 1881:

> Faraday in his mind saw lines of forces traversing all space where the mathematicians saw centres of force attracting at a distance: Faraday saw a medium where they saw nothing but distance; Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids.

The concepts of field and field lines are sources of confusion among physics students at university level. This fact calls for an educational strategy that not only reflects the inherent theoretical structure but also considers the cognitive difficulties encountered by students. The likely increase in the use of computers and computer graphics in physics education and elsewhere calls for a new form of literacy: the ability to interpret field lines. Our aim is to shed some light on the way students apprehend, comprehend, and use fields and field lines. We will only deal with the interaction between a static electric field and a small point-charge.

II. INTRODUCTORY EXPERIMENT

As an introductory experiment students were given a pen-and-pencil test from the collection of physics problems by Wilson and Hacket[1] consisting of a drawing of field lines in the 2D space between three conductors, A and C being charged, B having no net charge. See Fig. 1.

Question: *The figure above is a perpendicular cut through long, parallel pieces of metal. A and C are charged, B is neutral. There are no currents; the system is stationary. In this figure a number of electric field lines are drawn. Find all the errors in the figure and explain why the field lines cannot be drawn in this way.*

The test was given to 566 second-year students as part of the final examination of a 40-h compulsory nonmajor course in electricity and magnetism at the Royal Institute

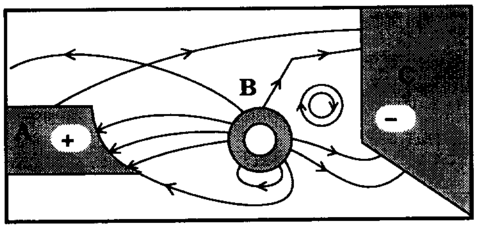

Fig. 1. Some erroneous field-lines between two charged objects and one uncharged object.

of Technology, Stockholm (Kungliga Tekniska Högskolan, KTH). 545 responses were received. Three items from the test were subject to a more detailed analysis. See Fig. 2: L = the Loop, a field line forming a closed loop, K = the Kink, a field line making a sharp bend, and X = two field lines crossing (X-ing) over. The students' responses to the given erroneous field lines are designated: D = error noted, explanation based on basic principles ("Divine"), C = error noted, correct explanation ("Correct"), B = error noted, physical explanation attempted, but false ("Botched"), A = error noted, explanation naive, ("Anthropomorphic or tautological"), and O = error not observed ("Oh!")

A. Results of introductory experiment

The result of the analysis is shown in Table I as percentages and presented graphically in the stacked column in Fig. 3.

B. Students' responses

Typical examples of students' responses are: LA: "...loops of field lines don't exist...a field line doesn't form closed loops...field lines cannot bite themselves in their tail...," LB: "there is no conductor to give a field like this...," i.e., a magnetic field, LC: "...must go from one positive charge to a negative charge...," LD: "...would make the potential non-unique...can only exist when induced by a time-dependent magnetic field...," KA: "field lines don't make sharp bends ...are not allowed to...shall not...must not make sharp bends...," KB: "...field lines are continuous (!)...field lines always take the shortest path-...such a sharp bend would need a huge force...,", KC: "...the tangent must be unique in all points...," and XC: "...the tangent must be unique in all points...."

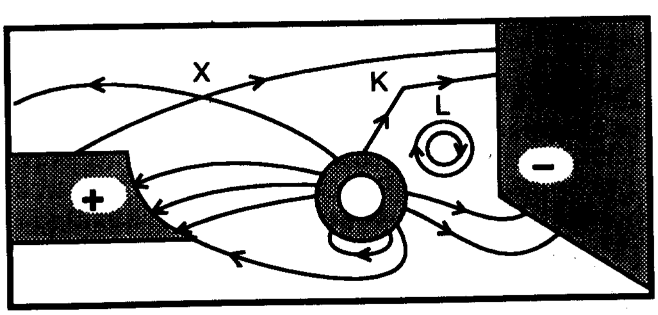

Fig. 2. The three field-line error for which students' responses were analyzed.

Table I. Percentages for different responses from the students.

	L	K	X
D	8	2	1
C	44	4	3
B	9	8	0
A	29	46	11
O	10	40	85

C. Comments about students' responses

The most alarming score is XO: 85%. The students did not react to the field lines crossing each other. The XO score together with the low score for KC: 4% indicates that the students do not master the mathematical concepts of continuity of a function and its derivatives well enough to apply them in a non math class context. Other disturbing scores are LA: 29% and KA: 46%. Too many students give naive explanations; they appear to be relying on intuition rather than application of what they have been taught in class.

The number of references to formalities such as "...field lines must go from plus to minus..." is alarmingly high. These answers have however been considered correct and included in the category LC.

III. INTERVIEWS

In order to find out more about the students' way of reasoning 87 exploratory interviews were conducted. The students interviewed entered the course two years after those tested in the introductory experiment described above. The interviews were made shortly after the examination of their course in electricity. The students were chosen randomly, about one third of the course participants.

The interviews are structured around nine different pictures of field lines and/or force vectors. The pictures are given in Fig. 4.

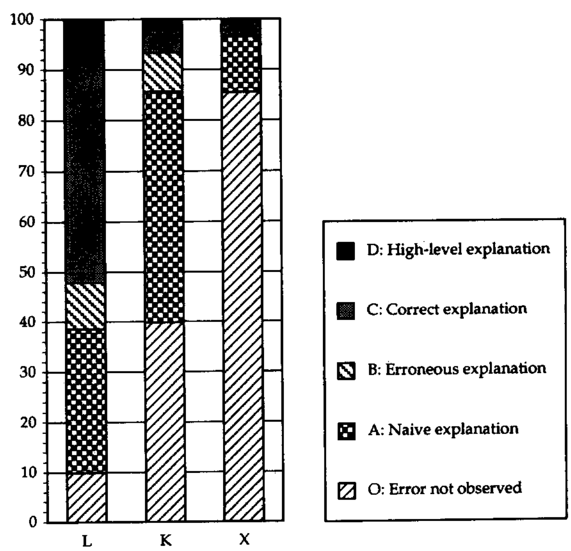

Fig. 3. A stacked-column representation of the data from Table I.

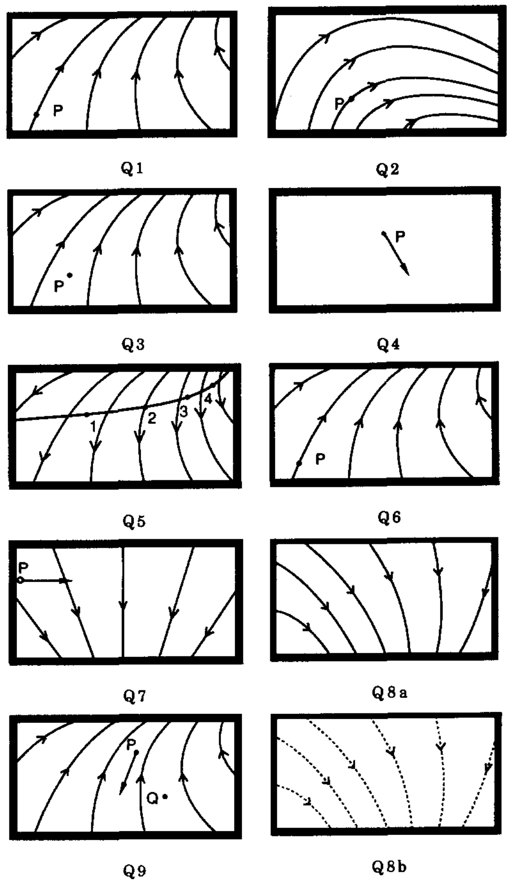

Fig. 4. The nine probing questions used in the interview.

A. Questions and analysis

1. Q 1-3: Draw a force vector on the given charge in the given point in the given field.

The questions are straightforward and most students did not have any problem in drawing an arrow along the tangent. Many of them, however, drew arrows carelessly with little concern about length, rather as a mere indication of direction. **11%** exhibited an unexpected but consistent behavior: They extrapolated the field lines to a supposed pole and drew the force vector straight toward that pole. **10%** started by drawing the force vector opposed to the field, which is also correct since no information about the sign of the given (test) charge was provided. When asked about their choice of direction none of them referred to a conscious assumption about the charge being negative; some referred to visual memories of electron trajectories from textbooks.

21% claimed in Q 2, when asked to be more precise, that the force vector is part of the field line: They insisted on drawing a curved arrow. When faced with the straight long arrow in Q 4, none of the students went back and corrected their answers to Q 1–3. (No transfer of knowledge.) It is interesting to note that **28%** tried to find the direction of the force vector by using the right-hand rule for the **magnetic** force. **16%** soon realized their mistake but the others who persisted had to be told they were on the wrong track.

2. Q 4: Draw field lines that can account for the given force vector in the given point.

Here, the students were asked to draw field lines matching a given force vector. **9%** had confused ideas, but **79%** drew straight equidistant field lines and **12%** curved equidistant lines as their first option. When they were asked about other possibilities **13%** drew an inhomogeneous field whereas **24%** maintained the straight equidistant field lines as the only feasible option. Thus only **13%** of the students considered an inhomogeneous field as an answer to this question although they had been given such fields in the previous questions. The heavy emphasis in textbooks on the homogeneous electric field between two parallel capacitor plates might be the reason for this one-sidedness.

3. Q 5: Draw force vectors on the given charge for the given positions on the given trajectory 1-2-3-4 in the given field.

The drawing in Q 5 contains a trajectory in addition to the field lines. This addition caused considerable confusion. **16%** relapsed to the right-hand rule for magnetic field or drew the force vector as a radius of curvature. No one drew force vectors downward. The fact that the upward acceleration requires an upward force is so obvious that the students tacitly accepted that this implies that the charge is negative. **33%** drew force vectors with the same length for all the points on the trajectory. **31%** were governed by the available space within the frame rather then by the increasing field line density: The vector at point 4 was thus drawn **shorter** than that at point 1. **20%** drew vectors reflecting the increasing field strength along the trajectory. This category contained two subgroups: The **7%** who drew all the force vectors inside the frame and the **13%** who felt free to extend the vector outside the frame. The latter all seemed to be aware of the distinction between the (force) vector space and the Euclidean (trajectory) space. One of them was a sailing instructor who explained the similarity between this task and showing his trainees how to estimate the strength and direction of the wind from the wave pattern on the water. This is a good illustration of "learning in meaningful situations" à la Ivan Illich.

4. Q 6: Draw a likely trajectory for a particle with zero initial velocity in the given point in the given field.

Here, the students produced the most clear-cut but erroneous response: **76%** made the trajectory follow the field line. **7%** drew it toward the supposed pole and **6%** were completely confused. Only **11%** offered a reasonable trajectory. The negative responses to this question is remarkable. Why, despite heavy courses in mechanics, do the students confuse force with trajectory?

5. Q 7: Draw likely trajectories for two particles having the same mass but opposite charge and with the initial velocity indicated in the given point in the given field.

33% gave a satisfactory response. **48%** gave the same curvature for both particles even though one of them goes into a region with a higher field line density. From this group **16%** started the trajectories at the point of the arrow or completely detached from it. **10%** made the trajectory bend more when it entered a region with a weaker field. **8%** were confused.

6. Q 8: Modify the given field line pattern when the source charges are increased by a factor 1.5. (Use the drawing Q 8b, where the original field-lines are given as dotted curves.)

The answers to this question fall into four categories.

(a) **16%** who said that the field lines do not change and drew them the same as before.

(b) **33%** who claimed that the line density now was 1.5. This category contained (b1) 18% who drew what they claimed. (b2) 15% who drew something else than they claimed.

(c)**22%** who claimed that the line density now was 2.25. This category contained (c1) 6% who drew what they claimed. (c2) 16% who drew something else than they claimed.

(d)**29%** other suggestions.

Thus only 18% gave a completely satisfactory response to this question, i.e., (b1).

7. Q 9: Draw a force vector on the charge in the point Q when the force vector on the same charge in another point P is given.

This was a summative question phrased so that responses could reflect any learning that had taken place during the interview. **42%** drew a vector in the right direction but without any consideration to its length. **6%** drew the second vector in the same direction as the first one. **52%**, however, drew a vector in the right direction and distinctly shorter than the given one.

IV. CONCLUSION

The results show that instead of using arguments in terms of the mathematical concepts of uniqueness and continuity, proportionality and isomorphic mapping, the students not too seldom give field lines an almost human character. They attach far too much reality to the field lines and often treat them as isolated entities in the Euclidean space, not as a set of curves representing a vectorial property of that space. The hierarchical sequence between the concepts (charge geometry—field line—force vector(—velocity vector)—trajectory) is not fully understood. We suggest that this **confusion by representation** is, at least partly, the cause of the well known "misconception" about the force concept as reported by, e.g., Johansson[2] and by Viennot:[3] It is not unlikely that students when asked to draw force vectors in a given point on a picture of a trajectory for a bouncing ball simply were not aware that they were asked to add one type of representation on top of another. Further studies in this field will consequently have to look into the role that representations play in the students' concept-formation process.

V. RECOMMENDATIONS

Results from this experiment, in particular its first introductory part, show that many students are unable to transfer knowledge from one academic subject, math, to another, physics. The fact that our students are taught mechanics and physics by different departments (a not uncommon feature for engineering schools) might be the reason for the confused answers to the questions Q 5–7. Our block structured, discipline oriented curriculum is evidently not conductive to a holistic approach to learning. If learning outcomes are to be improved, then we have to consider some sort of problem based learning in which the concepts are developed in a wider context.

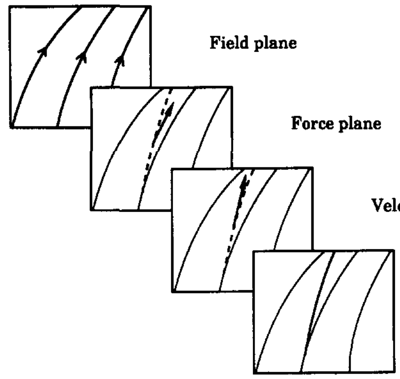

Fig. 5. Four isomorphic planes.

Few text books make the "transition" from force vectors to field lines clear. One good treatment is known to us, Purcell's *Electricity and Magnetism*.[4] In the chapter on electrostatics, the forces around two charges are represented in two ways: one with a great number of vectors and the other with field lines.

It is not a new discovery that students have shaky ideas about vectors as mathematical entities and show subsequent confusion between vectors representing different concepts. Arons[5] has noted similar problems and suggests the use of different arrows when representing the different concepts in the same picture: "an ordinary arrow for force, a single-half-headed arrow for velocity, and a double-half-headed arrow for acceleration." (p. 69). The use of color codes when feasible would serve the same purpose. When working with over-head transparencies we recommend that one use different sheets for different representations and apply the over-lay technique: see Fig. 5.

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[2]B. Johansson, *Krafter vid rörelser.* Report 1981:14 from Pedagogiska Institutionen, Göteborgs Universitet, Sweden.

[3]L. Viennot, "Spontaneous reasoning in elementary dynamics," Eur. J. Sci. Educ. **1**, 205–222 (1979).

[4]Edward M. Purcell, *Electricity and Magnetism* (Berkeley Physics Course, McGraw-Hill, New York, 1985), 2nd ed. Vol. 2, pp. 18–19.

[5]A. B. Arons, *A Guide to Introductory Physics Teaching* (Wiley, New York, 1990), pp. 68–69.